A Survey on Results of Fixed Point Theorems and Their Application in New Research Scenario

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Abstract – In this article, we present a historical brief survey on the development of classical fixed point results and their applications in the fields of pure and applied mathematics.

Keywords – Contraction, Fixed point, Brouwer's fixed point, Metric fixed point, Tarski's fixed point.

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1.1 INTRODUCTION

Fixed point theorems are the most important tools for proving the existence and uniqueness of the solutions of differential equations, integral equations, partial differential equations, a system of linear algebraic equations and various inequalities etc. representing phenomena arising in different fields, such as steady state temperature distribution, chemical reactions, neutron transport theory economic theories, flow of liquids. Moreover the usefulness of the concept for applications has been increased by the development of accurate and efficient techniques for computing fixed points.

Although Brower (1912) proved the first fixed point theorem but the credit of making the concept useful and popular goes to Polish Mathematician Stefan Banach (1922) who proved famous contraction mapping theorem.

The term Metric fixed point theory refers to those points theoretic results in which geometric conditions on the underlying spaces and/or mapping plays a crucial role. Obviously there can be no clear line separating this branch of fixed point theory from either the topological or set-theoretic branches since metric methods are always couched in at least a metric spaces framework, usually in a Banach space.

Fixed point: Let a mapping T : X→ X. A point x $\in$ X which satisfies T(x)=x is called a fixed point of the mapping T.

Fixed point is also called invariant point. In mathematics A fixed point is a point that does not change upon application of map. An ancient method of solving equation of type T(x)=x is the method of iteration. The motivation for this method is here. If T mapping is continuous and if sequence $\{x_n\}$ converge say $x_n \to x$ then

$$x = \lim_{n\to\infty} x_{n+1} = T(\lim_{n\to\infty} x_n) = T(x)$$

In 1912, A Dutch mathematicianLuitzenEgbertus Jan Brouwer (1881-1966) proved a fixed point theorem. The most famous result in fixed point theory was of Brouwer's which state that The continuous mapping of an n-dimensional element into itself has a fixed point. Let T:X→X be a continuous function from a non empty compact convex set $X \subset R^n$, then there is some $x^* \in X$ such that $T(x^*) = x^*$

In 1922, A Polish mathematician Stefan Banach (1892-1945) proved Banach fixed theorem. It is also known as the contraction mapping theorem. Contraction mapping principle is an important tool in the theory of Metric spaces. It guarantees the existence and uniqueness of fixed point of certain self-maps of metric spaces and provides a constructive method to find fixed points.

The first Contractive definition is due to Banach in 1922 which state that If T is a mapping of complete metric space into itself satisfying

$$d(Tx, Ty) \le \alpha d(x, y) \forall x, y \in X$$

and $0 \le \alpha < 1$. Then T admits a unique fixed point.

In 1930, Juliusz schaunder proved a fixed point theorem known as Schauder fixed point theorem. It is an extension of Brouwer's fixed point theorem to topological vector space , which may be of infinite dimension. It asserts that if K is a convex subset of a topological vector space V and T is a continuous mapping of K into itself. So that T (K) is continued in a compact subset of K then T has a fixed point.

A function T can be guaranteed to have a fixed point if:

i. Continuous.

ii. Map must set into itself (i.e *f*: X→X).

iii. X must be compact and convex.

Banach Contraction Mapping Principal is Remarkable in its simplicity. In particular the wealth of application of Banach contraction mapping principle is astonishingly diverse. Yet it is perhaps the most widely applied fixed point theorem in all of analysis. This is because the contractive Condition on mapping in simple and easy to test, because it required only a complete Metric space for because it provide a constructive algorithm.

Consider the cosine function on [0, 1]. Graphs of y = cos x and y = x intersect once over [0, 1], which means the cosine function has a fixed point in [0,1]. We will show this point can be obtained through iteration.

Since cosine is decreasing on [0,1] and cos 1≈ ·54, cos([0,1]) ⊂ [0,1]. To show cos x is a contraction mapping on [0, 1], we will use the mean-value theorem: for any differentiable function f, f(x)-f(y) = f′(t)(x-y) for some t between x and y, so bounding the derivative of f will give us a contraction constant.

Taking f(x) = cos x,

cos x - cos y = cos'(t) (x - y) = (-sin t)(x - y)

for some t between x and y. Thus

│ cos x - cos y│= │ sin t││x -y│Since sine is increasing on [0, 1] , │ sin t│= sin t ≤ sin 1 ≈ ·84147. Therefore

│cos x - cos y│ ≤ ·8415│x - y│

so cosine is a contraction mapping on [0, 1], which is complete. Hence there is a unique a ∈ [0, 1] with cos a = a, a ≈ ·739.

