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Equilibrium-State for Constant Concentration at Inlet Boundary in Porous Media Having Viscosity-Contrast

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Abstract – In porous media, displacement flows where two fluids are different in viscosity is much important for industry and biological applications. Due to viscosity-contrast between the fluids, instability is developed at the interface which is known as viscous fingering instability. In this work, we discussed about the equilibrium solution of the viscous-fingering instability problem. Advection-diffusion equation is obtained in equilibrium state. Hence, advection-diffusion is discussed in this paper which is fundamental in understanding of in- stability problem. Our result shows that in the early time, solution is not symmetric about the fluid-fluid interface. Our finding suggest that equilibrium state is time-dependent.

Keywords: Advection-Diffusion Equation, Inlet Boundary Condition, Symmetric, Fluid-Fluid Interface.

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1. INTRODUCTION

Recently, work in porous media has various important application. Porous media consists of interconnected voids [1]. Most of the medium are porous medium, e.g. sand, coal etc. Boundary plays an crucial role in each problem. So in this work, we discussed the constant concentration at inlet boundary. In porous media, various instability obtained at the fluid-fluid interface due to difference in various-various properties e.g. density fingering due to density difference [2], viscous fingering due to viscosity difference [1], Kelvin-Helmotz instability due to shear effect [3]. Tan and Homsy [4] discuss the linear stability analysis of viscous fingering instability. Equilibrium state is always important to study linear stability analysis. In stability analysis, perturbation to be given at the interface and we have to discuss how to perturbation to be behaved. Before that analysis of equilibrium state is important and hence discussed in this paper. Equilibrium state is found time-dependent in our study. Thus in this paper, various aspect of advection-diffusion equation is discussed. Literature about the solution of advection-diffusion equation is available [5, 6]. But relation to viscosity-constast problem is discussed in this paper. This paper is organized as: In section 2, mathematical modeling is discussed. Then in section 3, equilibrium state of the problem is discussed as follow result and conclusion in section 4 and 5 respectively.

2. MATHEMATICAL MODELLING

We considered the rectilinear displacement process, in which one fluid is displaced by second fluid through inlet boundary. Concentration of solute in solvent for one fluid (initially inside the domain) is assumed c_2 whereas for second fluid it is assumed c_1. In addition, in our study following assumptions are considered:

1. Both the fluids are assumed incompressible, neutrally buoyant.
2. Viscosity of injected fluid is assumed μ_1 and viscosity of concentration c_2 is assumed μ_2.
3. In addition, porous media is 2D homogenous and having constant permeability (κ) and porosity ϵ.
4. Here, most important assumption which we assumed is viscosity is assumed as function of concentration. Both the fluids are assumed of same density. Schematic of our problem is shown in figure 1, where fluid of concentration *c*1 is injected with velocity *U* in x-direction and displaces

already filled second fluid of concentration $c2$.

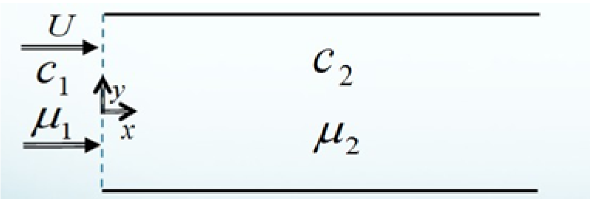

Figure 1: Schematic of flow configuration where fluid is injected through inlet boundary

In this case, we have the viscosity contrast between the fluids and hence concentration difference. Conservations law to be holds for any physical problem. Conservation of mass in governed by divergence-free velocity field due to incompressible fluids. In porous media, conservation of momentum is governed by linear relation between pressure gradient and Darcy's velocity which is known as Darcy's law. Since, dilute species evolved in the study, so conservation of dilute species is governed by convection-diffusion equation. Thus, mathematically the governing equations are described as [1]:

$$\nabla \cdot \dot{u} = 0, \qquad (1)$$

$$\nabla p = -\frac{\mu(c)}{\kappa}\dot{u} \qquad (2)$$

$$\frac{\partial c}{\partial t} + \dot{u}\cdot\nabla c = D\nabla^2 c \qquad (3)$$

where D is the dispersion coefficient which is assumed constant and isotropic, $\dot{u}$ is the Darcy's velocity, p is the hydrodynamics pressure. Domain of the given problem is $(x, y) \in [0, \infty) \times (-\infty, \infty)$

Boundary conditions are important for any physical problem. Concentration is constant at the inlet for our problem and velocity is $\dot{u} = (U, 0)$ at the inlet. At transverse boundary, no-slip boundary condition is implemented whereas no-flux for concentration. At outlet, no-flux due to far-field is implemented for concentration and ∂v = 0 at outlet. Thus, the given boundary conditions are mathematical written as:

$$\dot{u} = (U, 0),\ c = 0 \text{ at } x = 0\ \forall y \qquad (4)$$

$$\dot{u} = (1, 0),\ \frac{\partial c}{\partial x} \to 0 \text{ as } x \to \infty \qquad (5)$$

$$\frac{fv}{fy} = 0,\ \frac{\partial c}{\partial y} \to 0 \text{ as } y \to \pm\infty \qquad (6)$$

