Recent Developments and challenges of Generalized Quadratic 3-Dimensional Assignment Problem (GQ3AP): the Multi-Story Facility Optimization

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Abstract – The quadratic assignment problem (QAP) is known as one of the most interesting and challenging problems in combinatorial optimization. we propose two exact algorithms for the GQAP (generalized quadratic assignment problem). The facility space requirements, the location available space, the facility installation costs, the flows between facilities, and the distance costs between locations, one must assign each facility to exactly one location so that each location has sufficient space for all facilities assigned to it and the sum of the products of the facility flows by the corresponding distance costs plus the sum of the installation costs is minimized. The paper includes modeling new applications of the multi-story space assignment problem (MSAP) and the crossdock door assignment problem (CDAP), and developing solution methods for an innovative assignment problem, the generalized quadratic 3-dimensional assignment problem (GQ3AP).

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INTRODUCTION

Many facilities have sure capabilities or website conditions that require them to deal with their occupants on more than one testimonies. Examples of those facilities are hospitals, workplace buildings, hotels, factories, warehouses, branch stores, oil structures, naval vessels, and so 4th. Multi-story centers have to be designed to provide an efficient and safe environment to fulfill the needs of the humans and equipments that occupy it. The facility stream is vital to its infrastructure and layout, because it connects the interactive components of the facility and serves because the pathway for building offerings as well as for emergency egress. In constructing evacuation problems, the design of the stairwell widths, configuration of the landings, variety of stairwells, their area, exits and egress paths are all significantly related. It is a travesty that people are often trampled in a push to get out of the slender exists in sports stadia and in burning buildings.

The multi-story area venture problem (MSAP) can be defined as finding departments within a multi-tale facility, on the way to reduce the inter-branch interplay cost and the cost to evacuate the constructing. As might be argued later, this hassle is a generalization of each the generalized quadratic

venture problem (GQAP) and the quadratic three-dimensional mission trouble (Q3AP). This bankruptcy begins with the advent of the thing issues GQAP (more info given in Section 2.2) and Q3AP (greater information given in Section 2.4).
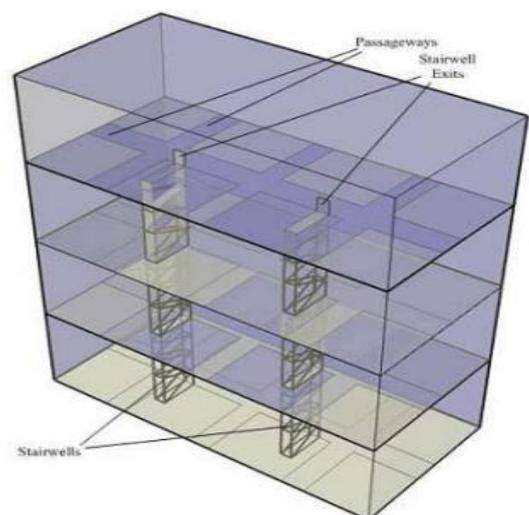

Figure 1. Multi-story facility.

The GQAP covers a broad elegance of issues that involve the minimization of a total pair-wise

interplay value among M entities and wherein placement of these entities into N feasible destinations is dependent upon current aid capacities at each vacation spot. These issues consist of finding the assignment of facilities to fixed places given restricted capacities at every possible place. The Lee and Ma (2004) version of the GQAP can be stated as an practical equipment region trouble where it is desired to discover M equipments amongst N fixed locations, where for every pair of equipments (i, k) there's a certain drift of commodities f (i, ok) and for every pair of places (j, n) there's a corresponding distance d (j, n). The two-manner interaction transportation fee between equipments i and k, given that i is assigned to location j and ok is assigned to location n, is f (i, k). d (j, n) + f (ok, i)· d (n, j). The goal is to locate an undertaking that minimizes the sum of all such transportation costs given the resource constraints.

Since the GQAP is quite new, little paintings has been executed to use it to constructing layout or facility vicinity. Notable is the work through Meller and Bozer (1997) at the multi-ground facility format problem (MFFLP). They described the power format trouble as locating a viable, non-overlapping association of departments (with given vicinity necessities inclusive of length and shape) inside a facility (building) to limit the interplay value between departments. Interaction price between departments is stated because the glide times the distance between those departments. In preferred, departments have unequal location requirements, and a number of them can be confined a priori to sure places within the building. They proceeded to solve the MFFLP in levels. Stage one assigned departments to floors in a try to decrease the inter-floor handling fee, which became formulated as a GQAP. Stage decided the format of every floor based on the assignments of level one. They argued that this approach not only comes up with near surest solutions, however that it additionally has control appeal, because there are many management concerns that are not easy to quantify as expenses.

Regarding the Q3AP, the interest in it stems from the truth that it's far carried out to issues in which the goal is to reduce linear and quadratic expenses related to a couple of independent simultaneous one-to-one assignments. Such a hassle arises in the layout of wi-fi conversation systems, in which a digital message is repeated two times. During each of the repeats, the task of facts words to transmitted symbols is changed. The Q3AP models the problem of optimizing the 2 assignments in the sort of way that the transmission mistakes are minimized. In the layout of multi-tale buildings, it is vital to simultaneously assign facilities to places and the same facilities to escape exits in the face of quadratic charges. Whereas, the assignments within the Q3AP are one-to-one, the assignments in multi-story building design tend to be many-to-one.