A number of authors have defined contractive type mappings on a complete metric space X which are generalizations of the well-known Banach contraction, and which have the property that each such mapping has a unique fixed point. The fixed point can always be found by usingPicard iteration, beginning with some initial choice x_0 ∈ X.These are following some different Contraction mapping theorems proved by many mathematicians.

In 1922, Banach fixed point Theorem or Contraction mapping theorem[28]state that, If *T:R→R* and X is complete Metric Space. Then there exist an $\alpha \in [0,1)$ such that for each an $x, y \in X$ such that

$$d(Tx, Ty) \le \alpha d(x, y).$$

The conclusion is that T has unique fixed point.

In 1961, Eddstien[18] proved Contraction mapping condition: let $T: X \to X$ be mapping and

$$d(Tx, Ty) \le d(x, y). \quad \forall\, x, y \in X$$

In 1968, R. Kannan[21] who proved a fixed point theorem for map $T: R \to R$

Satisfying the following inequality there exist a number, $0 \le \alpha < \frac{1}{2}$ such that, for each $x, y \in X$

$$d(Tx, Ty) \le \alpha[d(x, Tx) + d(y, Ty)].$$

In 1969,S.P. Singh[29] proved there exists a positive integer p and a number a, 0 < a<1/2such that, for each x,y ∈ X,

$$d(f^p(x), f^p(y)) < a[d(x, f^p(x)) + d(y, f^p(y))].$$

In 1971, S. Reich [30]proved there exist non negative numbers a, b, c satisfying a + b + c<1, for all x,y ∈ X,

$$d(f(x), f(y)) < a d(x, f(x)) + b\, d(y, f(y)) + c\, d(x, y).$$

In 1972, Zamfirescu[36] proved there exist real numbers α, β ,γ , 0 < α< 1,

0 <β , γ <1/2 such that, for each x, y ∈ X, at least one of the following is true:

i. d(f(x),f(y)) < α d(x ,y),

ii. d (f(x),f(y)) < β[d(x ,f(x)) + d(y, f(y))

iii. d (f(x),f(y)) < γ[d(x, f(y))+ d(y, f(x))].

In 1972 , Bianchini[22] proved there exists a number h, 0 < h < 1, such that, for each x , y ∈ X

d(f(x),f(y)) < h max{d(x,f(x)),d(y,f(y))).

In 1972, C.L.Yen[9] proved there exist positive integers p, q and a number α, 0 < α< 1, such that, for each x, y ∈ X,

$d(f^p(x),f^q(y)) < \alpha\, d(x,y)$.

In 1972, Sehgal's Condition for contractive mapping [38] state that

$$d(Tx,Ty) < \max\{d(x,y), d(x,Tx), d(y,Ty)\}$$

In 1973, G. E. Hardy and T. D. Rogers[13] proved there exist non negative constants a, satisfying $\sum_{i=1}^{5} a_i < 1$ such that , for all x,y $\in$ X,

$d(f(x),f(y)) < a_1\, d(x,y) + a_2\, d(x,f(x)) + a_3\, d(y,f(y)) + a_4\, d(x,f(y)) + a_5\, d(y,f(x))$.

In 1974, A new idea was given by Ciric [15]using following contractive definition:There exist a number, $0 \leq \alpha < 1$ such that, for each $x, y \in X$.

$$d(Tx,Ty) \leq \alpha m(x,y)$$

Where

$$m(x,y) = \max\{d(x,y), d(x,T_x), d(y,T_y), d(x,T_y), d(y,T_x)\}.$$

In 1976, Fisher [1]proved the result with

$$d(Tx,Ty) \leq \alpha[d(Ty,x) + d(Tx,y)]$$

In 1977, The mathematician jaggi[10] introduced the rational expression first time as

$$d(Tx,Ty) \leq \alpha\, d(x,y) + \beta\, d(x,Tx)\frac{d(y,Ty)}{d(x,y)},\ \text{for all } x,y \in X,\ x \neq y,\ 0 \leq \alpha+\beta < 1$$

