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Mathematical Modeling of Rayleigh-Taylor Instability in Porous Media using COMSOL

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***Abstract –** Rayleigh-Taylor instability (RTI) is frequently observed in nature like oil extraction process, -sequestration and in the laboratory like chromatographic separation techniques. Suppression and control of RTI to get desired result is very important aspect. Mathematical modelling as well as various numerical methods to get results are available in literature. But in this work, we studied the problem using COMSOL and consistent results are obtained. Our finding shown that less instability is obtained in mapped mesh as compared to triangular meshing. Also, our finds suggest that forward finger and backward finger length are almost same.*

***Keywords:** Modelling, Coupling, Darcy's Law, Instability.*

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1. INTRODUCTION

Rayleigh-Taylor Instability (RTI), is a hydrodynamics instability phenomena which is observed when the heavy fluid lie on the lighter fluid [1]. More density of upside fluid tends to penetrate the less dense fluid, and hence the instability is observed at the fluid-fluid interface. This phenomenon is occurred due to density difference and finger like patterns are obtained at the interface. It has various applications in oil recovery, CO_2-sequestration, groundwater contamination [2, 3]. Porous media is medium through which fluid flows, it consists of inter-connected voids of solid matrix. Most of the media are porous media e.g. sand, chalk, coal etc. So, study of RTI is important and studied by various authors in different-different situations [4, 5]. They took the Navier-Stokes equation for the conservation of momentum. For porous media, Darcy in 1958 give the explicit simple relation known as Darcy's law for conservation of law. In this work, we simulate the problem using Darcy's law. Mathematical modelling to model RTI in porous media are discussed by various authors in the literature [6], which is presented in this paper. Manickam [7] analysed vertical miscible displacement flows with monotonic viscosity and density profiles using linear stability analysis and numerical simulations. In this paper, we used the COMSOL software to capture the instability. Kim [3] studied the linear stability analysis for Rayleigh-Taylor instability of a miscible slice in a porous medium. In electrical imaging and fluid modelling of convective fingering in a shallow water-table aquifer, COMSOL implementation is done by Van Dam *et al.* [8]. To our best knowledge, COMSOL implementation is not implemented yet to solve RTI problem and used first time in this work. Coupling of various is easy to implement in COMSOL and discussed in this paper. In section 2, mathematical modelling is discussed which is available in literature. Then in section 3, use of COMSOL multiphysics is discussed and list of parameters are described. Then in the section 4, results and conclusions are discussed.

2. MATHEMATICAL MODELING

A fluid of viscosity μ_1, having density ρ_1 lie on the fluid of viscosity μ_0 having density ρ_0 in a two-dimensional rectangle porous media of width *W* and height *H* (see figure [1]). Concentration of solute in solvent (*c*) for upper and lower fluid are respectively c_1 and c_0. To non-dimensionlised the concentration $c \to (c-c_0)/(c_1-c_0)$ is used. Hence, finally we have *c* = *1* mol/m^3 for fluid of density ρ_1 and *c* = *0* mol/m^3 for fluid of density ρ_0. Besides, we have the following assumption for our model,

1. 2D Homogeneous porous media having constant porosity ϵ and permeability κ
2. Fluids are incompressible and miscible in nature
3. Isotropic and constant Dispersion D_f.

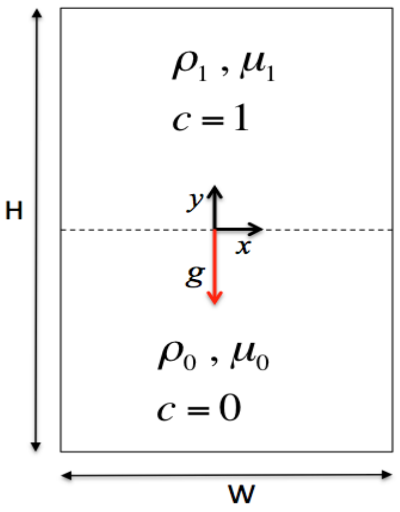

Figure 1. Schematic diagram of 2D flow where dashed line show the unperturbed fluid-fluid interface

To describe the fluid-flow in porous media: conservation of mass, conservation of momentum and conservation solute species laws to be holds. So for our problem, conservation of mass is governed by equation of continuity for incompressible fluid and conservation of momentum in porous media is governed by Darcy's law with gravity term. Conservation of solute species is described by convection-diffusion equation. So the governing equations of the described problem are written mathematically [6, 9] as follow:

$$\nabla \cdot \vec{u} = 0 \qquad (1)$$

$$\vec{u} = -\frac{\kappa}{\mu(c)}(\nabla p + \rho(c) g \nabla D) \qquad (2)$$

$$\varepsilon \frac{\partial c}{\partial t} + \vec{u} \cdot \nabla c = \varepsilon D_f \nabla^2 c \qquad (3)$$

where D = y-W is the elevation, $\vec{u} = (u, v)$ is the Darcy's velocity, p is the hydrodynamics pressure, and g is the gravitational constant *(=9.8 m^2/s)*. Density(ρ) and viscosity(μ) are the function of concentration c and assumed the following relation for our problem:

$$\rho(c) = \rho_0 + \beta c \qquad (4)$$

$$\mu(c) = \mu_0 e^{Rc} \qquad (5)$$

where $\beta = \rho_1 - \rho_0$, $R = \ln(\mu_2/\mu_1)$ is log-mobility ratio.

Suitable initial conditions are described by

$$c(x, y, 0) = \begin{matrix} 1, & if\ y > 0 \\ 0, & if\ y < 0 \end{matrix} \qquad (6)$$

$$p(x, y, 0) = 0 \qquad (7)$$

Boundary conditions:

No flux conditions are imposed on the all four boundaries for concentration.

