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Understanding the Concept of Different Stochastic Inventory Models

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Abstract – This paper displays a mathematical model for an inventory control system in which customers' demands and providers' service time are considered as stochastic parameters. Various aspects related to inventory theory are discussed in this article. A concise survey on stochastic process and stochastic inventory model is given for better understanding. We are taken two model, In the principal model we analyse a (s, S) generation Inventory system and in the model-II we stretch out these outcomes to perishable inventory system assuming that the life-time of each item pursues exponential distribution with parameter θ. There are several basic considerations engaged with deciding an inventory approach that must be reflected in the mathematical inventory model.

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I. INTRODUCTION

Inventory has been characterized by Monks, as idle assets that have certain monetary value. Usually, it is an important part of the speculation arrangement of any creation system. Keeping an inventory for future sales and using it whenever necessary is normal in business. For example, Retail firms, wholesalers, manufacturing companies and blood donation centers generally have a stock on hand. Quite often, the demand rate is chosen by the amount of the stock level. The motivational impact on the general population is caused by the nearness of stock at times. Large quantities of merchandise displayed in markets according to seasons, motivate the customers to purchase more. Either inadequate stock or stock in abundance, the two situations get misfortune to the manufacturer.

The cutting edge inventory theory offers a variety of economical and mathematical models of inventory systems together with various techniques and approaches aimed at achieving an optimal inventory arrangement. The main strides in applying a systematic inventory control are outlined as pursues.

- Formulating a mathematical model by depicting the behaviour of the inventory system.
- Looking for an optimal inventory arrangement as for the model.
- Utilizing an electronic information processing system to maintain a record of the present inventory levels.
- Utilizing this record of current inventory levels, applying the optimal inventory arrangement to indicate when and the amount to replenish inventory.

The mathematical inventory models utilized with this approach can be partitioned into two broad categories—deterministic models and stochastic models—according to the predictability of demand included. The demand for an item in inventory is the quantity of units that should be withdrawn from inventory for some utilization (e.g., sales) amid a particular period. On the off chance that the demand in future periods can be forecast with considerable precision, it is reasonable to utilize an inventory approach that assumes that all forecasts will always be totally accurate. This is the case of known demand where a deterministic inventory model would be utilized. Nonetheless, when demand cannot be anticipated great, it becomes necessary to utilize a stochastic inventory model where the demand in any period is a random variable rather than a known constant. There are several basic considerations engaged with deciding an inventory strategy that must be reflected in the mathematical inventory model.

II. COMPONENTS OF INVENTORY MODELS

Because inventory approaches affect profitability, the decision among strategies relies on their relative profitability. As already found in Examples 1 and 2, a portion of the costs that decide this profitability are (1) the requesting costs, (2) holding costs, and (3) shortage costs. Other relevant factors incorporate (4) revenues, (5) salvage costs, and (6) discount rates. These six factors are depicted thusly underneath.

The cost of requesting an amount z (either through purchasing or creating this amount) can be spoken to by a function c(z). The easiest type of this function is one that is straightforwardly proportional to the amount requested, that is, c z, where c speaks to the unit cost paid. Another regular assumption is that c(z) is made out of two parts: a term that is legitimately proportional to the amount requested and a term that is a constant K for z positive and is 0 for z 0. For this case,

c(z) cost of ordering z units

$$= \begin{cases} 0 & \text{if } z = 0 \\ K + cz & \text{if } z > 0, \end{cases}$$

where K setup cost and c unit cost.

The constant K includes the administrative cost of ordering or, when producing, the costs involved in setting up to start a production run.

III. CLASSIFICATION OF CLASS OF INVENTORIES

The classification of the class I, II, III, IV and V of inventories are discussed in form of Table 1.

Table 1 Classification of inventories

Class	Inventory I_0	Supply Process a(Q)	Demand b (Q)
I	Raw Material	Supplier	Production
II	Work in process	Production	Production
III	Finished goods	Production	Wholesaler
IV	Wholesale	Manufacturer	Retailer
V	Retailer	Wholesaler	Consumer

The inventory on hand at any time 't' is given by

$$I(t) = I_0 + \int_0^t [a(Q) - b(Q)]dQ$$

Where

$$a(Q) = \; supply\ rate\ /\ unit\ time$$

$$b(Q) \; = \; demand\ rate\ /\ unit\ time$$

$$I_0 = \; initial\ or\ starting\ inventory\ level.$$

In an inventory system, on the off chance that the supply and demand is from a solitary source, then it is called a solitary station model. On the off chance that there are many supply sources and similarly several wellsprings of demand and various stations operate simultaneously then it is called a system of parallel stations model. A system of stations is called a progression of station model, if the yield of one station is the contribution for the next, which are in arrangement. The answer for any model relies on these three characteristics. On the off chance that the supply and demand namely a(Q) and b(Q) are constant after some time, then it is called a static system, otherwise it is called a dynamic one. The inventory issues in real life situation, is conceptualized as a stochastic model and includes the optimization of inventory issue.