In 1977, Rhoades's Condition for Contractive mapping[7] states that

$$d(Tx,Ty) < \max\{d(x,y), d(x,Tx), d(y,Ty), [d(x,Ty) + d(y,Tx)]/2\}$$

In 1980, The mathematician jaggi and Das[11] obtained some fixed point theorem with mapping satisfying

$$d(Tx,Ty) \leq \alpha\, d(x,y) + \beta\, d(x,Tx)\frac{d(y,Ty)\, d(y,Ty)}{d(x,y) + d(x,Tx) + d(y,Tx)}$$

for all x, $y \in X, x \neq y, \alpha+\beta < 1$

In 1982, B.Fisher [5] proved that let (X, d) and (Z,σ) be complete metric spaces. If S is a continuous mapping of X into Z, and R is a continuous mapping of Z into X satisfying the inequalities:

$d(RSx,RSx') \leq c\max\{d(x,x'), d(x,RSx), d(x',RSx'), \sigma(Sx,Sx')\}$

$\sigma(SRz,SRz') \leq c\max\{\sigma(z,z'), \sigma(z,SRz), \sigma(z',SRz'), d(Rz,Rz')\}$

for all x, x' in X, and z, z' in Z, where $0 \leq c < 1$, then RS has a unique fixed point u in X and RS has a unique fixed point w inZ.

In 1983, N.P.Nung[20] proved that Let (X,d_1) , (Y,d_2) and (Z,d_3) be complete metric spaces and suppose T is a continuous mapping of X into Y, S is a continuous mapping of Y into Z and R is a continuous mapping of Z into X satisfying the inequalities:

$d_1(RSTx,RSy) \leq c\max\{d_1(x,RSy), d_1(x,RSTx), d_2(y,Tx), d_3(Sy,STx)\}$

$d_2(TRSy,TRz) \leq c\max\{d_2(y,TRz), d_2(y,TRSy), d_3(z,Sy), d_1(Rz,RSy)\}$

$d_3(STRz,STx) \leq c\max\{d_3(z,STx), d_3(z,STRz), d_1(x,Rz), d_2(Tx,TRz)\}$

for all x in X, y in Y and z in Z, where $0 \leq c < 1$. Then RST has a unique fixed point u in X, TRS has a unique fixed point v in Y and STR has a unique fixed point w inZ.

In 1996, R.K.Jain , H.K.Sahu and B.Fisher[24] proved Let (X, d), (Y,ρ)and (Z, σ) be complete metric spaces and suppose T is a continuous mapping of X into Y, S is a continuous mapping of Y into Z and R is a continuous mapping of Z into X satisfying the inequalities:

$d(RSTx,RSTx') \leq c\max\{d(x,x'), d(x,RSTx), d(x',RSTx'), \rho(Tx,Tx'),$

$\sigma(STx,STx')\}$

$\rho(TRSy,TRSy') \leq c\max\{\rho(y,y'), \rho(y,TRSy), \rho(y',TRSy'), \sigma(Sy,Sy'),$

$d(RSy,RSy')\}$

$\sigma(STRz,STRz') \leq c\max\{\sigma(z,z'), \sigma(z,STRz), \sigma(z',STRz'), d(Rz,Rz'),$

$\rho(TRz,TRz')\}$

for all x, x' in X, y, y' in Y and z, z' in Z where $0 \leq c < 1$. Then RST has a unique fixed point u in X, TRS has a unique fixed point v in Y and STR has a unique fixed point w inZ.

In 2000, D.Turkoglu and B.Fisher [12] proved let (X,d_1) and (Y,d_2) be complete metrics spaces, let F be mapping of X into B(Y) and let G be mapping of Y into B(X) satisfying the inequalities

$\delta_1(GFx,GFx^1) \leq c \max\{d_1(x,x^1), \delta_1(x,GFx),\delta_1(x^1, GFx^1), \delta_2(Fx,Fx^1)\}$

$\delta_2(FGy,FGy^1) \leq c \max\{ d_2(y,y^1), \delta_2(y,FGy), \delta_1(y^1,FGy^1), \delta_1(Gy,Gy^1)\}$

for all x,x^1 in X and y,y^1 in Y, where $0 \leq c < 1$. If F is continuous, then GF has a unique fixed point z in X and FG has a unique fixed point *w* in Y.