The multi-story space assignment problem (MSAP)[ ]

This subsection begins with the essential assumptions, definitions, and notation underlying the mathematical components of the MSAP.

Assumptions: (1) Distribution of the occupants on every ground is uniform and their tour time to and thru the hall and stairwell segments is a nation based function. (2) The footprint of the ability is a polygon or can be intently approximated by means of a simple polygon. The polygon need no longer be convex by means of it is closed and bounded. (3) The journey distance metric is believed to be rectilinear and the area per ground is given. The structural device either is pre-described or else is to be designed after this evaluation.

Definition: (1) Occupants are the those who occupy the facility and have to be evacuated. There are a total of O occupants $O = \sum_{i=1}^{M} O_i$ Where M is the overall quantity of departments. (2) The movement topology is the gathering of corridors and stairwells on each ground. This is either recognized or has to be generated.

Given M departments, in which for every branch i the variety of human beings O_i in step with department and the quantity of space required for the branch a_i is understood, the primal problem right here is to allocate the M departments to N flooring, such that the sum of all one-manner evacuation charges are minimized. Certain constraints need to be met as no department can be cut up between extraordinary floors and the to be had space on every ground won't be handed. A secondary, however vital, trouble is to inspire assignments that decrease the sum of all transportation expenses, as for every pair of departments (i, k) a certain glide of gadgets $f_{(i,k)}$ is thought and for each pair of flooring (j, n) a corresponding price to transport one unit of go with the flow between floors d (j, n) is understood.

Both primary and secondary troubles are dealt within the multi-story area undertaking hassle (MSAP), defined under.

Given the following parameters,

M number of departments needed to be placed,

N number of available floors,

P number of stairwells,

a_{ij} square feet of floor area needed by department i, if assigned to floor j,

A_j maximum usable floor area for floor j,

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f_{ik} flow units from department i to department k ,

d_{jn} cost per flow unit travel between floor j and floor n ,

e_{jp} distance between floor j and stairwell exit p from building,

λ_{ip} arrival rate of persons in the evacuation from department i , if assigned to stairwell p ,

μ_p service rate capacity of stairwell p ,

x_{ij} binary variable, $x_{ij} = 1$ iff department I is assigned to floor j ,

y_{ip} binary variable, $y_{ip} = 1$ iff department I is assigned to stairwell p .

The problem is to minimize the cost function

$$\min \quad \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{k=1}^{M}\sum_{n=1}^{N} \left(f_{ik} d_{jn} + \lambda_{ip} e_{jp} \right) x_{ij} y_{ip} x_{kn} \qquad (5\text{-}1a)$$

$$\text{s.t.} \quad \sum_{i=1}^{M} a_{ij} x_{ij} \leq A_j \qquad\qquad j = 1,...,N , \qquad (5\text{-}1b)$$

$$\sum_{j=1}^{N} x_{ij} = 1 \qquad\qquad i = 1,...,M , \qquad (5\text{-}1c)$$

$$\sum_{i=1}^{M} \lambda_{ip} y_{ip} \leq \mu_p \qquad\qquad p = 1,...,P , \qquad (5\text{-}1d)$$

$$\sum_{p=1}^{P} y_{ip} = 1 \qquad\qquad i = 1,...,M , \qquad (5\text{-}1e)$$

$$x_{ij} \in \{0,1\} \qquad\qquad i = 1,...,M; j = 1,...,N , \qquad (5\text{-}1f)$$

$$y_{ip} \in \{0,1\} \qquad\qquad i = 1,...,M; p = 1,...,P . \qquad (5\text{-}1g)$$

Equation (5-1b) makes positive that the potential of every ground isn't exceeded, (4-1c) makes certain that each branch receives assigned to simplest one ground, (5-1d) makes sure that the capacity of every stairwell isn't always surpassed, and (5-1e) makes certain that every department is assigned to most effective one number one stairwell.

The MSAP, as given in equations (5-1), combines a Q3AP with a GQAP, thus making it an NP-difficult problem. The strategies developed for fixing the Q3AP and the GQAP are an amazing basis for growing heuristics in addition to exact methods for generating sub-most suitable and most desirable solutions.

Optimization of cross dock layout design-

Many agencies utilize complicated distribution networks to transport numerous varieties of merchandise to meet the customers call for. To be competitive, they are trying to form the product moving inside and outside of the warehouse within the maximum cost-powerful, efficient, and well timed way. Cross docking is a valuable logistics method used in the retail and trucking industries to swiftly consolidate shipments from disparate assets and understand economics of scale in outbound transportation.

As said in Bartholdi and Gue (2004), cross docking basically eliminates the high priced handling and garage capabilities of a warehouse, at the same time as nonetheless allowing it to serve its receiving and transport capabilities. The idea is to transfer shipments immediately from incoming to outgoing trailers, without garage in between. Shipments typically spend less than 24 hours in a cross dock, once in a while less than an hour. With the procedure of transferring shipments from the receiving dock (inbound) to the transport dock (outbound), bypassing storage, cross docking reduces stock wearing value, transportation fee, and different prices related to cloth coping with. Cross docking is an important logistics strategy for businesses within the retail, grocery, and other distribution industries, together with less-than-truckload (LTL) trucking companies, which are looking for to consolidate shipments to obtain transportation economics.