IV. CLASSIFICATION OF INVENTORY CONTROL MODEL

The examination on inventory control deals with two sorts of issues, for example, single-item and multi-item issues. Concerning the process of demand for single-items, the mathematical inventory models are partitioned into two large categories deterministic and stochastic models which is appeared in figure 1.

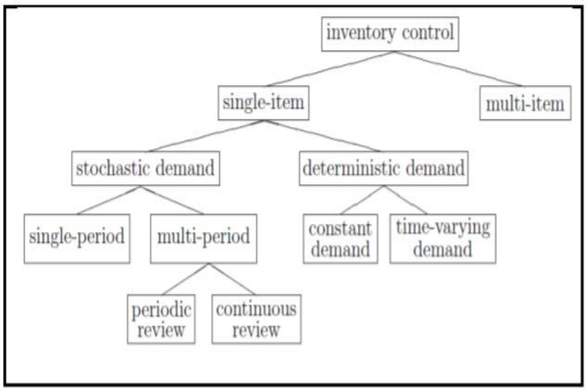

Figure 1: Classification of inventory control model

In single-item stochastic models, the rate of demand for items supplied by the system is viewed as known with uncertainty and it is called stochastic demand and when the demand is known with certainty it is viewed as deterministic. Also in single-item, the deterministic demand is either a constant quantity i.e., deterministic static model or a known function of time i.e., deterministic dynamic model. Multi-period is further subdivided into occasional survey and persistent audit.

Many of the available stochastic models and their answers are utilized here to conceptualize some fascinating new issues and settle them. The issues which are conceptualized on certain hypothetical assumptions are in Inventory Control, Reliability Theory and Queuing theory. All these disciplines depend increasingly more for their improvement and sophistication, the utilization of advanced probability theory for which stochastic process is a basic structure. Many of the real life issues which are administered by chance mechanism are profoundly included with the idea of stochastic process. An important aspect in the theory of stochastic process is the renewal theory which is from the mathematical view point and at the same time is a handy apparatus to take care of many issues of stochastic process.

One of the inventory models that have as of late gotten restored attention is the Newsboy issue and Base stock system issue. Hadley G et.al and Hanssman F have been credited for the seminal work on the classical rendition of these issues. Their models have been the foundation for many ensuing works by stretching out the original models to other various scenarios and applications. Nevertheless, despite its importance and the various publications related to the Newsboy issue or the multi-item Newsboy model and its variations remain limited.

The basic issue of inventory control or inventory management is to decide the optimal stock size and optimal reorder measure. Determination of the time to reorder is also an inquiry. An exceptionally detailed and application situated treatment of this subject is seen in Hanssman F.

The Inventory Level

The inventory level relies upon the relative rates of stream all through the system. Characterize y(t) as the rate of input stream at time t and Y(t) the cumulative stream into the system. Characterize z(t) as the rate of output stream at time t and Z(t) as the cumulative stream out of the system. The inventory level, I(t) is the cumulative input less the cumulative output.

$$I(t) = Y(t) - Z(t) = \int_0^t y(x)dx - \int_0^t z(x)dx$$

Figure 2 represents the inventory for a system when the rates vary with time.

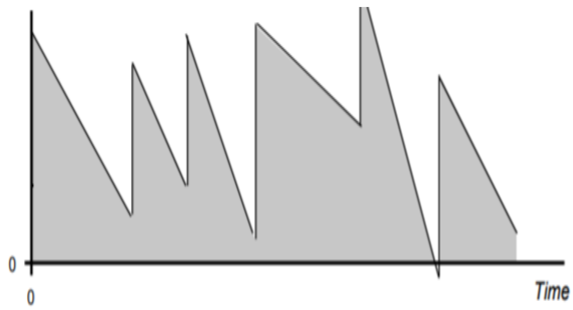

Figure 2. Inventory fluctuations as a function of time

The figure may speak to a raw material inventory. The stream out of inventory is a relatively persistent activity where individual items are placed into the creation system for processing. To replenish the inventory, a request is placed to a provider. After some delay time, called the lead time, the raw material is conveyed in a ton of a predefined amount. Right now of conveyance, the rate of input is infinite and at other times it is zero. Whenever the instantaneous rates of input and output to a part are not the same, the inventory level changes. When the input rate is higher, inventory develops; when output rate is higher, inventory declines.

Usually the inventory level remains positive. This relates to the nearness of on hand inventory. In cases where the cumulative output surpasses the cumulative input, the inventory level is negative. We call this a backorder or shortage condition. A backorder is a put away output prerequisite that is conveyed when the inventory finally becomes positive. Backorders may just be workable for certain systems. For example, if the item is not immediately available the client may go elsewhere; alternatively, a few items may have an expiration date like an airline seat and can just be backordered up to the day of departure. In cases where backorders are unthinkable, the inventory level is not allowed to end up negative. The demands on the inventory that happen while the inventory level is zero are called lost sales.

Variability, Uncertainty and Complexity

There are many reasons for variability and uncertainty in inventory systems. The rates of withdrawal from the system may rely upon client demand which is variable in time and uncertain in amount. There may be comes back from customers. Parts may be conveyed with imperfections causing uncertainty in quantities conveyed. The lead time associated with a request for replenishment relies upon the capabilities of the provider which is usually variable and not known with certainty. The reaction of a client to a shortage condition may be uncertain.