In 2003, S.Č.Nešić proved let (X,d),(Y,ρ) and (Z,σ) be complete metric spaces and suppose T is a mapping of X into Y , S is a mapping of Y into Z and R is a mapping of Z into X satisfying the inequalities

$d^2(RSy,RSTx) \leq c \max \{d(x,RSy)\rho(y,Tx), \rho(y,Tx)d(x,RSTx),$

$d(x,RSTx)\sigma(Sy,STx),\sigma(Sy,STx)d(x,RSy)\}$

$\rho^2(TRz,TRSy) \leq c \max \{\rho(y,TRz)\sigma(z,Sy),\sigma(z,Sy)\rho(y,TRSy),$

$\rho(y,TRSy)d(Rz,RSy),d(Rz,RSy)\rho(y,TRz)\}$

$\sigma^2(STx,STRz) \leq c \max \{\sigma(z,STx)d(x,Rz),d(x,Rz)\sigma(z,STRz),$

$\sigma(z,STRz)\rho(Tx,TRz), \rho(Tx,TRz)\sigma(z,STx)\}$

for all x in X , y in Y and z in Z , where $0 \leq c < 1$. If one of the mappings R,S,T is continuous, then RST has a unique fixed point u in X , TRS has a unique fixed point v in Y and STR has a unique fixed point w in Z.

In 2009,LuljetaKikinaand KristaqKikina [16] proved let (X, d_1), (Y, d_2), (Z, d_3) and (U, d_4) be complete metric spaces. Let $T : X \rightarrow Y$, $S : Y \rightarrow Z$, $R : Z \rightarrow U$ and $Q : U \rightarrow X$ be four mappings satisfying the following inequalities:

$d_1(QRSy,QRSTx) \leq cF_1(x, y)/ G_1(x, y)$

$d_2(TQRz,TQRSy) \leq cF_2(y, z)/ G_2(y, z)$

$d_3(STQu, STQRz) \leq cF_3(z, u)/ G_3(z, u)$

$d_4(RSTx,RSTQu) \leq cF_4(u, x)/ G_4(u, x)$

for all $x \in X$, $y \in Y$, $z \in Z$ and $u \in U$ for which $G_1(x, y) \neq 0$, $G_2(y, z) \neq 0$, $G_3(z, u) \neq 0$, $G_4(u, x) \neq 0$, where $0 \leq c < 1$ and

$F_1(x, y) = \max \{d_1(x,QRSTx)d_4(RSy, RSTx); d_1(x,QRSTx)d_3(Sy, STx);$

$d_1(x,QRSTx)d_2(y, TQRSy); d_1(x,QRSy)d_2(y, Tx)\}$

$F_2(y, z) = \max \{d_2(y, TQRSy)d_1(QRz,QRSy); d_2(y, TQRSy)d_4(Rz,RSy);$

$d_2(y, TQRSy)d_3(z, STQRz); d_2(y, TQRz)d_3(z, Sy)\}$

$F_3(z, u) = \max \{d_3(z, STQRz)d_2(TQu, TQRz); d_3(z, STQRz)d_1(Qu,QRz);$

$d_3(z, STQRz)d_4(u,RSTQu); d_3(z, STQu)d_4(u,Rz)\}$

$F_4(u, x) = \max \{d_4(u,RSTQu)d_3(STx, STQu); d_4(u,RSTQu)d_2(Tx,TQu);$

$d_4(u,RSTQu)d_1(x,QRSTx); d_4(u,RSTx)d_1(x,Qu)\}$

$G_1(x, y) = \max \{d_1(x,QRSy), d_1(x,QRSTx), d_2(Tx,TQRSy)\}$

$G_2(y, z) = \max \{d_2(y, TQRz), d_2(y, TQRSy), d_3(Sy, STQRz)\}$

$G_3(z, u) = \max \{d_3(z, STQu), d_3(z, STQRz), d_4(Rz,RSTQu)\}$

$G_4(u, x) = \max \{d_4(u,RSTx), d_4(u,RSTQu), d_1(Qu,QRSTx)\}$

then QRST has a unique fixed point $\alpha \in X$, TQRS has a unique fixed point

$\beta \in Y$, STQR has a unique fixed point $\gamma \in Z$ and RSTQ has a unique fixed

point $\delta \in U$.

A mapping may not have fixed point. It may have a unique fixed Point. It may have more than one or even infinitely many fixed point.

i. Let T is mapping from X to itself , Such that $T : X \rightarrow X$ defined as

$$T(x) = \begin{cases} x + \frac{1}{3}, & 0 \leq x \leq \frac{1}{3} \\ \frac{1}{4}, & \frac{1}{3} < x \leq 1 \quad \forall x \in X = [0,1] \end{cases}$$

In this, T(x) has no fixed Point.

ii. A mapping $T: \mathbb{R} \rightarrow \mathbb{R}$ defined by $T(x) = \frac{x}{3}$ has unique fixed point.

iii. A mapping $T: \mathbb{R} \rightarrow \mathbb{R}$ define as $Tx = x^2$ has two fixed Point.