Research subjects at the design and operational issues in cross docking include the scale of the cross dock, the quantity of doorways, the width of the dock, shapes for exceptional dock sizes, and assigning inbound and outbound trailer doors which is addressed here. Peck (1983), Tsui and Chang (1990, 1992), and Bartholdi and Gue (2000) have studied the optimization fashions to put out cross docks. A right format places excessive-float trailers close to one another, however now not so closes as to purpose congestion.

The cross dock door assignment problem (CDAP)-

The cross dock door undertaking hassle (CDAP) was proposed in Tsui and Chang (1990, 1992) with the objective of minimizing the weighted distance among incoming and outgoing trailers primarily based on door-to-door distance.

Given the following parameters,

M number of origins,

N number of destinations,

f_{ik} the number of trips required by the material handling equipment to move items originating from i to the cross dock door where freight destined for k is being consolidated,

d_{jq} the distance between receiving door j and shipping door q ,

x_{ij} binary variable, $x_{ij} = 1$ iff originals assigned to receiving door j,

y_{kq} binary variable, $y_{kq} = 1$ iff destination is assigned to shipping door q.

The formulation of CDAP is then

$$\min \sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{k=1}^{N}\sum_{q=1}^{N} f_{ik} d_{jq} x_{ij} y_{kq} \tag{5-2a}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} x_{ij} = 1 \qquad j = 1,...,M, \tag{5-2b}$$

$$\sum_{j=1}^{M} x_{ij} = 1 \qquad i = 1,...,M, \tag{5-2c}$$

$$\sum_{k=1}^{N} y_{kq} = 1 \qquad q = 1,...,N, \tag{5-2d}$$

$$\sum_{q=1}^{N} y_{kq} = 1 \qquad k = 1,...,N, \tag{5-2e}$$

$$x_{ij} \in \{0,1\} \qquad i,j = 1,...,M, \tag{5-2f}$$

$$y_{kq} \in \{0,1\} \qquad k,q = 1,...,N. \tag{5-2g}$$

Equation (5-2b) makes certain that each receiving door is assigned to most effective one beginning, (5-2c) makes sure that every starting place gets assigned to most effective one receiving door, (5-2d) makes certain that each delivery door is assigned to best one destination, and (5-2e) makes positive that every destination is assigned to most effective one delivery door.

Peck (1983) gave comparable models however additionally considered special varieties of freight and fabric managing systems. Bartholdi and Gue (2000) described models that manual a neighborhood seek ordinary in assigning vacation spot trailers to terminal doors if you want to minimize the total hard work value. Their layout models balanced the price of moving freight from incoming trailer to outgoing trailers with the cost of delays because of special varieties of congestion.

One can view the CDAP as a Q3AP (P.M. Hahn, private communication, February thirteen, 2006) if the variety of receiving doors is equal to the quantity of transport doorways, because the Q3AP minimizes the value over permutation matrices of the equal size. In that case, the Q3AP six-dimensional cost matrix may be rewritten as

$$c_{ijpknq} = f_{ik} d_{jq}, \quad \text{where} \quad \mathbf{F} = \left[f_{ik} \right] \quad \text{and} \quad \mathbf{D} = \left[d_{jq} \right]$$

and are N×N matrices. To check this speculation, I tailored the exact Q3AP answer algorithm to resolve a length M=N=10.CDAP test instance. This take a look at example turned into solved precisely in only 2,197.7 seconds on a Sun Ultra 10 pc with a unmarried 360 MHz CPU and required the evaluation of simply 4,162 nodes. It is thrilling to notice that a Q3AP check problem of the identical size (in which all six-dimensional price coefficients are arbitrary)

took about 4 times longer to solve at the same notebook and required the evaluation of about 2.5 instances of the nodes. To alter the Q3AP to solve a CDAP with M no longer same to N , the product fik d jq is manipulated in a manner that forces the answer of the Q3AP to be a solution of a CDAP whose X and Y matrices are of different sizes. To accomplish this, it is essential to set

$$f_{ik} = \infty \qquad \text{if } (i > M, M < N) \text{ or } (k > N, N < M), \tag{5-3a}$$

$$d_{jq} = \infty \qquad \text{if } (j > M, M < N) \text{ or } (q > N, N < M), \tag{5-3b}$$

$$f_{ik} d_{jq} = 0 \qquad \text{if } \begin{array}{l}(i > M, j > M, M < N) \\ \text{or } (k > N, q > N, N < M)\end{array}. \tag{5-3c}$$

It is easy to reveal that the limitations for the Q3AP are similar to those for the CDAP. Thus, it has been shown that the CDAP is a unique case of the Q3AP. One should be careful that, whilst the usage of the Q3AP to solve a CDAP, one have to take the transpose of the feasible Q3AP answers with a view to get the corresponding CDAP solutions.