Inventory systems are often perplexing with one part of the system encouraging another. Figure 3 demonstrates a straightforward serial manufacturing system delivering a solitary item.

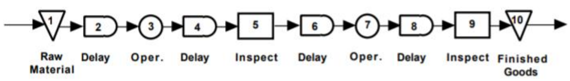

Figure 3. A manufacturing system with several locations for inventories

We distinguish planned inventories in Fig. 3 as reversed triangles, particularly the raw material and finished goods inventories. Material passing through the generation process is often called work in process (WIP). These are materials waiting for processing as in the delay squares of the figure, materials experiencing processing in the operation squares, or materials experiencing investigation in the examination squares. All the components of inventory add to the cost of creation as far as handling and speculation costs, and all require management attention.

For our analysis, we will often think of one as segment of the system separate from the remainder, particularly the raw material or finished goods inventories. In reality, rarely can these be managed independently. The material leaving a raw material inventory does not leave the system, rather it streams into the remainder of the creation system. Similarly, material entering a finished goods inventory originates from the system. Any analysis that enhances one inventory independent of the others must give not exactly an optimal solution for the system overall.

Solution of an Inventory Problem

The inventory issue is real life situation is conceptualized as a mathematical, stochastic model. In doing as such, the two costs namely the cost of abundance inventory which is also known as the holding cost is incorporated into the model. On the off chance that the demand is more than the supply the shortage may arise and hence the shortage cost is incorporated. In addition to this the cost of reordering is also incorporated. The goal of obtaining the optimal solution is to decide the solution which limits the overall cost. It is known as the optimal approach. Another issue is to decide the optimal reorder estimate as well the time at which the reordering is to be made.

It may be seen that the demand relies on many factors like market conditions, availability of substitutes and so forth. Hence it is not under the control of the decision maker. Then again the supply is under the control of the decision maker and hence called the control variable. In many issues of inventory control the optimal size of the supply is a matter of intrigue. Hence the optimal solution is often the determination of the supply measure. A similar approach is to decide the time of reorder and quantity or reorder. On the off chance that the demands as well as the supply are probabilistic in nature, then the probability distributions are taken into account and the normal cost is discovered. The solution which limits the normal cost is the optimal solution.

It may be noticed that the ongoing approach to locate the optimal solution takes into consideration another fact. The demand distribution may experience a parametric change, after a particular value of the random variable associated with the model. The time when the change happens is called the truncation point. Sometimes after the truncation point the distribution of demand which is a random variable can experience a change of distribution itself. Such facts are also incorporated in the model and the optimal solution is inferred.

V. STOCHASTIC PROCESS

Stochastic process is worried about the grouping of occasions represented by probabilistic laws. Many applications of stochastic process are available in Physics, Engineering, Mathematical Analysis and other disciplines. At times, arising in certain enterprises or military installations not just the demand for a particular commodity is a stochastic variable however its supply as well. In these cases it is advantageous to consider the inventory level coming about because of the interaction of supply and demand as a stochastic variable. The variation of the inventory level in time can be considered as a stochastic process.

On the off chance that the process is ergodic, the total inventory cost over a certain time may be spoken to as a function of the mean inventory level. This mean level can then be manipulated so as to limit the total inventory cost. In case of a stochastic process, on the off chance that a particular requesting strategy is presented, then the resultant fluctuating inventory level is a stochastic phenomenon. Also it becomes an issue to investigate the transient and stationary characteristics of the basic stochastic process.

As of late, the issue of how to decide ideal mean inventory levels has arisen habitually in large industrial concerns, where it appears to be a result of the institutional framework of the cutting edge firm. In many of the integrated companies of today, the rule of decentralized management has turned into an entrenched fact. This has driven with necessity as a rule to the practice of sub-optimization, because if a large industrial enterprise is subdivided for administrative reason into several rather independent acting departments, for example, generation, transportation, manufacturing, distribution, sales-department and so forth.

Uncertainty plays an important job in most inventory management situations. The retail merchant needs enough supply to satisfy client demands, however requesting a lot of increases holding costs and the risk of misfortunes through out of date quality or spoilage. A less request increases the risk of lost sales and unsatisfied customers. For example, the water assets manager must set the amount of water put away in a supply at a level that balances the risk of flooding and the risk of shortages.

The inventory model in which the stochastic nature of demand is explicitly perceived is dealt. In inventory theory, demand for the item is viewed as one of the features of uncertainty. In this thesis, the demand is assumed to be obscure and the probability distribution of demand is known. Mathematical derivation decides the optimal strategies as far as the distribution and choosing an appropriate distribution for the examination is important.

