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A Review on Development of Thermo Elasticity

Mankesh Sheoran*

Assistant Professor, Department of Mathematics, Govt. College, Baund Kalan, Charkhi Dadri-127306, Haryana, India

Abstract – This paper deals with the review on development of thermo elasticity. It is demonstrated the types of thermo elasticity i.e. uncoupled, coupled and generalised thermo elasticity.

The basic equation of two temperature thermo elasticity in context of lord and Shulman theory and Green and Nagdhi theories of generalised thermo elasticity are reviewed. Relevant literature on two temperature thermo elasticity and state space approach is also reviewed.

Keywords: elasticity; thermo elasticity; two-temperature, generalised thermo elasticity, state-space.

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INTRODUCTION

1.1 Elasticity:

The theory of elasticity was extended to include thermal effects. A body is said to be elastic if it regain its original shape when the force causing deformation are removed. The elastic properties is characterised mathematically by certain functional relationships connecting forces and deformation. An elastic solid that undergoes only an infinitesimal deformation and for which the governing material law is linear is called a linear elastic solid. In the theory of linear elasticity we are concerned with an ideal material governed by Hooke's Law (1678), which represents a linear relationship between the stress and strain. Hooke's law has influenced the scientific thoughts for a considerably long period for the classical linear infinitesimal theory of elasticity and its results agreed with experiment quite well. During the 150 years period following the discovery of Hooke's law the growth of science of elasticity proceeded from a synthesis of solution of special problems.

1.2 Thermo Elasticity:

The theory of thermo elasticity is concerned with the influence of the thermal state of an elastic solid upon the distribution of strain. Thermo elasticity deals with the dynamical system whose interactions with the surrounding include not only mechanical work and external work but the exchange of heat also. Changes in temperatures causes thermal effects on materials. Some of these thermal effects include thermal stress, strain, and deformation. If the thermal energy of a material decreases, the material will shrink or contract. Thus, thermo elasticity is based on temperature changes induced by expansion and compression of the test part. Thus, the theory of thermo elasticity is concerned with predicting the thermo mechanical behaviour of elastic solids. It represents a generalization of both the theory of elasticity and theory of heat conduction in solids.

However, the theory was based on independence of the thermal and mechanical effects. The total strain was determined by superimposing the elastic strain and the thermal expansion caused by the temperature distribution only. The theory thus did not describe the motion associated with the thermal state, nor did it include the interaction between the strain and the temperature distributions. Thermo elasticity was simulators by the various engineering sciences. A remarkable progress in the field of aircraft and machine structure has given rise to numerous problems in which thermal stresses play a role of primary importance.

1.3 Theory of Uncoupled Thermo Elasticity:

The theory of thermo elasticity deals with the effect of mechanical and thermal disturbances on an elastic body. In the nineteenth century, Duhamel [1] and Neumann [2] introduced the theory of uncoupled thermo elasticity. There are two shortcomings of this theory. First, the fact that the mechanical state of the elastic, body has no effect on the temperature is not in accordance with true _physical experiments. Second, the heat equation being parabolic predicts an infinite speed of propagation_ for the temperature, which again contradicts physical observations.

1.4 Theory of Coupled Thermo Elasticity:

Biot [3] formulated the theory of coupled thermo elasticity to overcome the Paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. In this theory, the equations of elasticity and of heat conduction are coupled. However, this theory shares the defect of the uncoupled theory in that it predicts infinite speeds of propagation for heat waves, i.e., when an elastic solid is subjected to a thermal disturbance , the effect is felt at a location far from the source, instantaneously.

1.5 Theory of Generalised Thermo Elasticity:

The generalized theory of thermo elasticity is one of the modified versions of classical uncoupled and coupled theory of thermo elasticity and has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermo elasticity.

In contrast to conventional thermo elasticity, non-classical theories came into existence during the last part of 20th century. These theories referred to as generalized thermo elasticity, were introduced in the literature is an attempt to eliminate the shortcomings of the classical thermo elasticity .There are many different theories of the generalized thermo elasticity. Lord and Shulman [4] formulated a theory by including a flux rate term and one relaxation time in the Fourier’s law of heat conduction and obtained a hyperbolic heat transport equation which ensures the finite speed of heat propagation. The generalized theory of thermo elasticity with two relaxation times was first introduced by Muller (1971) [5]. He adopted a more direct approach by considering restrictions on a class of constitutive equations with the help of his entropy inequality involving an entropy flux vector. Green and Lindsay[6] have developed a temperature rate dependent thermo elasticity by including temperature rate among the constitutive variables, which does not violate the classical Fourier's law of hear conduction when the body under consideration has a centre of symmetry, and this theory also predicts a finite speed of heat propagation.