Suitable initial conditions are described by

$$c(x, y, 0) = \begin{cases} 1, & if\ y > 0 \\ 0, & if\ y < 0 \end{cases} \qquad (7)$$

$$p(x, y, 0) = 0 \qquad (8)$$

3. EQUILIBRIUM-STATE

Equilibrium state is always important for any further study. Equilibrium state is the ideal-situation. Here, assumptions are fluids is flowing in the axial-direction with uniform velocity U and only axial diffusion is happening. Hence, the convection-diffusion equation becomes advection-diffusion one-dimensional linear equation. So, we end up with the following equations:

$$\dot{u} = (U, 0) \qquad (9)$$

$$\frac{dp}{dx} = -\mu(c) \qquad (10)$$

$$\frac{fc}{ft} + U\frac{fc}{fx} = D\frac{f^2c}{fx^2} \qquad (11)$$

In this work, we will discuss the solution of advection-diffusion equation. Using Laplace transform, solution of advection-diffusion equation can obtained. As follow [6, 7], solution is given as:

$$c(x,t) = c_2 + (c_1 - c_2)\left[\frac{1}{2}\,erfc\left(\frac{-x-Ut}{2\sqrt{Dt}}\right) + \frac{e^{Ux/D}}{2}\,erfc\left(\frac{-x+Ut}{2\sqrt{Dt}}\right)\right] \qquad (12)$$

For further analysis of linear-stability analysis and non-linear simulation of problem, understanding of base- state flow is very important. In linear-stability analysis, disturbances are given around the equilibrium point and then to see the behavior of disturbances around the equilibrium point [3]. Thus, equilibrium state is much important for linear-stability analysis.

4. RESULTS

In the result part, we will discuss the various cases as follow:

Special case 1: By putting $U = 0$, second term of the solution is zeros and pure diffusive profile is obtained which is the solution of advection-diffusion equation [11] when $U = 0$. It is found that profile diffuses with the rate of $\sqrt{t}$.

Special case 2: Assume the injected fluid concentration is $c1 = 0$. This case is known as the absorbing boundary case. In this case, there are hole at the inlet boundary and some mass is absorbed by the inlet boundary. That's why it is called as absorbing boundary case. Without loss of generality, we assume $c1 = 0$ and $c2 = 1$ for further analysis.

Effect of injected velocity on advection-diffusion equation is shown in figure 2. It is clear as the injection velocity value increases, diffusive-profile is only shift to right direction. In this case, in the early time, profile does-not follow the $\sqrt{t}$ relation.

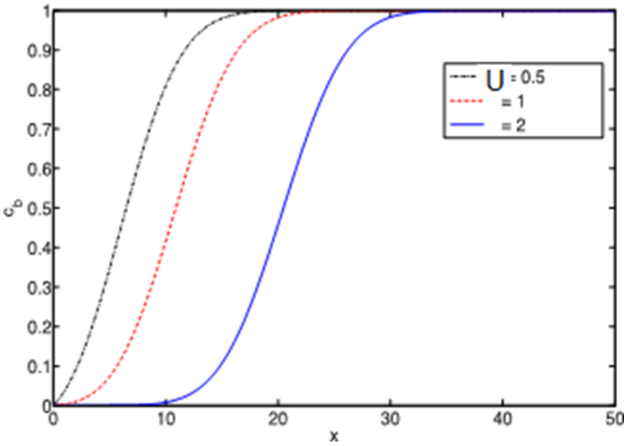

Figure 2: Effect of injected velocity on equilibrium concentration at *t* = 10, where *D* = 1

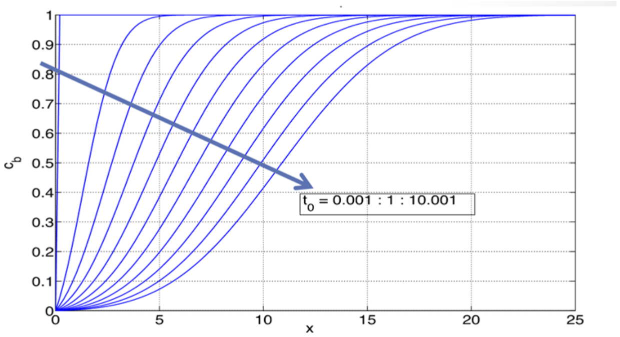

Figure 3: Temporal evolution of equilibrium concentration, where *D* = 1, *U* = 1

Temporal evolution of advective-diffusive profile is shown in figure 3. It is clear as time propagates, solution diffuses as well as advected. In the early time, boundary effects is there and so symmetry in the solution does not follow. Whereas for late time, solutions are symmetric around the interface position. Also, it is clear from analytic solution of advection-diffusion equation, second term of solution (12) approaches to zeros. Hence, solution is symmetric around the position of the interface which is $x = Ut$.

5. CONCLUSION

In this work, mathematical modeling of viscous fingering instability discussed. Importance of Equilibrium-state and inlet boundary conditions is discussed. So in this work, equilibrium solution of advection-diffusion equation is obtained and discussed.

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