Now remember a extra trendy version of the CDAP. First, right here are some observations about the restrictions of formulations (5-2) and (5-3): matrices X and Y are necessarily rectangular. But the drift matrix F and the distance matrix D aren't. The fact that X and Y are rectangular is proscribing. There could now not be very many practical situations in which each starting place could have its own receiving door and each vacation spot have its own delivery door. Thus, a more fashionable form with extra parameters has been defined (P.M. Hahn, private communication, February 23, 2006),

J number of receiving doors,

Q Number of shipping doors.

The formulation becomes

$$\min \sum_{i=1}^{M}\sum_{j=1}^{J}\sum_{k=1}^{N}\sum_{q=1}^{Q} f_{ik} d_{jq} x_{ij} y_{kq} \tag{5-4a}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} s_i x_{ij} \le S_j \qquad j = 1,...,J, \tag{5-4b}$$

$$\sum_{j=1}^{J} x_{ij} = 1 \qquad i = 1,...,M, \tag{5-4c}$$

$$\sum_{k=1}^{N} t_k y_{kq} \le T_q \qquad q = 1,...,Q, \tag{5-4d}$$

$$\sum_{q=1}^{Q} y_{kq} = 1 \qquad k = 1,...,N, \tag{5-4e}$$

$$x_{ij} \in \{0,1\} \qquad i = 1,...,M; j = 1,...,J, \tag{5-4f}$$

$$y_{kq} \in \{0,1\} \qquad k = 1,...,N; q = 1,...,Q. \tag{5-4g}$$

Equation (5-4b) makes certain that the capability of each receiving door isn't exceeded, and (5-4c) makes certain that each origin receives assigned to only one receiving door. While (5-4d) makes

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positive that the potential of every shipping door isn't always exceeded, and (5-4e) makes sure that each destination is assigned to best one transport door.

PROBLEM DEFINITION FOR THE GQ3AP

Professor Hahn additionally defined a brand new magnificence of challenge hassle, the generalized quadratic three-dimensional challenge trouble (GQ3AP), which combines the Q3AP with the GQAP and which allows to deal with the MSAP in addition to the CDAP (Guignard et al. 2006). The GQ3AP is given by minimizing the goal characteristic

$$\min \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{k=1}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P} c_{ijpknq} u_{ij} u_{kn} w_{ip} w_{kq} \qquad (5\text{-}5a)$$

Subject to the following constraints on u

$$\sum_{i=1}^{M} s_{ij} u_{ij} \leq S_j \qquad j=1,2,...,N, \qquad (5\text{-}5b)$$

$$\sum_{j=1}^{N} u_{ij} = 1 \qquad i=1,2,...,M, \qquad (5\text{-}5c)$$

$$u_{ij} \in \{0,1\} \qquad i=1,2,...,M; j=1,2,...,N, \qquad (5\text{-}5d)$$

In which S_j is the available aid at location j and s_{ij} is the aid requirement of device i , and problem to the subsequent constraints on w

$$\sum_{i=1}^{M} t_{ip} w_{ip} \leq T_p \qquad p=1,2,...,P, \qquad (5\text{-}5e)$$

$$\sum_{p=1}^{P} w_{ip} = 1 \qquad i=1,2,...,M, \qquad (5\text{-}5f)$$

$$w_{ip} \in \{0,1\} \qquad i=1,2,...,M; p=1,2,...,P, \qquad (5\text{-}5g)$$

Where T_p is the available resource at area p and t_{ip} is the aid requirement of device i .

Now, if the six-dimensional price matrix C may be written as

$$c_{ijpknq} = f_{ik} d_{jn} + \lambda_{ip} e_{jp}, \qquad (5\text{-}6)$$

In which all of the notations are described simply as before inside the MSAP.

Then the GQ3AP goal characteristic will become

$$\min \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{k=1}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P} \left(f_{ik} d_{jn} + \lambda_{ip} e_{jp} \right) u_{ij} u_{kn} w_{ip} w_{kq}. \qquad (5\text{-}7a)$$

But, the function being minimized can be simplified as

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{k=1}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P} \left(f_{ik} d_{jn} + \lambda_{ip} e_{jp} \right) u_{ij} u_{kn} w_{ip} w_{kq}$$

$$= \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{k=1}^{M}\sum_{n=1}^{N} \left(f_{ik} d_{jn} + \lambda_{ip} e_{jp} \right) u_{ij} u_{kn} w_{ip} \sum_{q=1}^{P} w_{kq} \qquad (5\text{-}7b)$$

$$= \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{k=1}^{M}\sum_{n=1}^{N} \left(f_{ik} d_{jn} + \lambda_{ip} e_{jp} \right) u_{ij} w_{ip} u_{kn}$$

That is the same objective feature as for the MSAP. And the restrictions for the GQ3AP are essentially similar to for the MSAP.

Now don't forget the case of the CDAP. If one rewrites the six-dimensional cost matrix C as

$$c_{ijpknq} = f_{ik} d_{jq}, \qquad (5\text{-}8)$$

Wherein F = [f_{ik}] , i , okay = 1,..., M and D = [d_{jq}] ,j = 1,..., J ; q = 1,...,Q , then it is possible to recast the CDAP as a GQ3AP, in which D is the receiving/transport distance in cross docking facility and F is the number of moving journeys of products, just as earlier than. The distinction in (5-8) is that the range of origins is identical to the variety of locations (a perhaps extreme dilemma).

