Multicriteria Decision-Making Techniques: A Review

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Abstract – Decisions are taken every second of our lives. Our world is connected with each other via social media or other telecommunication medium and one decide to whom one wish to connect. Businesses also makes regular decisions. In the global economy, businesses can exploit foreign markets as easily as their own domestic markets. They usually apply scientific methods to decide which new market they should explore. MCDM methods are one such method which can provide optimal solution under specified criteria restrictions. This paper presents a short review to the most common and rank generating methods namely SAW, TOPSIS, VIKOR, ELECTRE. At the end it demonstrates the results obtained by SAW, TOPSIS and ELECTRE on a simple use case of HOTEL SELECTION for summer vacations with results and a conclusion.

Keywords: SAW Technique, TOPSIS Technique, VIKOR Technique, ELECTRE Technique, MCDM Technique

INTRODUCTION

MCDM is relatively a very new technique but it has gained a wide popularity. MCDM techniques were introduced about 30 years ago. One of the reasons of MCDM popularity is associated with the rise of computers. Computers allows complex MCDM problems to be structurally broken down in solvable parts. Businesses need to make critical operational decisions which require scientific understanding of the criteria involved and the constraints associated with the decision. Businesses with global market reach have huge information flow on which decisions needs to be made for efficient business operation. MCDM techniques presents a logical, rational result which has mathematical base. MCDM problems generally have multiple alternative with multiple criteria on which the alternatives are evaluated. MCDM problems are very common. It usually has criteria with some criterion conflicting with each other.

LITERATURE REVIEW

Karni, Pedro and VM Rao (1990) [1] have compared the MCDM techniques namely SAW, ELECTRE, Weighted Linear Assignment Method (WLAM) with respect to AHP. They used three real-life scenarios and then compared the results obtained from other MCDM techniques w.r.t. AHP. They found out that results given out by SAW and ELECTRE are similar in comparison to AHP whereas WLAM gives drastically different result.

Xiaozhan Xu (2001) [2] introduced a superiority and inferiority ranking (SIR) method. This method requires two new types of information namely the superiority and the inferiority information. The information is used to derive two flows the superiority flow and the inferiority flow. These flows are used to rank the alternatives partially or completely according to the decision maker's wish. SIR method not only represents a MCDM technique but a general MCDM approach because it uses new information extracted from decision matrix instead of directly utilizing the decision matrix.

Karamouz, Zahraie & Kerachian (2009) [3] presented a paper that provides a master plan and an extensive case study for water resources pollution control in Isfahan Province in Iran. They estimated the effect on pollution by the agricultural, domestic, and industrial sectors using the two MCDM techniques: SAW & AHP. Different quality indicators, their values and relative weights were determined using engineering judgement and expert opinions. Error estimation in engineering judgement were also considered & evaluated using AHP method. The main objective of doing the study wss to reduce water pollution by half within 10 years' time. Authors also noted that instead of MCDM technique, economic analysis can also be used using economic data. But

MCDM techniques have advantage since economic data can be fudged or unavailable whereas MCDM can easily derive results using expert's opinion, available limited data & engineering judgements.

Van Wijk, Klungel, Heerdink et al. (2006) [4] performed a study to assess a MCDM model which should generate scores based on a set of characteristics related to different drug class or drug alternatives. The scores can then be used for comparison between them and hence encourages evidence based pharmacotherapy. Their model used SAW and TOPSIS MCDM techniques. Values for weights were given by medical practitioners and internists residing in The Netherlands.

Mei-Tai Chu, Joseph Shyu, Gwo-Hshiung Tzeng Rajiv Khosla (2007) [5] applied MCDM techniques namely SAW, TOPSIS and "VlseKriterijumska Optimizacija I Kompromisno Resenje" (VIKOR) to ascertain the goals & measurable insights of knowledge communities by comparing the decisions taken in the group discussions. All three above mentioned methods were applied on an empirical case. Their result showed that SAW and TOPSIS MCDM techniques gives similar rankings but TOPSIS can also distinguish qualities better. TOPSIS and VIKOR have similar success setting priorities by weight. VIKOR however produce different rankings than TOPSIS and SAW techniques but it allows to choose appropriate strategies easily. They concluded the paper by stating both TOPSIS and VIKOR methods are suitable for assessing problems of similar nature, provide excellent results close to real values, and allows extensive analysis.

Sheng-Hshiung Tsaur, Te-Yi Chang, Chang-Hua Yen (2002) [6] performed a study to evaluate Quality of Service (QoS) provided by airline services. Airline services are difficult to measure since the related attributes are intangible. Authors applied AHP for obtaining criteria weights and TOPSIS for ranking. Authors found out that the most important QoS concerns are tangible in nature i.e. the physical aspects and the least QoS concerns are the empathy aspects. They suggest that airline should maintain their physical features up to a certain point and keep renovation necessary.

