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An Algorithm Based on Exponential Modified Cubic B-Spline Functions for Numerical Solution of Fisher's Equation

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Abstract – This paper deals with an algorithm for the simulation of Fisher's equation. In this paper the algorithm is developed with the help of Exponential Modified Cubic B-spline and differential quadrature method for the simulation of Fisher's Equation. Fisher's equation combines diffusion with logistic nonlinearity. The numerical simulation of these type of equations is very important as these equations occur in logistic population growth models, neurophysiology and nuclear reactions. Herein, the developed exponential modified cubic B-spline functions based differential quadrature method (EMCB-DQM) is applied on the Fishers' equation. Some practical problems are solved to check the accuracy and utility of the EMCB-DQM.

Key Words, Fisher's equation, Non-linear diffusion equation, Exponential Modified Cubic B-spline Functions, Differential Quadrature Method, Runge-Kutta Method.

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1. INTRODUCTION

The Fisher's equation was proposed by Fisher [8] as a model for the spatial and temporal propagation of a virile gene in an infinite medium. This equation also represents a one-dimensional reaction diffusion model for the evolution of the infected population. The equation is defined by

$$\frac{\partial u}{\partial t'} = D\frac{\partial^2 u}{\partial x'^2} + ku(1-u), \qquad (1.1)$$

where t' represents the time and $x' \in (-\infty, \infty)$ represents the position. The diffusion and reactive processes are parameterized by a diffusion coefficient *D*, and a reactive coefficient, *k*. It has been shown [5] that with the appropriate boundary conditions Fisher's equation will support traveling waves of the form $u(x' - c't')$ moving in the positive *x*-direction, provided that the speed $c' > 2\sqrt{kD}$. The equation (1.1) used to model many problems in mathematical biology [18]. By the change of the variables,

$$t = kt', \qquad x = x'\left(\frac{k}{D}\right)^{\frac{1}{2}}$$

The equation (1) becomes

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u(1-u), \qquad (1.2)$$

and traveling wave solutions exist for dimensionless $c \geq 2$ [13].

In equation (1.2), energy released by non-linear term balances energy consumed by diffusion, resulting in traveling waves or fronts [12].Traveling wave fronts have important applications in chemistry, biology and medicine [25]. Such wave fronts were first studied by Fisher in 1930s by studying the equation (2).

The Fisher's equation (2) occurs in chemical kinetics [15] and neutron population in a nuclear reaction. Moreover, the same equation also occurs in logistic population growth models [5], flame propagation, neurophysiology, autocatalytic chemical reactions, and branching Brownian motion processes.

The mathematical properties of the Fisher's equation and a lot of discussion are available in the literature. Brazhnik and Tyson [5], Kawahara and Tanaka [12] and Larson [14] provide excellent summaries of Fisher's equation. Wang [29] has presented the exact and explicit solitary wave solutions for the generalized Fisher's equation.

Ablowitz and Zepetelle [2] have also presented explicit solutions of Fisher's equation for a special wave speed. Wazwaz and Gorguis [1] discuss the analytic study of Fisher's equation by using Adomain decomposition method. But the numerical solutions of the Fisher's equation were not present till 1974s in the literature. First numerical solutions of Fisher's equation were presented by Gazdag and Canosa [10] with a pseudo-spectral approach. After that, a number of researchers have solved Fisher's equation numerically. Parekh and Puri [19] and Twizell et al. [27] have presented the implicit and explicit finite differences algorithms to discuss the numerical study of Fisher's equation. Hagstrom and Keller [11] have developed asymptotic boundary conditions by using centered finite difference algorithm. Tang and Weber [26] proposed a Galerkin finite element method and Rizwan-Uddin [24] compared the nodal integral method and non standard finite-difference schemes. Carey and Shen [9] used a least-squares finite element method and Al-Khaled [3] proposed sinc collocation method. Mickens [16] proposed a best finite-difference scheme for Fisher's equation. Qiu and Sloan [21] used a moving mesh method for numerical solution of Fisher's equation. Olmos and Shizgal [7] proposed a pseudo spectral method for the numerical solution of Fisher's equation. Recently, Mittal and Sumit Kumar [17] have studied Fisher's equation by applying wavelet Galerkin method.