Then the objective is to limit the characteristic under.

$$\sum_{i=1}^{M}\sum_{j=1}^{J}\sum_{p=1}^{Q}\sum_{k=1}^{M}\sum_{n=1}^{J}\sum_{q=1}^{Q} f_{ik} d_{jq} u_{ij} u_{kn} w_{ip} w_{kq}$$

$$= \sum_{i=1}^{M}\sum_{j=1}^{J}\sum_{p=1}^{M}\sum_{k=1}^{J}\sum_{n=1}^{Q} f_{ik} d_{jq} u_{ij} u_{kn} w_{kq} \sum_{p=1}^{Q} w_{ip} \qquad (5\text{-}9)$$

$$= \sum_{i=1}^{M}\sum_{j=1}^{J}\sum_{k=1}^{M}\sum_{q=1}^{Q} f_{ik} d_{jq} u_{ij} w_{kq} \sum_{n=1}^{J} u_{kn}$$

$$= \sum_{i=1}^{M}\sum_{j=1}^{J}\sum_{k=1}^{M}\sum_{q=1}^{Q} f_{ik} d_{jq} u_{ij} w_{kq}$$

Similarity between (5-9) and (5-4a) is placing, as the only difference being that in (5-9) the higher degrees of i and k are identical.

To allow the GQ3AP to clear up a CDAP in which the wide variety of origins isn't same to the wide variety of destinations, it is vital to vicinity restrictions at the product f_{ik} d_{jq} in a manner that forces an answer of the GQ3AP to be an answer of a CDAP. Namely, one must set

$$f_{ik} = 0 \qquad \text{if } (i>M, M<N) \text{ or } (k>N, N<M). \qquad (5\text{-}10)$$

Finally, it's far necessary to assure that the restrictions for this changed GQ3AP are certainly similar to for the CDAP. Thus, it's been proven that the CDAP is a special case of the GQ3AP.

REFORMULATION-LINEARIZATION TECHNIQUE (RLT) ON THE GQ3AP

Two equivalent GQ3AP formulations-

If one takes out the linear phrases with coefficients b_{ijp} that are blanketed within the (5-5a), then the formulation of GQ3AP is

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$$\min \left\{ \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P} b_{ij} u_{ij} w_{ip} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{\substack{k=1 \\ k\neq i}}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P} c_{ijpknq} u_{ij} u_{kn} w_{ip} w_{kq} \right\},$$

$$: u \in U, v \in W; u, v \text{ binary}$$ (5-11a)

$$\text{where}\quad u \in U \equiv \left\{ u \geq 0 : \sum_{i=1}^{M} s_{ij} u_{ij} \leq S_j, \forall (j=1,...,N); \sum_{j=1}^{N} u_{ij} = 1, \forall (i=1,...,M) \right\},$$ (5-11b)

$$\text{and}\quad w \in W \equiv \left\{ w \geq 0 : \sum_{i=1}^{M} t_{ip} w_{ip} \leq T_p, \forall (p=1,...,P); \sum_{p=1}^{P} w_{ip} = 1, \forall (i=1,...,M) \right\}.$$ (5-11c)

An equivalent formulation of the GQ3AP can be stated as

$$\min\quad \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P} b_{ij} x_{ijp} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{\substack{k=1 \\ k\neq i}}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P} c_{ijpknq} x_{ijp} x_{knq}$$ (5-12a)

$$\text{s.t.}\quad \sum_{j=1}^{N}\sum_{p=1}^{P} x_{ijp} = 1 \qquad i=1,...,M,$$ (5-12b)

$$\sum_{i=1}^{M}\sum_{p=1}^{P} s_{ij} x_{ijp} \leq S_j \qquad j=1,...,N,$$ (5-12c)

$$\sum_{i=1}^{M}\sum_{j=1}^{N} t_{ip} x_{ijp} \leq T_p \qquad p=1,...,P,$$ (5-12d)

$$x_{ijp} \in \{0,1\} \qquad i=1,...,M; j=1,...,N; p=1,...,P.$$ (5-12e)

The GQ3AP in (5-12) may be created from (5-11) by letting $u_{ij} w_{ip} = x_{ijp}$ and $u_{kn} w_{kq} = x_{knq}$. The equivalence of the two goal functions is trivially real, so are the restrictions on binary variables. Constraints (5-12b) come from multiplying the second equality constraints in (5-11b) with the second equality constraints in (5-11c) as:

$$\sum_{j=1}^{N} u_{ij} \sum_{p=1}^{P} w_{ip} = \sum_{j=1}^{N}\sum_{p=1}^{P} u_{ij} w_{ip} = \sum_{j=1}^{N}\sum_{p=1}^{P} x_{ijp} = 1$$

Constraints (5-12c) follow from multiplying the primary inequality constraints in (5-11b) with the second one equality constraints in (5-11c). Constraints (5-12d) result from

$$\sum_{i=1}^{M} s_{ij} u_{ij} \sum_{p=1}^{P} w_{ip} = \sum_{i=1}^{M}\sum_{p=1}^{P} s_{ij} u_{ij} w_{ip} = \sum_{i=1}^{M}\sum_{p=1}^{P} s_{ij} x_{ijp} \leq S_j.$$