This theory was extended by Sherief and Dhaliwal [7] for an anisotropic media. In this theory, a modified law of heat conduction including both the heat flux and its time derivative replaces the conventional Fourier's law.

Green and Nagdhi [8-10] proposed three new thermoelastic theories based on entropy equality which was referred to as G-N theory of type I, II, III. The constitutive assumption for the heat flux vector is different in each theory .The nature of these three types of constitutive equations is such that when the respective theories are linearized, type I is same as the classical heat conduction theory (based on Fourier’s heat conduction law), type II predicts the finite speed for heat propagation involving no energy dissipation and type III indicates the propagation of thermal signals with infinite/finite speed.

In dealing with classical or generalized thermoelastic problems in most situations , the displacement potential function approach is used .However Bahar and Hetnarski[11] outlined several disadvantages of the potential function approach .These may be summarized in the fact that the boundary and initial conditions of the problem are not related directly to the potential function , as it has no physical meaning explicitly .Finally it was found that many integral representations of physical quantities are convergent in the classical sense, while their potential function representations only converge in the mean .To get rid of these difficulties Bahar and Hetnarski introduced the state space formulation in thermoelastic problems .This state space approach has been further developed in Sherief[12] to include the effect of heat sources. Ezzat [13] has discussed the developments in the theory of thermo elasticity and fluid mechanics. He has also reviewed the method of matrix exponential, which constitutes the basis of the state space approach in the same work.

TWO TEMPERATURE THEORY OF THERMO ELASTICITY:-

Chen and Gurtin [14] and Chen et al. [15-16] have formulated a theory of heat conduction in deformable bodies, which depends on two distinct temperatures, the conductive temperature ϕ and the thermodynamic temperature T. The two-temperature theory involves a material parameter $a^* > 0$. The limit $a^* \rightarrow 0$ implies that $\phi \rightarrow T$ and hence classical theory can be recovered from two-temperature theory. The two-temperature model has been widely used to predict the electron and phonon temperature distributions in ultra-short laser processing of metals. The two temperatures T and ϕ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body .The two temperature thermo elasticity theories has gained much attention of the researchers in the recent years. Youssef [17] developed a new theory of generalized thermo elasticity by taking into account the theory of heat conduction in deformable bodies, which depends on two distinct temperatures, the conductive temperature and the thermodynamic temperature where the difference between these two temperatures is proportional to the heat supply. Abbas and Youssef [18] analysed a finite element model of two-temperature generalized magneto-thermo elasticity. Banik and Kanoria [19] studied the effects of three-phase-lag on two-temperature generalized thermo elasticity for infinite medium with spherical cavity. Ezzat et al. [20] introduced both modified Ohm's and Fourier's laws to the equations of the linear theory of magneto-thermoviscoelasticity involving two-temperature theory, allowing the second sound effects obtained the exact formulas of temperature, displacements, stresses, electric field, magnetic field and current density. Singh and Bala [21] studied the reflection of P and SV waves from the free surface of a two-temperature thermoelastic solid half-space.

Deswal and Kalkal [22] studied magneto-thermoelastic tractions in an initially stressed isotropic homogeneous elastic half-space with two temperatures using mathematical methods under the purview of the LS theory. Boley and Tolin [23], while studying the transient coupled thermoplastic boundary-value problem in half-space, concluded that two temperatures and strains are the foundation for representation of the results in the form of the wave-pulse response which occurs instantaneously throughout the body. Youseff and Al-Harby [24] investigated various problems on the basis of the two temperature thermo elasticity with relaxation time and showed that the obtained results are qualitatively different as compared to those in the case of one-temperature thermo elasticity.

Mukhopadhyay and Kumar [25] have reported several problems based on two-temperature generalized thermoelastic model and have analyzed that two-temperature models show qualitatively different results as compared to the one-temperature models. It is realized that the models of two-temperature thermo elasticity may be of more relevance to real situations. Therefore, it is believed that theses non-classical theories will be found more applicable to the technological problems when the experimental and theoretical research will progress further. Very recently, Quintanilla [26] has proved that when the dual-phase-lag heat conduction law is combining with two temperature theory then it is a well posed problem andis more realistic in nature. In this context he has given the basic equations of a two-temperature thermo elasticity theory. The uniqueness results for the heat conduction equation are also provided in this note.

REFERENCES:

1. J. M. C. Duhamel (1838). Me moiresur lecalcul des actions mol secularised evelopp ees par les changements de temperaturedans les corps solids, M emoirs par Divers Savans (Acad. Sci. Paris), 5, pp. 440-498.

2. K. E. Neumann (1841). Die Gesetze der Doppelbrechung des Lichts in comprimirtenoderungleichf ormigerwarmtenunkrystallinisc henK orpern, Pogg. Ann., Physics Chem., 54, 1841, 449-476.