VI. STOCHASTIC INVENTORY MODEL

Often, there is some worry about the relation of demand amid some time period which is relative to the inventory level at the start of the time frame. On the off chance that the demand is not exactly the initial inventory level and there is an inventory remaining at the finish of the interval then the condition of abundance brings about. On the off chance that the demand is greater than the initial inventory level, then the condition of shortage acquires. Sooner or later, the inventory level is assumed to be a positive value Z. Amid some interval of time, the demand is a random variable Q with PDF f(Q) and CDF F(Q). The mean and standard deviation of this distribution are μ and σ respectively. With the given distribution, the probability of a shortage Ps and the probability of overabundance PE are figured. For a constant distribution, PE and Ps is given as

$$P_E = P\{Q \le Z\} = \int_0^Z f(Q)dQ = F(Z)$$

$$P_S = P\{Q > Z\} = \int_Z^\infty f(Q)dQ = 1 - F(Z)$$

In some cases it may be interesting to obtain expected shortage. This depend on whether the demand is greater or less than

Items short = $\begin{cases} 0 & , \text{if } Q \le Z \\ Q - Z, & \text{if } Q > Z \end{cases}$

Then $\psi_1(Q)$ is the expected shortage and is

$$\psi_1(Q) = \int_Z^\infty (Q - Z) f(Q)dQ$$

Similarly, for excess, the expected excess is $\psi_2(Q)$

$$\psi_2(Q) = \int_0^Z (Z - Q) f(Q)dQ$$

Also the expected excess can be represented in terms of $\psi_1(Q)$

$$\psi_2(Q) = \int_0^\infty (Z - Q) f(Q)dQ - \int_z^\infty (Z - Q) f(Q)dQ$$

$$= Z - \mu + \psi_1(Q)$$

Hence, this concept of stochastic process has similarity with the model discussed in Hanssman F which is the prime motivation behind this research work.

Probability Distribution for Demand

The one feature of uncertainty considered in this segment is the demand for items from the inventory. We assume that demand is obscure, however that the probability distribution of demand is known. Mathematical derivations will decide optimal approaches as far as the distribution.

- Random Variable for Demand (x): This is a random variable that is the demand for a given timeframe. Care must be taken to perceive the period for which the random variable is characterized because it contrasts among the models considered.

- Discrete Demand Probability Distribution Function (P(x)): When demand is assumed to be a discrete random variable, P(x) gives the probability that the demand equals x.

- Discrete Cumulative Distribution Function (F(b)): The probability that demand is not exactly or equal to b is F(b) when demand is discrete.

$$F(b) = \sum_{x=0}^{b} P(x)$$

- Continuous Demand Probability Density Function (f(x)): When demand is assumed to be continuous, f(x) is its density function. The probability that the demand is between a and b is

$$P(a \le X \le b) = \int_a^b f(x)dx.$$

We assume that demand is nonnegative, so f(x) is zero for negative values.

- Continuous Cumulative Distribution Function (F(b)): The probability that demand is less than or equal to b when demand is continuous.

$$F(b) = \int_0^b f(x)dx$$

- Standard Normal Distribution Function (x) and (x): These are the density function and cumulative distribution function for the standard normal distribution.

- Abbreviations: In the accompanying we abbreviate probability distribution function or probability density function as pdf. We abbreviate the cumulative distribution function as CDF.

Selecting a Distribution

An important modelling decision concerns which distribution to use for demand. A typical assumption is that individual demand occasions happen independently. This assumption leads to the Poisson distribution when the normal demand in a time interval is small and the normal distribution when the normal demand is large. Let a be the average demand rate. Then for an interval of time t the normal demand is at. The Poisson distribution is then

$$P(x) = \frac{(at)^x e^{-(at)}}{x!}.$$

When at is large the Poisson distribution can be approximated with a normal distribution with mean and standard deviation

$$\mu = at \text{, and } \sigma = \sqrt{at}.$$

Values of F(b) are evaluated using tables for the standard normal distribution.

Obviously other distributions can be assumed for demand. Regular assumptions are the normal distribution with other values of the mean and standard deviation, the uniform distribution, and the exponential distribution. The latter two are helpful for their analytical simplicity.

VII. COMPARISION OF MODELS

Model-I

In this model the inventory system starts with S units of the item on stock and creation unit is in OFF mode. When the inventory level reaches s, because of demands from primary or orbital customers, the system is immediately switched on to ON mode ie. generation starts. The time required to create one unit of the item is exponentially distributed with parameter J.L. When inventory level reaches zero, the approaching customers join an orbit of finite capacity M (if it is not full) and attempt their karma after some time. In this way customers who experience the system when inventory level is zero and orbit full are lost. Demands arrive according to a Poisson process with rate A. Each orbital client attempt to access the service counter to such an extent that the entomb retrial times pursue exponential distribution with parameter k, when there are k customers in the orbit. On the off chance that atleast one unit of the item is available the demand will be met immediately; otherwise the client come back to the orbit. The creation will remain in ON mode until the inventory level reaches to S. Let I(t), t ~ 0, be the inventory level at time t.

N(t), t ~ 0, be the number of customers in the orbit at time t.