Serafim Opricovic, Gwo-Hshiung Tzeng (2004) [7] provided a comparative analysis of two MCDM techniques i.e. TOPSIS & VIKOR. TOPSIS & VIKOR both are aggregating function that produces output that are close to ideal. Their main difference are as follows: VIKOR uses ranking index using a unique measure close to ideal value whereas TOPSIS choses alternative that is closest to the ideal solution and farthest from the negative ideal solution. TOPSIS also don't consider the relative importance of the distances from these points. Another contrast that they point is that VIKOR uses linear normalization & doesn't depend on the dimension of the criterion whereas the TOPSIS method uses vector normalization to eliminate the dimensions of criterion & may depend on the dimension of the criterion.

SAW

SAW (Simple Additive Weighted) was first proposed by Harsanyi in 1955 [8]. SAW is known as Weighted Linear Combination method or scoring method. This method is widely used due to its simple nature of calculation and faster rate to achieve results. This method works as follows: An intermediate score is calculated for each criterion by taking the product of the dimensional values assigned to alternative of a criteria with the relative weights assigned to that criteria by the decision maker and finally summing all the products for all criteria.

The mathematical expression of SAW is given below:

$$\mathrm{v}(a_n) = \frac{1}{N}\sum_{k=1}^{k} w_k . v_k\left(f_k(a_n)\right)$$

$$\text{with } w_k \geq 0 \text{ and } \sum_{k=1}^{k} w_k = 1$$

here w_k denotes the weight assigned to the criterion k. k means the total number of criteria considered.

$$f_k(a_n) \rightarrow \max,$$

$$v_k\left(f_k(a_n)\right) = \frac{f_k(a_n) - \min\left(f_k(a_n)\right)}{max\left(f_k(a_n) - \min\left(f_k(a_n)\right)\right)}$$

Weights are taken in SAW such that their sum always equals to 1. SAW method also allows to adjust the criteria according to the decision makers needs so that it can represent best possible solutions. Since SAW technique is basically addition of weighted value, it is very intuitive for the decision makers.

TOPSIS

TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) is a multi-criteria decision analysis method, which was first introduced by Hwang and Yoon in 1981 [9]. It uses Euclidean distance algorithm to find optimal solution. It finds an alternative whose Euclidean distance with the ideal solution is minimum and with the negative alternative solution is maximum in a multi-dimensional computing space. TOPSIS has numerous

advantage over other methods. It is a very simple method which is easy to use and to program it. Its number of steps to derive result remain same irrespective of the number of alternatives presented. The Steps required in TOPSIS are as follows:

Step 1. Create the decision matrix

A =

	X_1	X_2	…	X_n
Y_1	X_{11}	X_{12}	…	X_{1n}
Y_2	X_{21}	X_{22}	…	X_{2n}
…	…	…	…	…
Y_m	Y_{m1}	Y_{m2}	…	Y_{mn}

Step 2. Normalize the decision matrix A using given formula:

$$p_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{j} x_{ij}{}^2}}$$

Step 3. Calculate the weighted normalized decision matrix by multiplying weight to the result as:

$$V_{ij} = w_{ij}.p_{ij}$$

Step 4. Determine the ideal solution and negative ideal solution

$$B^* = \left\{\left(\max v_{ij} \mid j \in J\right), \left(\min v_{ij} \mid j \in J'\right)\right\}$$

$$B^- = \left\{\left(\min v_{ij} \mid j \in J\right), \left(\max v_{ij} \mid j \in J'\right)\right\}$$

Step 5. Calculate the distance to ideal & negative ideal solutions

$$S_i^* = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^*)^2}$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}$$

Step 6. Determine the relative proximity to the ideal solution

$$C_i^* = \frac{S_i^-}{S_i^* + S_i^-}, 0 \leq C_i^* \leq 1$$

Step 7. Calculate the total values and then find the value that is closest to 1.

ELECTRE

The ELECTRE (ELimination and Choice Expressing REality) [10] methods is a collection of decision-making techniques that focuses on the partial aggregation based on the development of relations by comparing each pair of actions. In this method, instead of creating a cost function and then searching its optimum value here the comparison of every pair takes place in such a way that between every pair a preferred action is selected. Finally, a rank matrix is created, and we will get a set of preferences of one action over another. These strategies have the preferences of tolerating circumstances of uniqueness with qualitative and non-numerical criteria. To use ELECTRE method the list of actions (from which we want best), the list of criteria (attributes of every action), the evaluation of each action by criterion and the weight of each criterion (how much any attribute is preferable) should be known. Also in ELECTRE methods, we get Concordance matrix, Discordance Matrix, over ranking matrix and Ranking table. Concordance matrix is created by calculating the concordance indices for each pair of alternatives. Similarly Discordance matrix is created. If value v of row i is higher than value v of row j, the component k can be classified as the concordance set Cij or the discordance set Dij. The concordance set Cij and the discordance set Dij can be obtained:

$$Cij = \{k|vik \geq Vjk\}$$

$$Dij = \{k|vik < vjk\}$$

The sum of each component's weight forms a concordance matrix C, as shown in:

$$C = [cij]mxm,$$

$$Cij = \frac{\sum_{k \in Cij} wk}{\sum_{k1}^{n} wk}$$

The discordance matrix can be found as:

$$dij = \frac{maxk \in Dij\{|vik - vjk|\}}{maxk \in S\{|vik - vjk|\}}$$

We convert the result from concordance and discordance matrix in binary form and after ANDING both we get over ranking matrix. Finally based on ranking table final ranks are created. Sometimes it's not necessary we get results from the ranking matrix.