Iv. An identity mapping $T: \mathbb{R} \rightarrow \mathbb{R}$ defined as $Tx = x$ has infinite many fixed point.

Fixed Point Theorem gives the condition under which maps have solution. As stated Previously, If T is a function which maps a set X into itself; i.e. $T: X \rightarrow X$, a fixed point of mapping is element $x \in X$. Such that T(x)=x. If the system of equations for which a solution is sought is of the form G(x) =0, then if the function G should be respresented as G(x)=T(x)-x. A fixed point of T is a solution to G(x) =0. Fixed Point is also known as an invariant Point.

The concept of fixed point play a key role in analysis of fixed point Theorem which are mainly used in the study of existence theory of differential equation, integral equation , partial differential equation and in many other disciplines.

A fixed Point Theorem that asserts the every function that satisfies some given property must have a fixed point. Metric fixed Point theory in an important mathematical subject, because of variational and linear inequalities optimization and approximation theory. Fixed Point Theorems give the conditions under which maps have solutions.

Fixed point technique has been applied in many other fields as biology, chemistry, economics, engineering, game theory and physics. In economics, theory use in a format role predicting how the game will he played and it explain the strategy which produce the most favorable outcomes for player. To fixed Point Theory could we use in such communication network space. The method can be applied not just to numerical equation but also to equation involving vector or function. Fixed point theory is after to use to prove the existence of solution differential equation. It is also used in applied Mathematics.

The fixed Point Theorem itself is a beautiful mixture of analysis, topology and geometry. It refers to those fixed points theoretical results in which geometric conditions on the underlying spaces play a crucial role. For the past many years, Metric fixed point Theory has been flourishing area for many mathematicians. Brain Fisher, P.P. Murthy, R.K. Namdeo, S.Jain, R.K Sahu, V.Papa and many more have given their contribution in the development of fixed point theory on metric spaces. So there exist a vast literature on the topic and this is very active field of research at present.

1.2 PRELIMINARIES

Metric Space:-The term Metric is derived from the word “METOR”(measure). The concept of a metric space is essential due to a French mathematician Maurice Frechet (1878-1973), through the definition presently in use is given by the German Mathematician Felix Hausdraff (1868-1942) in 1914. Frechet introduced the notation of metric space in his doctorate thesis, presented to the University of Paris in 1906.

1.2.1 Definition

A Metric space is an ordered pair (X, d) where X is a non-empty Set and d is a metric on X or also called distance function or simple distance, i.e a function $d: X\text{x}\, X \to R$ such that for any, the following: $\forall\, x, y, z \in X$

1. $d(x,y) \geq 0 \quad (non-negative)$
2. $d(x,y) = 0 \quad iff x = y$
3. $d(x,y) = d(x,y) \quad (symmetric)$
4. $d(x,y) \leq d(x,z) + d(z,y) \quad (Triangleinequality)$

An essential feature is that the fact that for any two points in a metric space, there is defined a positive Number called distances the points.

1.2.1 Example

Let X be a non empty set and *R* be the set of real number

Let the mapping *d:X X→R* defined by

$$d(x,y) = \begin{cases} 0, & if x = y \\ 1, & if x \neq y \end{cases} \forall\, x, y \in X$$

Then d is a metric on X, called discrete metric on X.

1.2.2 Definition

Let *(X,d)* be Metric space. A map *T: X → X* is said to be lipschitzian if there exist a constant a > o such that

$$d(Tx,Ty) \leq \alpha d(x,y) \forall x, y \in X$$

Here, *a* is called lipschiz constant.

1.2.3 Definition

Let *(X,d)* be Metric space. A map $T: X \to X$ and there exist a number $\alpha \in [0,1)$ such that

$$d(Tx,Ty) \leq \alpha d(x,y) \forall x, y \in X$$

Then, *T* is called contraction of X into X or T is called contraction map.

1.2.3 Example

Let a map $T: X \to X$ is defined by $\mathrm{T(x)} = {5x}/{6}$ is contraction mapping. for

$$|T(x) - T(y)| = \left|{}^{5x}/_6 - {}^{5y}/_6\right|$$

$$= {}^{5x}/_6 \, |x - y|$$

$$\leq \alpha|x - y| \text{for } {}^{5}/_6 \leq \alpha < 1$$

So T is contraction map from X to X.