Define

$$X(t) = \begin{cases} \text{I if the system is in ON mode} \\ \text{o if the system is in OFF mode} \end{cases}$$

To get Gontinuous time Markov process, we consider {(I (t), X (t), N (t)), t ~ O} whose state space is E = El U E2 where,

$$El = \{(i,0,N) : i = s+1, s+2, \ldots, S;\ N = 0,1,\ldots,M\}$$

$$E2 = \{(i,1,N) : i = 0,1,2,\ldots,S-l;\ N = O,l,\ldots,M\}$$

The infinitesimal generator of the process is given by

$$\tilde{A} = (a(i)j,k{:}\ l,m,n))j(i,j,k),(l,m,n) \in E \ where$$

$$a(i,j,k:l,m,n) = \begin{cases} \lambda & \text{if } i = s+2,\cdots,S;\ j=0,k=0 \\ & l=i-1;m=j,n=k \\ \lambda & \text{if } i=s+1;j=0,k=0,1,\cdots,M \\ & l=i-1;m=1;n=k \\ \lambda & \text{if } i=1,2,\cdots,S-1;j=1;k=0,1,\cdots,M \\ & l=i-1;m=j,n=k \\ k\gamma & \text{if } i=1,2,\cdots,S-1;j=1,k=1,2,\cdots,M \\ & l=i-1;m=j;n=k-1 \\ \lambda & \text{if } i=0;j=1,k=0,1,\cdots,M-1 \\ & l=i;m=j;n=k+1 \\ \mu & \text{if } i=0,1,\cdots,S-2;j=1,k=0,1,\cdots,M \\ & l=i+1;m=j;n=k \\ \mu & \text{if } i=S-1;j=1,k=0,1,\cdots,M \\ & l=i+1;m=0;n=k \\ -(\lambda+\mu) & \text{if } i=0,1,\cdots,S-1;j=1,k=0,1,\cdots,M \\ & l=i;m=j;n=k \\ -(\lambda+\mu+k\gamma) & \text{if } i=0,1,\cdots,S-1;j=1,k=1,\cdots,M \\ & l=i;m=j;n=k \\ -(\lambda+k\gamma) & \text{if } i=s+1,\cdots,S;j=0,k=1,\cdots,M \\ & l=i;m=1;n=k \\ k\gamma & \text{if } i=s+1;j=0,k=1,2,\cdots,M \\ & l=i-1;m=1;n=k-1 \\ k\gamma & \text{if } i=s+2,\cdots,S;j=0,k=1,2,\cdots,M \\ & l=i-1;m=j;n=k-1 \end{cases}$$

$$\text{Write} A_{il} = (a(i,j,k;l,m,n))$$

Then the infinitesimal generator A can be convinicntly expressed as a pertitioned matrix $A = ((A_{il}))$ *where* A_{il} *is an* $(M+1)\ x\ (M+1)$ matrix which is given by

$$A_{il} = \begin{cases} A_1 & \text{if } i = s+2, \cdots, S; l = i-1 \text{ and production off or} \\ & i = 1, 2, \cdots, S-1; l = i-1 \text{ and production on or} \\ & i = s+1; l = i-1 \text{ and production off} \\ A_2 & \text{if } i = 0, 1, \cdots, S-2; l = i+1 \text{ and production on or} \\ & i = S-1; l = i+1 \text{ and production on} \\ A_3 & \text{if } i = s+1, \cdots, S; l = i \text{ and production off} \\ A_4 & \text{if } i = 1, 2, \cdots, S-1; l = i \text{ and production on} \\ A_5 & \text{if } i = 0; l = i \text{ and production on} \\ 0 & \text{otherwise} \end{cases}$$

With

$$A_1 = \begin{array}{c} M \\ M-1 \\ M-2 \\ \cdots \\ 2 \\ 1 \\ 0 \end{array} \begin{pmatrix} \lambda & M\gamma & 0 & 0 & 0 & 0 \\ 0 & \lambda & (M-1)\gamma & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 \\ \cdots & & & & & \cdots \\ 0 & 0 & 0 & \lambda & 2\gamma & 0 \\ 0 & 0 & 0 & 0 & \lambda & \gamma \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{pmatrix}$$

$$A_2 = \begin{array}{c} M \\ M-1 \\ \cdots \\ 1 \\ 0 \end{array} \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 \\ \cdots & & & \cdots \\ 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & \mu \end{pmatrix}$$

$$A_3 = \begin{array}{c} M \\ M-1 \\ \cdots \\ 1 \\ 0 \end{array} \begin{pmatrix} -(\lambda+M\gamma) & 0 & 0 & 0 \\ 0 & -(\lambda+(M-1)\gamma) & 0 & 0 \\ & & & \\ 0 & 0 & -(\lambda+\gamma) & 0 \\ 0 & 0 & 0 & -\lambda \end{pmatrix}$$

$$A_4 = \begin{array}{c} M \\ M-1 \\ \cdots \\ 1 \\ 0 \end{array} \begin{pmatrix} -(\lambda+\mu+M\gamma) & 0 & 0 & 0 \\ 0 & -(\lambda+\mu+(M-1)\gamma) & 0 & 0 \\ & & & \\ 0 & 0 & -(\lambda+\mu+\gamma) & 0 \\ 0 & 0 & 0 & -(\lambda+\mu) \end{pmatrix}$$

$$A_5 = \begin{array}{c} M \\ M-1 \\ M-2 \\ \cdots \\ 2 \\ 1 \\ 0 \end{array} \begin{pmatrix} -\mu & 0 & 0 & 0 & 0 & 0 \\ \lambda & -(\lambda+\mu) & 0 & 0 & 0 & 0 \\ 0 & \lambda & -(\lambda+\mu) & 0 & 0 & 0 \\ \cdots & \cdots & & & \cdots & \\ 0 & 0 & 0 & -(\lambda+\mu) & 0 & 0 \\ 0 & 0 & 0 & \lambda & -(\lambda+\mu) & 0 \\ 0 & 0 & 0 & 0 & \lambda & -(\lambda+\mu) \end{pmatrix}$$