Multiplying the 1st inequality constraints in (5-11c) with the second one equality constraints in (5-11b) as :

$$\sum_{i=1}^{M} t_{ip} w_{ip} \sum_{j=1}^{N} u_{ij} = \sum_{i=1}^{M}\sum_{j=1}^{N} t_{ip} u_{ij} w_{ip} = \sum_{i=1}^{M}\sum_{j=1}^{N} t_{ip} x_{ijp} \leq T_p.$$

Conversely, given the GQ3AP model in (5-12), by letting

$$x_{ijp} = u_{ij} w_{ip}, \quad \sum_{j=1}^{N}\sum_{p=1}^{P} x_{ijp}$$

becomes

$$\sum_{j=1}^{N}\sum_{p=1}^{P} u_{ij} w_{ip} = \sum_{j=1}^{N} u_{ij} \sum_{p=1}^{P} w_{ip} = \sum_{j=1}^{N} u_{ij} = \sum_{p=1}^{P} w_{ip} = 1, \qquad \sum_{i=1}^{M}\sum_{p=1}^{P} s_{ij} x_{ijp}$$

becomes

$$\sum_{i=1}^{M}\sum_{p=1}^{P} s_{ij} u_{ij} w_{ip} = \sum_{i=1}^{M} s_{ij} u_{ij} \sum_{p=1}^{P} w_{ip} = \sum_{i=1}^{M} s_{ij} u_{ij} \leq S_j, \qquad \text{and} \qquad \sum_{i=1}^{M}\sum_{j=1}^{N} t_{ip} x_{ijp}$$

becomes

$$\sum_{i=1}^{M}\sum_{j=1}^{N} t_{ip} u_{ij} w_{ip} = \sum_{i=1}^{M} t_{ip} w_{ip} \sum_{j=1}^{N} u_{ij} = \sum_{i=1}^{M} t_{ip} w_{ip} \leq T_p.$$

Therefore, the model of (5-11) can also be derived from the formulation (5-12).

The level-1 RLT formulation of the GQ3AP

The subsequent step is to transform the unique model of the GQ3AP into an equivalent linear zed mixed-integer programming version G3RLT, much like those effectively used on Q3AP and GQAP. 1st, multiply each of equality constraints (5-12b) and inequality constraints (5-12c)-(5-12d), written as $\sum_{n=1}^{N}\sum_{q=1}^{P} x_{knq} = 1$, $\sum_{k=1}^{M}\sum_{q=1}^{P} s_{kn} x_{knq} \leq S_n$ and $\sum_{k=1}^{M}\sum_{n=1}^{N} t_{kn} x_{knq} \leq T_q$, Through each of N^3 binary variables x_{ijp}. Append a lot of these new restrictions. Express the ensuing product inside the order $x_{ijp} x_{knq}$.Next, explicitly consist of the trivial regulations that $x_{ijp} x_{knq} = x_{knq} x_{ijp} \forall (i, j, p, ok, n, q), ok > i$. Then, linearize every prevalence of each product $x_{ijp} x_{knq}$ with a unmarried nonnegative non-stop variable $y_{ijpknq} = x_{ijp} x_{knq}$.

Finally, don't forget the answer shape of the GQ3AP, which shows that $y_{ijpinq} = 0 \, \forall (i, j, p, n, q; n \neq j$ or $q \neq p)$ and $y_{ijpijp} = x_{ijp} \, \forall (i, j, p)$. The degree-1 RLT formula G3RLT then outcomes in.

[G3RLT]

$$\min\quad \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P} b_{ij} x_{ijp} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{\substack{k=1 \\ k\neq i}}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P} c_{ijpknq} y_{ijpknq}$$ (5-13a)

$$\text{s.t.}\quad \sum_{n=1}^{N}\sum_{q=1}^{P} y_{ijpknq} = x_{ijp} \qquad i,k=1,...,M; j=1,...,N; p=1,...,P; k \neq i,$$ (5-13b)

$$\sum_{k=1}^{M}\sum_{q=1}^{P} s_{kn} y_{ijpknq} \leq S_n x_{ijp} \quad i=1,...,M; j,n=1,...,N; p=1,...,P,$$ (5-13c)

$$\sum_{k=1}^{M}\sum_{n=1}^{N} t_{kq} y_{ijpknq} \leq T_q x_{ijp} \quad i=1,...,M; j=1,...,N; p,q=1,...,P,$$ (5-13d)

$$y_{ijpknq} = y_{knqijp} \qquad i,k=1,...,M; j,n=1,...,N; p,q=1,...,P; k > i,$$ (5-13e)

$$\sum_{j=1}^{N}\sum_{p=1}^{P} x_{ijp} = 1 \qquad i=1,...,M,$$ (5-13f)

$$\sum_{i=1}^{M}\sum_{p=1}^{P} s_{ij} x_{ijp} \leq S_j \qquad j=1,...,N,$$ (5-13g)

$$\sum_{i=1}^{M}\sum_{j=1}^{N} t_{ip} x_{ijp} \leq T_p \qquad p=1,...,P,$$ (5-13h)

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$$y_{ijpknq} \geq 0 \qquad i,k=1,...,M; j,n=1,...,N; p,q=1,...,P; k \neq i ,(5\text{-}13\text{i})$$

$$x_{ijp} \in \{0,1\} \qquad i=1,...,M; j=1,...,N; p=1,...,P. \qquad (5\text{-}13\text{j})$$

Problem GQ3AP and G3RLT are equivalent within the following sense. Given any possible solution x to the GQ3AP, there exists a y such that (x, y) is feasible to the G3RLT with the identical objective cost. Conversely, for any feasible solution (x , y) to the G3RLT, the corresponding x is viable to the GQ3AP with the identical objective fee.