2. DQM

DQM is a numerical method for solving differential equations. It was firstly, introduced by Bellman et al. [4]. By this method, we approximate the derivatives of a function at any location by a linear summation of all the functional values at a finite number of grid points, then the equation can be transformed into a set of ordinary differential equations, if the equation is unsteady, otherwise a set of algebraic equations. The solutions can be obtained by applying standard numerical methods.

According to the DQM, the ***r th*** partial derivatives of a dependent function $u(x,t)$ can be approximated the formula given in [6]

$$u_x^{(r)}(x_i,t) \cong \sum_{j=1}^{N} w_{ij}^{(r)} u(x_j,t) \qquad i = 1,2,\ldots\ldots,N \qquad r = 1,2,\ldots\ldots,N-1 \tag{2.1}$$

where $u_x^{(r)}(x_i,t)$ indicate the ***r th*** order derivatives of $u(x,t)$ with respect to ***x*** at grid points x_i, $w_{ij}^{(r)}$ are the weighting coefficients related to $u_x^{(r)}(x_i,t)$. Bellman et al [4] proposed two approaches to compute the weighting coefficients $w_{ij}^{(r)}$. To improve Bellman's approaches in computing the weighting coefficients, many attempts have been made by researchers. One of the most useful approach is the one introduced by Quan and Chang [22, 23].After that, Shu's [6] general approach which was inspired from Bellman's approach, was made available in the literature. Recently, many types of DQMs have been developed in literature [30-35].

3. EXPONENTIAL MODIFIED CUBIC B-SPLINE DIFFERENTIAL QUADRATURE METHOD

Let us assume that the one dimensional domain [a, b] is discretized into N grid points $a = x_1 < x_2, \ldots\ldots < x_N = b$ uniformly with step size $\Delta x = x_{i+1} - x_i$. By the basic fundamental of DQM, the r[th]order spatial partial derivatives of the unknown $u(x,t)$ with respect to x are approximated at x_i, $i = 1,2,\ldots,N$ as follows

$$\frac{\partial^r u(x_i,t)}{\partial x^r} = \sum_{j=1}^{N} a_{ij}^{(r)} u(x_j,t), \; i = 1,2,\ldots,N, \tag{3.1}$$

where $a_{ij}^{(r)}$ are the weighting coefficients of the r[th]order spatial partial derivatives with respect to x.

In this work, exponential cubic B-spline basis functions are used to find the weighting coefficients of one and two dimensional problems.

3.1 Exponential cubic B-spline basis functions

The exponential cubic B-spline basis functions are defined as

$$E_i(x) = \frac{1}{h^3}\begin{cases} b_2\left((x_{i-2}-x) - \frac{1}{p}\left(\sinh\left(p(x_{i-2}-x)\right)\right)\right), & x \in [x_{i-2}, x_{i-1}) \\ a_1 + b_1(x_i - x) + c_1 \exp\left(p(x_i - x)\right) + d_1 \exp\left(-p(x_i - x)\right), & x \in [x_{i-1}, x_i) \\ a_1 + b_1(x - x_i) + c_1 \exp\left(p(x - x_i)\right) + d_1 \exp\left(-p(x - x_i)\right), & x \in [x_i, x_{i+1}) \\ b_2\left((x - x_{i+2}) - \frac{1}{p}\left(\sinh\left(p(x - x_{i+2})\right)\right)\right), & x \in [x_{i+1}, x_{i+2}) \\ 0, & \text{otherwise} \end{cases} \tag{3.2}$$