Thus we can write $\tilde{A}$ in the partitioned form as

$$\tilde{A} = \begin{array}{c} (S,0) \\ (S-1,0) \\ S-2 \\ \cdots \\ (s+1,0) \\ (S-1,1) \\ \cdots \\ (s+1,1) \\ (s,1) \\ (s-1,1) \\ \\ 2 \\ 1 \\ 0 \end{array} \begin{pmatrix} A_3 & A_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_3 & A_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & & & & & & & & & & \\ 0 & 0 & 0 & A_3 & 0 & 0 & A_1 & 0 & 0 & 0 & 0 \\ A_2 & 0 & 0 & 0 & A_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & & & & & & & & & & \cdots \\ 0 & 0 & 0 & 0 & 0 & A_4 & A_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_2 & A_4 & A_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_2 & A_4 & 0 & 0 & 0 \\ \cdots & & & & & & & & & \cdots & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_4 & A_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_2 & A_4 & A_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_2 & A_5 \end{pmatrix}$$

Model-II

In this model we expanded the aftereffect of model-I to a (s, S) generation inventory system where items delivered have random life-times which is exponentially distributed with parameter e. Also it is assumed that when inventory level is zero the arriving demands enter the orbit with probability {3 and with probability (1 - (3) it is lost for ever. All assumptions of model-I hold in this case also.

Model and Analysis

Let

I(t), t ~ 0, be the inventory level at time t.

N(t), t ~ 0, be the number of customers in the orbit at time t.

Define

$$X(t) = \begin{cases} 1 & \text{if the system is in ON mode} \\ 0 & \text{if the system is in OFF mode} \end{cases}$$

To get continuous time Markov chain, we consider $\{(I(t), X(t), N(t)), t \sim o\}$ whose state space is $E = El\ U\ E2$ where,

$$E_1 = \{(i, 0, N) : i = s+1, s+2, \cdots, S; N = 0, 1, \cdots, M\}$$

$$E_2 = \{(i, 1, N) : i = 0, 1, 2, 3, \cdots, S-1; N = 0, 1, \cdots, M\}$$

The infinitesimal generator A of the process has entries given by,

$A = (a(i,j,k : l,rn,n))j\ (i,j,k),(l,rn,n)\ E$ E, where

$$a((i,j,k:l,m,n)) = \begin{cases} \lambda & \text{if } i = 1,2,\cdots,S-1; j=1, k=0,1,\cdots,M \\ & l = i-1; m = j, n = k \text{ or} \\ & \text{if } i = s+2,\cdots,S; j=0, k=0,1,\cdots,M \\ & l = i-1; m = j, n = k \text{ or} \\ & \text{if } i = s+1; j=0, k=0,1,\cdots,M \\ & l = i-1; m = l, n = k \text{ or} \\ \lambda\beta & \text{if } i = 0; j=1, k=0,1,\cdots,M-1; l=0; m=j, n=k+1 \\ -\lambda\beta & \text{if } i = 0; j=1, k=0,1,\cdots,M-1; l=i; m=j; n=k \\ \mu & \text{if } i = 0,1,\cdots,S-2; j=1, k=0,1,\cdots,M \\ & l = i+1; m=j; n=k \text{ or} \\ & \text{if } i = S-1; j=1, k=0,1,\cdots,M; l=i+1; m=0; n=k \\ -\mu & \text{if } i = 0; j=1, k=M; l=i; m=j; n=k \\ k\gamma & \text{if } i = 1,2,\cdots,S-1; j=1, k=1,2,\cdots,M \\ & l = i-1; m=j; n=k-1 \text{ or} \\ & \text{if } i = s+2,\cdots,S; j=0, k=1,\cdots,M \\ & l = i-1; m=j; n=k-1 \text{ or} \\ & \text{if } i = s+1; j=0, k=1,\cdots,M; l=i-1; m=1; n=k-1 \\ i\theta & \text{if } i = 1,2,\cdots,S-1; j=1, k=0,1,2,\cdots,M \\ & l = i-1; m=j; n=k \text{ or} \\ & \text{if } i = s+2,\cdots,S; j=0, k=1,\cdots,M. \\ & l = i-1; m=j; n=k \text{ or} \\ & \text{if } i = s+1; j=0, k=1,\cdots,M; l=i-1; m=1; n=k \\ -(\lambda\beta+\mu) & \text{if } i = 0; j=1, k=0,1,\cdots,M-1; l=i; m=j; n=k \\ -(\lambda+i\theta) & \text{if } i = s+1,\cdots,S; j=0, k=0; l=i; m=j; n=k \\ -(\lambda+\mu+i\theta) & \text{if } i = 1,\cdots,S-1; j=1, k=0; l=i; m=j; n=k \\ -(\lambda+i\theta+k\gamma) & \text{if } i = s+1,\cdots,S; j=0, k=1,\cdots,M; l=i; m=j; n=k \\ -(\lambda+\mu+i\theta+k\gamma) & \text{if } i = 1,\cdots,S-1; j=1, k=1,\cdots,M; l=i; m=j; n=k \end{cases}$$