Proof.

For a given x pleasurable the constraints of the GQ3AP given in (5-12), compute yijpknq = xijp xknq ∀(i , j , p , k , n, q). It is trivially real that (x, y) is a viable approach to the G3RLT and the value of goal capabilities of GQ3AP and G3RLT matches as long as (five-12b)-(5-12e) are a part of the G3RLT. In order to prove the alternative course of equivalency, it is vital to expose that a feasible answer (x , y) to the G3RLT satisfies y_{ijpknq} = xijp xknq ∀(i , j , p , ok , n, q), ok ≠ i ,which guarantees that the goal values of GQ3AP and G3RLT are same. In truth, it is enough to reveal that if (x, y) is feasible to the G3RLT then y_{ijpknq} is 0 except x_{ijp} and x_{knq} are both same to at least one wherein case y_{ijpknq} is likewise 1.

If x_{ijp} = 0 for given i , j and p , then (5-13b)-(5-13d) and (5-13i) collectively mean that y_{ijphst} = 0 ∀ (h, s , t), h ≠ i .

If x_{knq} = 0 for given k , n and q , then (5-13b)-(5-13e) and (5-13i) together imply that y_{luwknq} = 0 ∀ (l , u , w),l ≠ okay .

Therefore, if x_{ijp} = x_{knq} = 0 for given i , j , p , okay ≠ i , n and q , then y_{ijpknq} = 0 .

Besides, if x_{ijp} = x_{knq} = 1 for given i , j , p , k ≠ i , n and q , then it must be proven that y_{ijpknq}= 1 .

$$\sum_{u=1}^{N} \sum_{w=1}^{P} x_{iuw} = 1 \; \forall (i)$$

From (5-13f) which implies that if x_{ijp} = 1, then

$$x_{iuw} = 0 \; \forall (u,w), u \neq j, w \neq p ; \qquad (5\text{-}14\text{a})$$

$$x_{ijw} = 0 \; \forall (w), w \neq p ; \qquad (5\text{-}14\text{b})$$

and $\quad x_{iup} = 0 \; \forall (u), u \neq j . \qquad (5\text{-}14\text{c})$

Thus, similarly as above, (5-13b)-(5-13d) and (5-13i) together imply that

$$y_{iuwhst} = 0 \; \forall (u,w,h,s,t), u \neq j, w \neq p, h \neq i ; \qquad (5\text{-}15\text{a})$$

$$y_{ijwhst} = 0 \; \forall (w,h,s,t), w \neq p, h \neq i ; \qquad (5\text{-}15\text{b})$$

and $\quad y_{iuphst} = 0 \; \forall (u,h,s,t), u \neq j, h \neq i . \qquad (5\text{-}15\text{c})$

So, $\quad y_{iuwknq} = 0 \; \forall (u,w), u \neq j, w \neq p, k \neq i ; \qquad (5\text{-}16\text{a})$

$$y_{ijwknq} = 0 \; \forall (w), w \neq p, k \neq i ; \qquad (5\text{-}16\text{b})$$

and $\quad y_{iupknq} = 0 \; \forall (u), u \neq j, k \neq i . \qquad (5\text{-}16\text{c})$

By (5-13b) and (5-13e), it turns out that,

$$\sum_{u=1}^{N} \sum_{w=1}^{P} y_{iuwknq} = x_{knq} \; \forall (i), i \neq k . \qquad (5\text{-}17)$$

Therefore,

$$\sum_{u=1}^{N} \sum_{w=1}^{P} y_{iuwknq} = \sum_{\substack{u=1 \\ u \neq j}}^{N} \sum_{\substack{w=1 \\ w \neq p}}^{P} y_{iuwknq} + \sum_{\substack{w=1 \\ w \neq p}}^{P} y_{ijwknq} + \sum_{\substack{u=1 \\ u \neq j}}^{N} y_{iupknq} + y_{ijpknq} = y_{ijpknq} = x_{knq} ,$$

$$(5\text{-}18)$$

Which means that if x_{knq} = 1 then y_{ijpknq} = 1 . This completes the proof.↑

The mathematical shape of G3RLT (5-13) can be comfortably exploited through Lagrangian duality. An vital step of establishing the Lagrangian dual of a combinatorial optimization problem is to partition the restrictions into units: the restrictions to be dualized into goal characteristic using suitable Lagrangian multipliers and the ultimate constraints which limit the incredibly smooth sub problems. The improvement of the Lagrangian twin for the GQ3AP is included in Section 5.4.