Where

$$a_1 = \frac{phc}{phc - s}, \; b_1 = \frac{p}{2}\left(\frac{c(c-1)+s^2}{(phc-s)(1-c)}\right), \; c_1 = \frac{1}{4}\left(\frac{\exp(-ph)(1-c)+s\left(\exp(-ph)-1\right)}{(phc-s)(1-c)}\right)$$

$$d_1 = \frac{1}{4}\left(\frac{\exp(ph)(c-1)+s\left(\exp(ph)-1\right)}{(phc-s)(1-c)}\right), \; b_2 = \frac{p}{2(phc-s)}, \; c = \cosh(ph), \; s = \sinh(ph).$$

In Eq. (3.2), the free parameter p is used to obtain different form of exponential cubic B-Spline functions. The set $\{E_{-1}, E_0, \ldots, E_N, E_{N+1}\}$ $\{E_0, E_1, \ldots, E_N, E_{N+1}\}$ is chosen in such a way that it forms a basis over the domain $a \le x \le b$. The values of exponential cubic B-splines and its derivatives at the nodal points are depicted in Table 1.

Table 1: Coefficients of the exponential cubic B-spline functions E_i and its derivatives at the node x_i.

	x_{i-2}	x_{i-1}	x_i	x_{i+1}	x_{i+2}
$E_i(x)$	0	$\frac{s-ph}{2(phc-s)}$	1	$\frac{s-ph}{2(phc-s)}$	0
$E_i'(x)$	0	$\frac{p(c-1)}{2(phc-s)}$	0	$-\frac{p(c-1)}{2(phc-s)}$	0
$E_i''(x)$	0	$\frac{p^2 s}{2(phc-s)}$	$-\frac{p^2 s}{phc-s}$	$\frac{p^2 s}{2(phc-s)}$	0

The basis exponential cubic B-spline basis functions are modified in such way that the resulting matrix system of equations is diagonally dominant. The exponential cubic B-spline basis functions are modified as

$$\left.\begin{array}{l}\phi_1(x)=E_1(x)+2E_0(x)\\ \phi_2(x)=E_2(x)-E_0(x)\\ \phi_m(x)=E_m(x)\ for\ m=3,...,N\text{-}2\\ \phi_{N-1}(x)=E_{N-1}(x)-E_{N+1}(x)\\ \phi_N(x)=E_N(x)+2E_{N+1}(x)\end{array}\right\}, \tag{3.3}$$

where $\{\phi_1,\ \phi_2,\ ...,\phi_N\}$ forms a basis in the region $a\le x\le b$.

3.2 To determine the weighting coefficients

Taking $r=1$ in Eq. (3.1) and substituting the values of $\phi_m(x),\quad m=1,2,...,N$, we get a system of linear equations

$$\phi_m'(x_i)=\sum_{j=1}^{N}a_{ij}^{(1)}\phi_m(x_j),\ \text{for } i,m=1,2,...,N. \tag{3.4}$$

With the help of Eq. (3.3) and Table 1, Eq. (3.4) reduces into a tri-diagonal system of equations

$$A\vec{a}^{(1)}[i]=\vec{R}[i],\ for\ i=1,2,...,M, \tag{3.5}$$

where $A=[\phi_{ij}]$ is the coefficient matrix of order N given by:

$$A=\begin{bmatrix}\frac{phc-ph}{phc-s} & \frac{s-ph}{2(phc-s)} & & & & & \\ 0 & 1 & \frac{s-ph}{2(phc-s)} & & & & \\ & \frac{s-ph}{2(phc-s)} & 1 & \frac{s-ph}{2(phc-s)} & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & \frac{s-ph}{2(phc-s)} & 1 & \frac{s-ph}{2(phc-s)} & \\ & & & & \frac{s-ph}{2(phc-s)} & 1 & 0\\ & & & & & \frac{s-ph}{2(phc-s)} & \frac{phc-ph}{phc-s}\end{bmatrix}$$