Define $A_{il} = (a(i,j,J.:;l,1n,n))$

Then the infinitesimal generator $\tilde{A}$ can be convinicntly expressed as a partitioned matrix

$A = ((A_{il}))$ where A_{il} is a $(M + 1)\, x\, (M + 1)$ matrix is given by

$$A_{il} = \begin{cases} A & \text{if } i = 0,1,\cdots,S-1; l = i+1 \text{ and the production on} \\ A_i & \text{if } i = s+1,\cdots,S; l = i-1 \text{ and the production off} \\ B_i & \text{if } i = s+1,\cdots,S; l = i \text{ and the production off or} \\ C_i & \text{if } i = 1,2,\cdots,S-1; l = i \text{ and the production on} \\ D & \text{if } i = 0; l = i \text{ and the production on} \\ D_i & \text{if } i = 1,\cdots,S-1; l = i-1 \text{ and the production on} \\ 0 & \text{otherwise} \end{cases}$$

With

$$A = \begin{array}{c} M \\ M-1 \\ M-2 \\ \dots \\ 2 \\ 1 \\ 0 \end{array} \begin{pmatrix} \mu & 0 & 0 & & 0 & 0 & 0 \\ 0 & \mu & 0 & & 0 & 0 & 0 \\ 0 & 0 & \mu & & 0 & 0 & 0 \\ \dots & & & & & & \dots \\ 0 & 0 & 0 & & \mu & 0 & 0 \\ 0 & 0 & 0 & & 0 & \mu & 0 \\ 0 & 0 & 0 & & 0 & 0 & \mu \end{pmatrix}$$

$$A_i = \begin{array}{c} M \\ M-1 \\ M-2 \\ \dots \\ 1 \\ 0 \end{array} \begin{pmatrix} (\lambda+i\theta) & M\gamma & 0 & 0 & 0 \\ 0 & (\lambda+i\theta) & (M-1)\gamma & 0 & 0 \\ 0 & 0 & (\lambda+i\theta) & 0 & 0 \\ \dots & & & & \\ 0 & 0 & 0 & (\lambda+i\theta) & \gamma \\ 0 & 0 & 0 & 0 & (\lambda+i\theta) \end{pmatrix}$$

$$B_i = \begin{array}{c} M \\ M-1 \\ \dots \\ 1 \\ 0 \end{array} \begin{pmatrix} -(\lambda+i\theta+M\gamma) & 0 & 0 & 0 \\ 0 & -(\lambda+i\theta+(M-1)\gamma) & 0 & 0 \\ & & & \\ 0 & 0 & -(\lambda+i\theta+\gamma) & 0 \\ 0 & 0 & 0 & -(\lambda+i\theta) \end{pmatrix}$$

$$C_i = \begin{array}{c} M \\ M-1 \\ \dots \\ 0 \end{array} \begin{pmatrix} -(\lambda+\mu+i\theta+M\gamma) & 0 & 0 \\ 0 & -(\lambda+\mu+i\theta+(M-1)\gamma) & 0 \\ \dots & & \\ 0 & 0 & -(\lambda+\mu+i\theta) \end{pmatrix}$$

$$D = \begin{array}{c} M \\ M-1 \\ M-2 \\ \dots \\ 2 \\ 1 \\ 0 \end{array} \begin{pmatrix} -\mu & 0 & 0 & & 0 & 0 & 0 \\ \lambda\beta & -(\lambda\beta+\mu) & 0 & & 0 & 0 & 0 \\ 0 & \lambda\beta & -(\lambda\beta+\mu) & & 0 & 0 & 0 \\ \dots & \dots & & & & & \\ 0 & 0 & 0 & & -(\lambda\beta+\mu) & 0 & 0 \\ 0 & 0 & 0 & & \lambda\beta & -(\lambda\beta+\mu) & 0 \\ 0 & 0 & 0 & \dots & 0 & \lambda\beta & -(\lambda\beta+\mu) \end{pmatrix}$$

$$D_i = \begin{array}{c} M \\ M-1 \\ M-2 \\ \dots \\ 2 \\ 1 \\ 0 \end{array} \begin{pmatrix} (\lambda+i\theta) & M\gamma & 0 & 0 & 0 & 0 \\ 0 & (\lambda+i\theta) & (M-1)\gamma & 0 & 0 & 0 \\ 0 & 0 & (\lambda+i\theta) & 0 & 0 & 0 \\ & & & & & \\ 0 & 0 & 0 & (\lambda+i\theta) & 2\gamma & 0 \\ 0 & 0 & 0 & 0 & (\lambda+i\theta) & \gamma \\ 0 & 0 & 0 & 0 & 0 & (\lambda+i\theta) \end{pmatrix}$$