3. M. A. Biot (1956). Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low frequency range. II. Higher frequency range, Journal of Acoustical Society of America, 28, pp. 168-191.

4. H. Lord and Y. Shulman (1967). A generalized dynamical theory of thermo elasticity, Journal of the Mechanics and Physics of Solids, 15, pp. 299-309.

5. I. M. Muller (1971). The Coldness, a universal function in thermoplastic bodies, Archive for Rational Mechanics and Analysis, 41, 1971, pp. 319-332.

6. A. E. Green and K. A. Lindsay (1972). Thermo elasticity, Journal of Elasticity, 2, pp. 1-7.

7. R. S. Dhaliwal and H. H. Sherief (1980). Generalized thermo elasticity for anisotropic media, Quarterly of Applied Mathematics, 33, pp. 1-8.

8. A. E. Green and P. M. Nagdhi (1991). A re-examination of the basic postulates of thermo mechanics, Proceedings of Royal Society of London: A, 432, pp. 171-194.

9. A. E. Green and P. M. Naghdi (1992). On undammed heat waves in an elastic solid. J. Thermal Stresses 15, 1992, pp. 253–264.

10. A. E. Green and P. M. Naghdi (1993). Thermo elasticity without energy dissipation, Journal of Elasticity, 31, pp. 189-209.

11. L. Y. Bahar and R. B. Hetnarski (1978). “State space approach to thermo elasticity”, vol. 2, no.1, pp.135-145.

12. H. H. Sherief (1993). “State space formulation for generalized thermo elasticity with one relaxation time including heat sources, “Journal of thermal stresses, vol. 16, pp.163-180.

13. M. A. Ezzat (2008). “State space approach to solids and fluids,” Canadian journal of Physics, vol.86, no.11, pp. 1241-1250.

14. P. J. Chen and M. E. Gurtin (1968) on a theory of heat conduction involving two temperatures, Z. Angew. Math. Phys. 19, pp. 614-627.

15. P. J. Chen., M. E. Gurtin and W. O. Williams (1968). A note on non-simple heat conduction, Z. Angew. Math. Phys. 19, pp. 969-970.

16. P. J. Chen., M. E. Gurtin and W. O. Williams (1969) on the thermodynamics of non-simple elastic materials with two temperatures, Z. Angew. Math. Phys., 20, pp. 107-112.

17. H.M Youssef (2006). Theory of two-temperature-generalized thermo elasticity, IMA Journal of Applied Mathematics, 71, pp. 383-390.

18. I. A. Abbas, .and H. M. Youssef (2009). Finite element analysis of two-temperature generalized magneto thermo elasticity Archive of Applied Mechanics, 79, pp. 917-925.

19. S. Banik and M. Kanoria (2012). Effects of three-phase-lag on two temperature generalized thermo elasticity for infinite medium with spherical cavity, Applied Mathematics and Mechanics, 33, pp. 483-498.

20. M. A. Ezzat, M. Zakaria and A. A. El-Bary (2012). Two Temperature theory in thermo-electric viscoelastic material subjected to modified Ohm's and Fourier's Laws, Mechanics of Advanced Materials and Structures, 19, pp. 453-464.

21. B. Singh and K. Bala (2012). Reflection of P and SV waves from the free surface of a two-temperature thermoelastic solid half-space, Journal of Mechanics of Materials and Structures, 7, pp. 183-193.

22. S. Deswal and K. K. Kalkal (2013). Two temperature magneto-thermo elasticity with initial stress: state space formulation, J.Thermodynamics. (pp. 754-798), pp. 1-13.

23. B. A. Boley and I. S. Tolin (1962). Transient coupled thermoplastic boundary value problem in the half-space, J. Appl.Mech., 29, pp. 637–646.

24. H. M. Youssef and H. A. Al-Harby (2007). State space approach of two temperature generalized thermo elasticity of infinite body with a spherical cavity subjected to different types of thermal loading, Arch. Appl. Mech., 77, pp. 675–687.

25. S. Mukhopadhyay and R. Kumar (2009). Thermoelastic Interaction on Two temperatures. Generalized thermo elasticity in an Infinite Medium with a Cylindrical Cavity, J. Therm. Stresses, vol. 32, pp. 341–360.

26. R. Quintanilla (2008). A Well Posed Problem for the Dual-Phase-Lag Heat Conduction, J. Therm. Stresses, vol. 31, pp. 260–269.

Corresponding Author

Mankesh Sheoran*

Assistant Professor, Department of Mathematics, Govt. College, Baund Kalan, Charkhi Dadri-127306, Haryana, India

mankesh.sheoran86@gmail.com