$\vec{a}^{(1)}[i]=\left[a_{i1}^{(1)},a_{i2}^{(1)},...,a_{iN}^{(1)}\right]^T$ is the weighting coefficient vector corresponding to knot point x_i, and the coefficient vector $\vec{R}[i]=\left[\phi_{1,i}',\ \phi_{2,i}',\ ...\ ...,\ \phi_{N-1,i}',\ \phi_{N,i}'\right]^T$ corresponding to knot point x_i, $i=1,2,...,N$ are evaluated as

$$\vec{R}[1]=\begin{bmatrix}-\frac{p(c-1)}{phc-s}\\ \frac{p(c-1)}{phc-s}\\ 0\\ 0\\ \vdots\\ 0\\ 0\end{bmatrix},\vec{R}[2]=\begin{bmatrix}-\frac{p(c-1)}{2(phc-s)}\\ 0\\ \frac{p(c-1)}{2(phc-s)}\\ 0\\ \vdots\\ 0\\ 0\end{bmatrix},\cdots,\vec{R}[N-1]=\begin{bmatrix}0\\ 0\\ \vdots\\ 0\\ -\frac{p(c-1)}{2(phc-s)}\\ 0\\ \frac{p(c-1)}{2(phc-s)}\end{bmatrix},\vec{R}[N]=\begin{bmatrix}0\\ 0\\ \vdots\\ 0\\ 0\\ -\frac{p(c-1)}{phc-s}\\ \frac{p(c-1)}{phc-s}\end{bmatrix}$$

We note that the coefficient matrix A is invertible. The tri-diagonal system of equations is solved for each knot point x_i $(i=1,2,...,N)$ using the Thomas algorithm, which gives the weighting coefficients $a_{i1}^{(1)},a_{i2}^{(1)},...,a_{iN-1}^{(1)},a_{iN}^{(1)}$ $(i=1,2,...,N)$ of the first order partial derivative.

The weighting coefficients $a_{ij}^{(2)},1\le i,j\le N$ for the second order and higher order partial derivatives are determined by the formula

$$\begin{cases}a_{ij}^{(r)}=r\left(a_{ij}^{(1)}a_{ii}^{(r-1)}-\frac{a_{ij}^{(r-1)}}{x_i-x_j}\right),\ for\ i\ne j\ \text{and}\ \ i=1,2,3,...,N;\quad r=2,3,...,N-1\\ a_{ii}^{(r)}=-\sum_{j=1,j\ne i}^{N}a_{ij}^{(r)},\ \ \text{for } i=j,\end{cases} \tag{3.6}$$

where $a_{ij}^{(r-1)}$ and $a_{ij}^{(r)}$ are the weighting coefficients of the $(r-1)^{\text{th}}$ and r^{th} order partial derivatives with respect to *x*.

4. SOLUTION OF FISHER'S EQUATION

In this section, we apply EMCB-DQM for three types of Fisher's equations and a nonlinear diffusion Fisher type equation, which correspond to some real physical process. In all cases, the domain of definition of the equation is discredited by selecting uniform grid points.

Case I: In this case, we take the Fisher's equation of the form [28]

$$u_t = u_{xx} + u^2(1-u) \quad , \quad 0<x<1 \qquad (4.1)$$

By applying EMCB-DQM, we approximate the spatial partial derivative of the above equation as follows

$$u_{xx}(x_i,t) \cong \sum_{j=1}^{N} a_{ij}^{(2)} u(x_j,t), \qquad i=1,2,\ldots,N \qquad (4.2)$$

where u_{xx} is the partial derivates of $u(x,t)$ *w.r.t. x* and b_{ij}'s are the weighting coefficients of the second order partial derivatives and *N* is the total number of grid points taken in the interval. Using (7) in (6), we get the following system of ordinary differential equations

$$\frac{du_i}{dt} = \sum_{j=1}^{N} a_{ij}^{(2)} u_j + u_i^2(1-u_i) \qquad i=1,2,\ldots,N \qquad (4.3)$$

where $u(x_i,t)$ is referred as u_i.