$i = 1,\cdots,S-1; l = i-1$ and production is on

So we can write the partitioned matrix as follows:

$$\tilde{A} = \begin{array}{c} (S,0) \\ (S-1,0) \\ \dots \\ (s+1,0) \\ (S-1,1) \\ \dots \\ (s+1,1) \\ (s,1) \\ (s-1,1) \\ \\ 2 \\ 1 \\ 0 \end{array} \begin{pmatrix} B_S & A_S & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & B_{S-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & & & & & & & & \\ 0 & 0 & B_{s+1} & 0 & 0 & A_{s+1} & 0 & 0 & 0 & 0 \\ A & 0 & 0 & C_{S-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & & & & & & & & \dots \\ 0 & 0 & 0 & 0 & C_{s+1} & D_{s+1} & 0 & 0 & 0 & 0 \\ B & 0 & 0 & 0 & A & C_s & D_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A & C_{s-1} & 0 & 0 & 0 \\ \dots & \dots & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_2 & D_2 & 0 \\ 0 & 0 & B & 0 & 0 & 0 & 0 & A & C_1 & D_1 \\ 0 & 0 & 0 & B & 0 & 0 & 0 & 0 & A & D \end{pmatrix}$$

VIII. CONCLUSION

From the investigation taken up on the various kinds of inventory models, it is quite fascinating to watch the changes in the optimal solutions when the models are suitably altered by incorporating a few changes in the models. The conceptualization of the models is by incorporating some real life-situations, which are acceptable. For example, the demand for any item or commodity can experience changes with the passage of times.

The most secured inventory theory techniques (models) in logical examinations are numerous item models with stochastic demand portrayal. The application of the procedures and methods created in this examination can be applied to any inventory condition where attrition lessens available cost of inventory level.

According to their assessment of the above analysis, the authors prescribe the utilization of the "Mathematica" platform, which is not hard to control and fills in as the basis for modelling various situations regarding an item demand. This platform is certainly suitable for companies that will settle certain issues via this platform. The authors propose modelling demand of the stochastic nature, where usability is at a higher level and it is conceivable to model various situations that the market may offer with regards to demand, which may not be exactly decided, and in this way can react to the actual conditions potentially happening on the market, independent of their susceptibility.

REFERENCES: -

1. Lukáš Polanecký, Xenie Lukoszová (2016). “Inventory Management Theory: a Critical Review” Littera Scripta, 2016, Volume 9, Issue 2, pp. 79-89

2. LUKÁŠ, L. and J. HOFMAN (2016). Generalized inventory models of EOQ type and their implementation in Mathematica. In: Production management and Engineering Science. 2301 EH Leiden, the Netherlands: CRC Press/Balkema, Taylor

and Francis Group, pp. 169- 174. ISBN 978-1-138-02856-2.

3. Reza Rashid, Seyed Farzad Hoseini, M. R. Gholamian, Mohammad Feizabadi (2015) "Application of queuing theory in production-inventory optimization" J Ind Eng Int (2015) 11: pp. 485–494

4. BAZAN, E., M. Y. JABER and S. ZANONI (2016). A review of mathematical invektory models for reverse logistics and the future of its modelling: An environmental perspective, Applied Mathematical Modelling [online]. 40, pp. 4151-4178. Available at: http://dx.doi.org/10.1016/j.apm.2015.11.027.

5. Rashid R, Bozorgi-Amiri A, Seyedhoseini S (2015). Developing a new stochastic competitive model regarding inventory and price. J Ind Eng Int, pp. 1–10

6. Kizito Paul Mubiru (2013). "An EOQ Model For Multi-Item Inventory With Stochastic Demand" International Journal of Engineering Research & Technology (IJERT) Vol. 2 Issue 7, July - 2013 IJERT ISSN: 2278-0181

7. Karimi-Nasab M. & Konstantaras I. (2013). An inventory control model with stochastic review interval and special sale offer. Eur J Oper Res 227: pp. 81–87

8. Taleizadeh A.A., Niaki S.T.A., Seyedjavadi S.M.H. (2012). Multi-product multi-chance-constraint stochastic inventory control problem with dynamic demand and partial back-ordering: a harmony search algorithm. J Manuf Syst 31: pp. 204–213

9. Herbert E. Scarf (2002). "INVENTORY THEORY" Vol. 50, No. 1, January–February 2002, pp. 186–191

10. Waldmann K. H. : "INVENTORY MODELS" OPTIMIZATION AND OPERATIONS RESEARCH – Vol. IV

11. Hanssman F. (1962). "Operations research in production and inventory control", John Wiley and Sons Inc, New York.

12. Hadley G. and Whitin T.M. (1961). "An optimal final inventory model", Management Science, Vol.7, Issue 2, pp. 179-183.

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