1.2.4 Definition

Let a map $T{:}X \to X$ is said to be Contractive map if $d(Tx, Ty) \leq d(x,y) \forall x, y \in X$ and $x \neq y$.

Also remember that every contraction map is contractive but converse is not true.

1.2.5 Definition

Let (X ,d) be metric space. A mapping $T{:}X \to X$ is said to be Expansive mapping if $d(Tx, Ty) > d(x,y) \forall x, y \in X$ x≠y

1.2.5 Example

Let a map $T: R \to R$ define by $T(x) = 3 + {}^{7x}/_4$ with usual metric space.

$$d(T(x), T(y)) = |T(x) - T(y)|$$

$$= \left|3 + {}^{7x}/_4 - 3 - {}^{7y}/_4\right|$$

$$= {}^{7}/_4 \, |x - y|$$

$$> |x - y| = d(x,y) \quad , \forall x, y \in R$$

So here T is expansive map.

1.2.6 Definition

Let *(X ,d)* be a metric space and T is mapping from X to itself. Then T is non expansive mapping if

$$d(Tx, Ty) \leq d(x,y) \forall x, y \in X$$

1.2.6 Example

Let a map $T: R \to R$ define by $T(x) = x + {}^{3}/_2$ with usual metric space.

$$d(T(x), T(y)) = |T(x) - T(y)|$$

$$= |x + {}^{3}/_2 - y - {}^{3}/_2|$$

$$= |x - y| \leq |x - y| = d(x,y) \, , \forall x, y \in R$$

Also remember that Every Contraction map is non expansive map but converse is not true.

1.2.7 Definition:

Let *(X*, d*)* be a Metric Space. A Sequence $\{x_n\}$ in X is said to converge to point *x* in *X* iff the following criterion is satisfied.

For each $\in > 0 \exists$ a positive integer $n_0(\in)$, Such that

$$d(x_n, x) < \in \qquad , \forall n \geq n_0.$$

1.2.8 Definition

A sequence $\{x_n\}$ in a Metric space *(X, d)* is said to be a Cauchy Sequence iff for each $\in > 0$ there exist a positive integer number $n_o(\in)$ such that

$$d(x_m, x_n) < \in \qquad , \forall n, m \geq n_0$$

i.e The distance between any two terms from some place onward becomes smaller and smaller.

Also remember that every convergent Sequence in a metric space is a Cauchy sequence. But converse need not be true.

1.2.8 Example

Let $X = (0,1]$ and d be usual metric on X. Let $\forall \{x_n\} = \left\{\frac{1}{n}\right\}, \; n \in \mathbb{N}$ be a sequence. It is an Cauchy sequence.

$$\text{But } \forall \lim_{n\to\infty} x_n = \lim_{n\to\infty} \left(\frac{1}{n}\right) = 0 \; \notin X = (0,1]$$

Hence $\{x_n\}$ is an Cauchy sequence which does not Converge to any point of the space.

1.2.9 Definition

metric space *(X, d)* is said to be complete iff every Cauchy sequence in X converges to point of X.

1.2.9 Example

Usual Metric space $(\mathbb{R}, d)$ is Complete.

1.2.10 Definition

Let *(X, d)* be Metric space. Let $A = \{G_\alpha, \alpha \in \Lambda\}$ be collection of subset of X. Then A is called Cover of the space X if

$$X = \bigcup_{\alpha \in \Lambda} G_\alpha$$

In particular , if each G_a is an open set in *X*, then *A* is called an open cover of the *X*. A cover *A* is called finite cover if it has finite many members, otherwise, it is called infinite Cover.

1.2.11 Definition

If A_1, A_2 be two cover of the metric space X such that $A_1 \subseteq A_2$. Then A_1 is called Sub cover of A_2 for X.

1.2.12 Definition

Let *(X, d)* be metric space and *X* is said to be compact set if every open covering of *X* is reducible to finite Sub covering.

OR

If $A = \{G_\alpha, \alpha \in \Lambda\}$ be open cover for the metric space X. Then X is compact if $\exists$ A finite many indicies $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n$ and finite

sub covering of *A* for *X* such that

$$X = \bigcup_{i=1}^{n} G_{\alpha_i}$$

1.2.12 Example

Every finite set in a metric space is compact.

1.3 CONCLUSION

Fixed point theorem are mainly used in the existence theory of differential equations, integral equations, Partial diff. Equations, random diff. Equations & in other related areas. Fixed points has a variety problems.

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