Case II: In this case, we choose the generalized Fisher's equation of the form [1]

$$u_t = u_{xx} + u(1-u^{\alpha}) \qquad (4.4)$$

Similarly, applying the DQM describe in section 2, we get the following system of ordinary differential equations

$$\frac{du_i}{dt} = \sum_{j=1}^{N} a_{ij}^{(2)} u_j + u_i(1-u_i^{\alpha}) \qquad i=1,2,\ldots,N \qquad (4.5)$$

where $u(x_i,t)$ is referred as u_i.

Case III: Here, Fisher's equations [1] of the following form is considered

$$u_t = u_{xx} + \alpha u(1-u) \qquad (4.6)$$

Same as the above cases, by applying the DQM on the equation (4.6), we get the following system of ODE

$$\frac{du_i}{dt} = \sum_{j=1}^{N} a_{ij}^{(2)} u_j + \alpha u_i(1-u_i) \qquad i=1,2,\ldots,N \qquad (4.7)$$

where $u(x_i,t)$ is referred as u_i.

Case IV: In this case, we consider the nonlinear diffusion equation of the Fisher's type [1]

$$u_t = u_{xx} + u(1-u)(u-a), \qquad 0<a<1, \qquad (4.8)$$

Same as the above cases, the equation (4.8) reduces into the following system of ordinary differential equations (ODE) after using DQM

$$\frac{du_i}{dt} = \sum_{j=1}^{N} a_{ij}^{(2)} u_j + u_i(1-u_i)(u_i-a) \qquad i=1,2,\ldots,N \qquad (4.9)$$

where $u(x_i,t)$ is referred as u_i.

Thus, in all the cases we get a system of nonlinear ordinary differential equations. The above system of ordinary differential equations are solved by Pike and Roe's fourth-stage RK4 scheme [20] where function f is not explicitly dependent on *t.*

5. NUMERICAL EXPERIMENTS AND DISCUSSIONS

In this section, we solved three problems of Fishers types in order to show the applicability and convergence of the proposed method.

Problem 1. We consider the Fisher's equation (4.1) in domain [0, 1] .The exact solution is given by [28]

$$u(x,t) = \left(1+\exp\left(\nu(x-\nu t)\right)\right)^{-1}, \quad \nu = \frac{1}{\sqrt{2}} \qquad (5.1)$$

Problem 2. We consider the generalized Fisher's equation (4.4) in domain [0, 1] .The exact solution given by [29]

$$u(x,t) = \left[-\frac{1}{2}\tanh\left\{\frac{\alpha}{2\sqrt{2\alpha+4}}\left(x-\frac{\alpha+4}{\sqrt{2\alpha+4}}t\right)\right\}+\frac{1}{2}\right]^{\frac{2}{\alpha}} \qquad (5.2)$$

Problem 3. We consider the nonlinear diffusion Fisher's type equation (4.6) in domain [0, 1]. The exact solution [1] is given by

$$u(x,t) = \frac{1}{2}(1+a) + \left(\frac{1}{2}-\frac{1}{2}a\right)\tanh\left[\sqrt{2}(1-a)\frac{x}{4} + \frac{(1-a^2)}{4}t\right] \qquad (5.3)$$

The initial and boundary conditions are taken from the exact solution of each problem. The results are plotted in the Figures 1-5 in form of absolute errors. The errors are computed at two different time steps 0.001 and 0.0005 respectively. It can be seen from the Figures that errors become half when we choose half step length. This fact show that the proposed method is converges with good accuracy. The figures show that absolute errors are decreasing as we march forward in time .Hence the solutions are converging to exact ones.