No flow, $\vec{u} = (0,0)$ on the vertical walls and on the upper boundary.

For consistent system, at lower boundary pressure condition is imposed as $p = 0$.

3. USE OF COMSOL MULTIPHYSICS

Darcy's Law (dl) and transport of dilute species in porous media (tds) of COMSOL multiphysics (trial version) 5.2a are used for the simulations. COMSOL "dl"-model has continuity equation and Darcy's law whereas "tds" model contains the convection-diffusion equation. Our model is two-way coupled. Darcy's velocity is used in the 'tds'-properties velocity field. Also, in the properties field of 'dl'-model, density and viscosity relations are used as described by the equation [4, 5]. Appropriate boundary and initial conditions are used and described early. Triangular as well as mapped meshes are used to discritised the domain.

List of parameters

Parameter	Symbol	Value and unit
Width of domain	W	1 mm
Height of domain	H	4 mm
Viscosity of the lower fluid	μ_0	1 mPa s
Log-mobility ratio	R	0
Diffusion coefficient	D_f	10^{-7} m^2/s
Porosity	ϵ	0.4
Permeability	κ	10^{-6} m^2
Density of Upper fluid	ρ_1	1000, 1010 kg/m^3
Density of Lower fluid	ρ_0	1000, 1010 kg/m^3

In our problem, domain is $(x,y) \in W \times H$. Log-mobility ratio R is taken 0 for pure Rayleigh-Taylor instability. Density of upper fluid and lower fluids are taken 1000 as well as 1010. In all the possible cases, results are discussed in the next section.

4. RESULTS AND CONCLUSIONS

First, we discuss the result obtained using the triangular mesh. In triangular meshes, interface is not flat. Thus, interface is perturbed automatically. When the lighter fluid lie on the heavier fluid (i.e. upper fluid has density 1000 and lower fluid has density 1010) and both having the same viscosity

(*R=0*), only diffusion at the interface is observed as shown in figure 2.

Whereas, when the heavier fluid lie on the lighter fluid having same viscosity, fingering instability is observed at the interface, which is known as Rayleigh-Taylor instability, as shown in figure 3. Thus, our COMSOL model is able to capture the physical realistic results as Manickam [9].

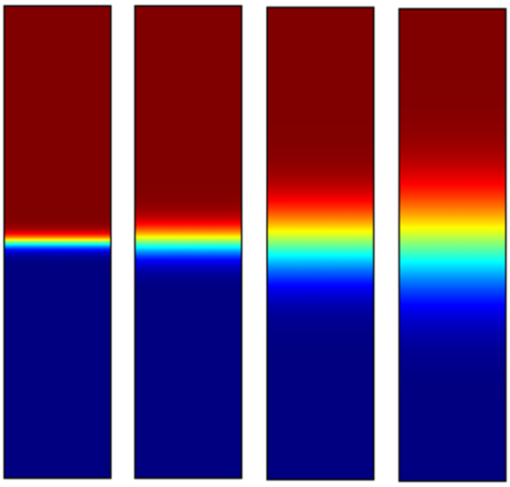

Figure 2. Concentration density plot at time 0.02, 0.1, 0.5 and 1 sec. respectively, where $R = 0, \rho_1 = 1000, \rho_1 = 1010.$

Also, RTI is observed when mapped mesh is used to descritised the domain. In mapped mesh, fluid-fluid interface appeared almost flat, but in COMSOL little inbuilt random perturbations are present at interface which triggers the instability. In case of triangular mesh, onset time of instability is early due to unstructured mesh, that introduce some extra perturbation as compare to mapped mesh. Also as shown in figure 3 and figure 4, forward finger and backward finger is of same length.

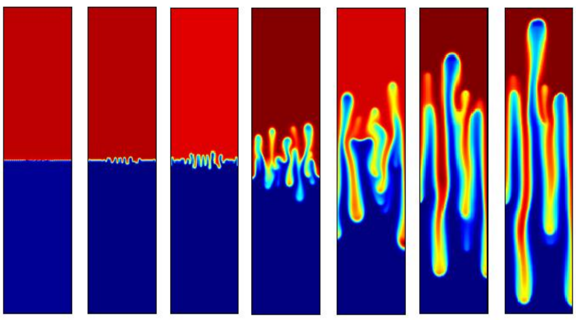

Figure 3. Concentration at time 0, 0.001, 0.002, 0.01, 0.02, 0.03 and 0.035 sec., respectively, where $R = 0, \rho_1 = 1010, \rho_1 = 1000.$

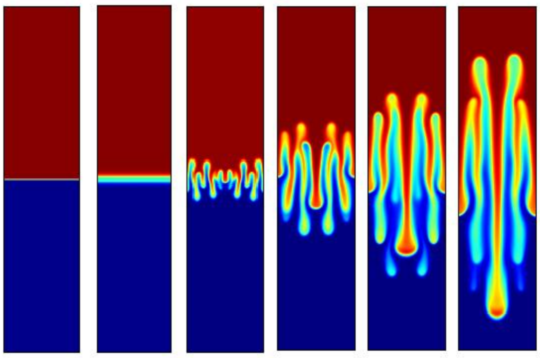

Figure 4. Concentration at time 0, 0.006, 0.01, 0.02, 0.03, and 0.04 sec., respectively, where $R = 0, \rho_1 = 1010, \rho_1 = 1000.$

In this paper, we perform the simulations for density-contrast fluids only by taking same viscosity. We model the problem for general case which can take care of viscosity-contrast as well as density contrast problem. Study of the coupled effect of R and RTI is one the future aspect.

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