VIKOR

The VIKOR [11] method is another MCDM which was developed by Serafim Opricovic in his Ph.D. dissertation in 1979 and it ranks actions and determines the closest ideal solution of the problem. The name VIKOR come from Serbian: VIseKriterijumska Optimizacija I Kompromisno Resenje meaning Multicriteria Optimization and Compromise Solution. The real applications were presented in 1998. VIKOR method is recognized internationally in 2004.

In this first beneficial (whose higher value is desired) and non-beneficial (whose lower value is desired) criterion are identified first. Next, we find the best and worst value for the criterion. For beneficial criterion, the max value is best, and min value is worst, but for non-beneficial this is opposite.

Then unity measure is calculated represented as Si and can be found as

$$Si = \sum_{j=1}^{m} \left(Wj * \frac{Xi^{+} - Xij}{Xi^{+} - Xi^{-}} \right)$$

Where Wj is weight for the criterion, Xi+ is the best value for that criterion and Xi- is the worst value for that criterion. Xij is the value in the cell. Next Ri known as individual regret is calculated as:

$$Ri = maxj \left(Wj * \frac{Xi^{+} - Xij}{Xi^{+} - Xi^{-}} \right)$$

Ri is the maximum value in that row. Next S*, R*, S^{-} and R^{-} are calculated which are minimum of Si, maximum of Si, minimum of Ri and maximum of Ri respectively. Finally, Qi is calculated using

$$Qi = v * \frac{Si - S^{*}}{S^{-} - S^{*}} + (1 - v) * \frac{Ri - R^{*}}{R^{-} - R^{*}}$$

Where v is weight for the strategy of maximum group utility, finally rank is given based on Qi value.

COMPARISON OF SAW, TOPSIS, AND ELECTRE

ELECTRE, TOPSIS and SAW methods were chosen to select the best destination for summer vacation based on various criterions like geographic location; Hotel cost daily, Number of nearby attractions in that location, how safe is the place for tourist, how is the food quality and cost of local transportation per kilometre.

Geographic location means the distance of the destination from home location; it is a numerical criterion. The average cost of staying in Hotel per day is also a quantitative criterion. A number of nearby attractions means how many tourist sites are present in that location this is also a quantitative quantity. Also, it was considered how much that location is safe. It's a qualitative quantity, so it was scaled from 1 to 5 where lower value means least secure. Similarly, food quantity was also scaled from 1 to 5. Likewise, local transportation was also considered as one of the criteria in which cost per kilometre is taken, and it is also a quantitative criterion.

Table 1: Criterion and weightage for our case study

Criterion	Weightage (in %)	Beneficial
Geographic Location	10	0
Hotel Cost	10	0
Number of attractions	15	1
Safety	35	1
Food Quality	15	1
Local Transportation	15	0

Now out of these criteria most important one is Safety, so it was given 35% weightage. Number of attractions, food quality, and local transportation were given 15% weightage each. Similarly, 10% weight given to geographic location and 10% to hotel cost. Now some quantities are beneficial, and some are non-beneficial. Beneficial means higher the value better the quantity. If quantity is beneficial 1 was assigned to it else 0 was assigned as shown in table 1.

Table 2: Actions and Values for our case study

Destination Criterion	1	2	3	4
Geographic Location	10	20	35	5
Hotel Cost	450	750	500	500
Attraction	10	25	60	25
Safety	1	2	3	1
Food Quality	1	2	3	4
Local Transportation	1	12	13	7

RESULT

When the SAW algorithm was executed with the data shown in table 2 as well with weights and beneficial criteria assigned as shown in table 1 then the following result was obtained:

Destination 3 > Destination 4 > Destination 1> Destination 2

When the TOPSIS algorithm was executed with the above data, we got:

Destination 3 > Destination 2 > Destination 4> Destination 1

Finally, the ELECTRE algorithm was executed with the given data, and yielded the rank matrix

0	0	0	0
1	0	0	1
1	1	0	1
1	0	0	0

From the rank matrix, it can be conclude that destination 2 > destination 1 and destination 2> destination 4. Also, it was conclude that destination 3> destination 1, destination 2, and destination 4. Also destination 4 > destination 1. So finally it can be conclude that

Destination 3 > Destination 2 > Destination 4> Destination 1

CONCLUSION

Different MCDM techniques were reviewed and later implemented a use case for SAW, TOPSIS, and ELECTRE in MATLAB. Results obtained with SAW and TOPSIS were different but same results for ELECTRE and TOPSIS even when ELECTRE does not require beneficial criteria the result obtained are same as it works with every pair of action and hence can be used with both beneficial and non-beneficial criteria.

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