6. CONCLUSION

In this proposed work, an exponential modified cubic B-spline functions based differential quadrature method (EMCB-DQM) is proposed for the solution of the Fisher's equation. The method is simple and straight forward which on application gives a system of ordinary differential equations. The resulting system of ordinary differential equations is solved by a four stage RK4 method. The method is applied on three test problems given in the literature. The absolute errors in the solutions are shown in the Figures which show the solutions are converging to exact ones, in some cases rapidly. It is found that the accuracy of the method depends on the number of grid points chosen for DQM and the O.D.E solver used to find the solution of resulting system of O.D.E. The strength of the method is its simple and easiness to apply.

7. REFERENCES

[1] Abdul-Majid Wazwaz and Alice Gorguis, (2004). 'An analytic study of Fisher's equation by using Adomain decomposition method', Appl. Math. Comput., Vol. 154, pp. 609-620.

[2] Ablowitz M. and Zepetella A. (1979). 'explicit solution of Fisher's equation for a special wave speed', Bull. Math. Biol., Vol. 41, pp. 835-840.

[3] Al-Khaled K. (2001). 'Numerical study of Fisher's reaction-diffusion equation by the sinc collocation method', J. Comput. Appl. Math., Vol. 137, pp. 245-255.

[4] Bellman R., Kashef B.G and Casti J. (1972). 'Differential quadrature: A technique for the rapid solution of nonlinear partial differential equations', J. Comput. Phys, Vol. pp. 1040-52.

[5] Brazhnik P. and Tyson J. (1999). 'On traveling wave solutions of Fisher's equation in two spatial dimensions," SIAM J. Appl. Math., Vol. 60, No. 2, pp. 371-391.

[6] Chang Shu (2000). 'Differential Quadrature and its Application in Engineering', Athenaeum Press Ltd., Great Britain.

[7] Daniel Olmos, Bernie D. and Shizgal, (2006). 'A pseudo spectral method of solution of Fisher's equation', J. Comput. Appl. Maths, Vol. 193, pp. 219-242.

[8] Fisher R.A. (1937). 'The wave of advance of advantageous genes', Ann. Eugenics, Vol. 7, pp. 355-369.

[9] Garey G.F. and Shen Y. (1937). 'Least-squares finite element approximation of Fisher's reaction-diffusion equation', Numer. Methods in Partial Differential Equations, Vol. 11, pp. 175-186.

[10] Gazdag J. and Canosa J. (1974). 'Numerical solution of Fisher's equation', J. Appl. Probab., Vol. 11, pp. 445-457.

[11] Hagstrom T. and Keller H.B. (1986). The numerical calculation of travelling wave solutions of nonlinear parabolic equations', SIAM J. Sci. Statist. Comput., Vol. 7, pp. 978-988.

[12] Kawahara T. and Tanaka M. (1983) 'Interactions of traveling fronts: an exact solution of a nonlinear diffusion equation', Phys. Lett. A, Vol. 97, No. 8, pp. 311-314.

[13] Kolmogorov A., Petrovsky I. And Piscounov N. (1937). 'Etude de l' equation de la diffusion avec croisance de la quantite de matiere et son application a un probleme biologique', Bull. Univ. Etat. Moscu. A, Vol. 1, pp. 1-25.

[14] Larson D.A. (1978). 'Transient bounds and time-asymptotic behavior of solutions to nonlinear equations of Fisher's type', SIAM J. Appl. Math., Vol. 34, pp. 93-103.

[15] Malfliet W. (1992). 'Solitary wave solutions of nonlinear wave equations', Am. J. Phys., Vol. 60, No.7, pp. 650-654.

[16] Mickens R.E., (1994). 'A best finite-difference scheme for Fisher's equation', Numer. Meth. Part. Diff. Eqns. Vol. 10, pp. 581-585.

[17] Mittal R.C. and Sumit Kumar (2006). 'Numerical study of Fisher's equation by wavelet Galerkin method', Int. J. Computer Math., Vol. 83, pp. 287-298.

[18] Murray J.D. (2002). 'Mathematical Biology I: An Introduction, third ed.', Springer, New York.

[19] Parekh N. and Puri S. (1990). 'A new numerical scheme for the Fisher's equation', J. Phys. A, Vol. 23, pp. L1085-L1091.

[20] Pike J and Roe PL (1985). 'Accelerated convergence of Jameson's finite volume Euler scheme using Van Der Houwen integrators', Comput Fluids, Vol. 13, pp. 223-236.

[21] Qiu Y. and Sloan D.M., (1998) 'Numerical solution of Fisher's equation using a moving mesh method', J. Comput. Phys., Vol. 146, pp. 726-746.

[22] Quan JR and Chang CT (1989a) 'New insights in solving distributed system equations by the quadrature methods-I', Comput Chem Engrg, Vol. 13, pp. 779-788.

[23] Quan JR and Chang CT (1989b). 'New insights in solving distributed system equations by the quadrature methods-II," Comput Chem Engrg, Vol. 13, pp. 1017-1024.

[24] Rizwan-Uddin, (2001). 'Comparison of the nodal integral method and non standard finite-difference scheme for the Fisher's equation', SIAM J. Sci. Comput., Vol. 22, pp. 1926-1942.

[25] Sherratt J., (1998). 'On the transition from initial data traveling waves in the Fisher-KPP equation," Dyn. Stab. Syst., Vol. 13, No. 2, pp. 167-174.

[26] Tang S. and Weber R.O., (1991). 'Numerical study of Fisher's equation by a Petrov-Galerkin finite element method', J. Austral. Math. Soc. Sci. B, Vol. 33, pp. 27-38.

[27] Twizell E.H., Wang Y. and Price W.G., (1990). 'Chaos free numerical solutions of reaction-diffusion equations', Proc. Roy. Soc. London Sci. A., Vol. 430, pp. 541-576.

[28] Verwer J.G., Hundsdorfer W.H. and Sommeijer B.P., (1990) 'Convergence properties of the Runge-Kutta-Chebyshev Method', Numer. Math, Vol. **57,** pp. 157-178.

[29] Wang X.Y. (1988). 'Exact and explicit solitary wave solutions for the generalized Fisher's equation', Phys. Lett. A, Vol. 131, No. 4/5, pp. 277-279.

[30] Ram Jiwari, Stefania Tomasiello and Francesco Tornabene (2018). A numerical algorithm for computational modelling of coupled advection-diffusion-reaction systems, Engineering Computations, 35 (3), pp. 1383-1401.

[31] Ram Jiwari, H. S. Shukla, M Tamsir and V. K. Srivastava (2018). A numerical algorithm for computation modeling of 3D nonlinear wave equations based on exponential modified cubic B-spline differential quadrature method, International Journal of Computer Mathematics, 95 (4), pp. 752-766.

[32] Ram Jiwari, Sukhveer Singh and Ajay Kumar (2017). Numerical simulation to capture the pattern formation of coupled reaction-diffusion models, Chaos, Solitons & Fractals, 103, pp. 422-439.

[33] Maria A. De Rosaa, Maria Lippiello, Ram Jiwari, Stefania Tomasiello (2016). A differential quadrature based procedure for parameter identification, Applied Mathematics and Computation, 290, pp. 460-466.

[34] Ram Jiwari (2015). Lagrange interpolation and modified cubic B-spline differential quadrature methods for solving hyperbolic partial differential equations with Dirichlet and Neumann boundary conditions, Computer Physics Communications, 193, pp. 55-65.

[35] Ram Jiwari and Jinyun Yuan (2014). A computational modeling of two dimensional reaction-diffusion Brusselator system arising in chemical processes, Journal of Mathematical Chemistry, 52, pp. 1535-1551.

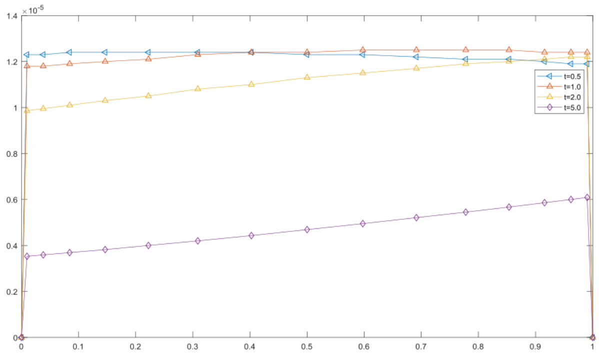

Figure 1. Absolute Errors in Example 1 for time step length 0.001 and N=17.

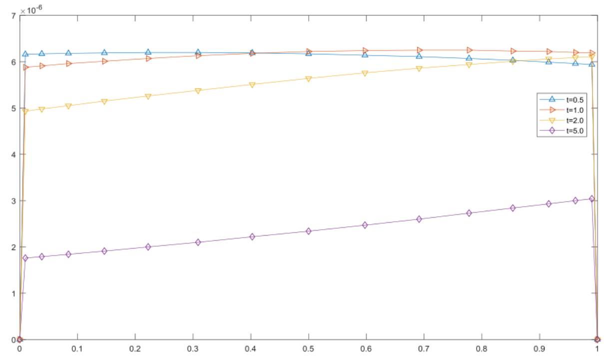

Figure 2. Absolute Errors in Example1 for time step length 0.0005 and N=17.

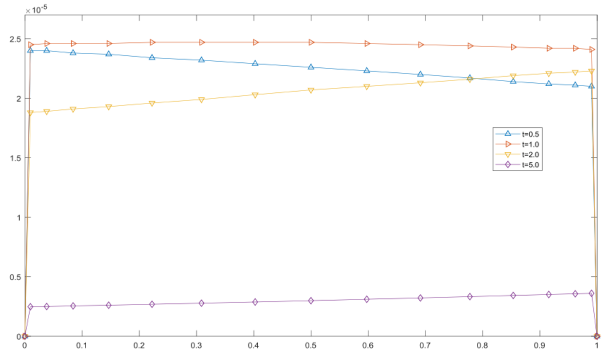

Figure 3. Absolute Errors in Example 2 for time step length 0.001 and N=17.

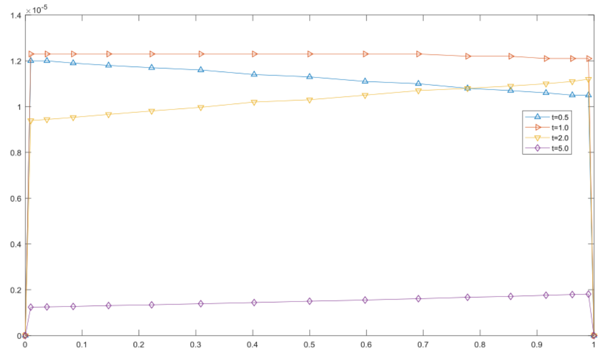

Fig 4. Absolute Errors in Example 2 for time step length 0.0005 and N=17.

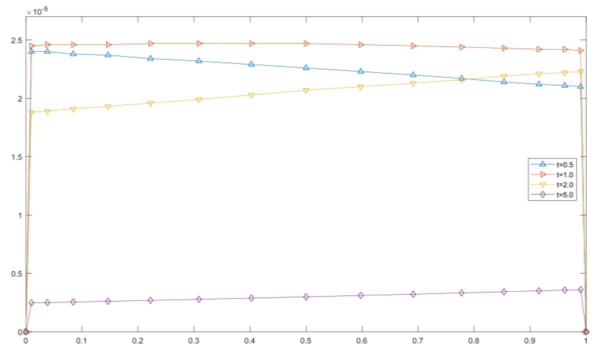

Figure 5. Absolute Errors in Example 3 for time step length 0.001 and N=17.

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