Identifying the GQ3AP solution matrix and cost matrix-

Now keep in mind the M 2 × N 2 × P2 matrix Y as a Kronecker made of the 3-dimensional M × N × P mission solution matrix X with itself, in view that mathematically the Kronecker product is an operation on two matrices of arbitrary length ensuing in a block matrix. That is,

$$Y = X \otimes X$$

$$= \left\{ \begin{bmatrix} x_{111}X & \cdots & x_{11P}X \\ \vdots & \ddots & \vdots \\ x_{1N1}X & \cdots & x_{1NP}X \end{bmatrix}_{i=1} \right.$$
$$\left. \begin{bmatrix} x_{M11}X & \cdots & x_{M1P}X \\ \vdots & \ddots & \vdots \\ x_{MN1}X & \cdots & x_{MNP}X \end{bmatrix}_{i=M} \right\}$$

$$= \left[y_{ijpknq} \right]_{M^2 \times N^2 \times P^2} = \left[y_{[ijp]} \right]_{M \times N \times P}$$

$$= \left\{ \begin{bmatrix} Y_{[111]} & \cdots & Y_{[11P]} \\ \vdots & \ddots & \vdots \\ Y_{[1N1]} & \cdots & Y_{[1NP]} \end{bmatrix}_{i=1} \right.$$
$$\left. \begin{bmatrix} Y_{[M11]} & \cdots & Y_{[M1P]} \\ \vdots & \ddots & \vdots \\ Y_{[MN1]} & \cdots & Y_{[MNP]} \end{bmatrix}_{i=M} \right\} \qquad (5\text{-}19\text{a})$$

where $\quad y_{ijpknq} = x_{ijp}x_{knq} = x_{knq}x_{ijp} = y_{knqijp}$

$$\forall (i,k=1,...,M; j,n=1,...,N; p,q=1,...,P), k \neq i , \qquad (5\text{-}19\text{b})$$

$$y_{ijpijp} = x_{ijp} \qquad \forall (i=1,...,M; j=1,...,N; p=1,...,P), \qquad (5\text{-}19\text{c})$$

$$y_{ijpinq} = 0 \qquad \forall (i=1,...,M; j,n=1,...,N; p,q=1,...,P), n \neq j \text{ or } q \neq p . \qquad (5\text{-}19\text{d})$$

As indicated in (5-19a), every sub matrix Y[ijp] of Y is an M × N × P 3-dimensional matrix. Here, (5-19b)-(5-19d) are indicated by using the answer structure X of the GQ3AP and the definition of Y . Figure 2 presents an instance of Y matrix and its corresponding X matrix for the GQ3AP of M = 4 , N =3 and P = 2 . The matrix Y is a partitioned matrix

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whose factors are composed of the Null matrix (all elements being 0) and matrix X , and furthermore Y famous a gross sample equal to that of the matrix X.
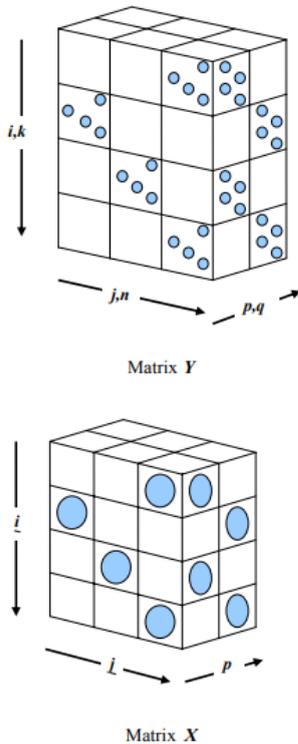

Matrix **Y**

Matrix **X**

Figure 2. Example of GQ3AP solution matrix *Y* and its *X* matrix.

Equation (5-19b) gives the complementary pairs inside the GQ3AP answer matrix Y , which may be very vital to the effectiveness of the dual-ascent manner for the GQ3AP lower bound calculation. It says that if an element y_{ijpknq} is worried in an venture (i.E., equal to at least one) then it has a complementary detail y_{knqijp} that is also equal to 1. These complementary pairs have an interesting belongings; they're continually in specific three-dimensional sub matrices that by no means occupy the equal ok − aircraft, here the k − aircraft is the plane along (n, q) in the sub matrix Y[ijp]. The equality among the pair creates a precious communication between sub matrices that can be exploited in calculating the GQ3AP lower bounds.

Certain elements of Y as indicated via (5-19d) are continually 0. These factors are referred to as "disallowed factors". Elements defined in (5-19c) are termed "linear-fee elements". Observe that if there are any 1's in a 3-dimensional submatrix Y[ijp] its linear-value detail is continually unity and vice versa. Observe, too, that one andonly one solidarity detail may additionally exist in every i − plane, which is the plane alongside (j, p) in matrix Y. These constraints follow directly from the definition of Y as the manufactured from 3-dimensional venture matrices. A linear-value is unique in another way; it has no complementary element due to the fact $y_{ijpijp} = x_{ijp} x_{ijp}$.

Now, suppose one arranges the M 2 × N 2 × P2 nonnegative value coefficient c_{ijpknq} (i , ok = 1,..., M ; j , n = 1,..., N ; p , q = 1,..., P) in an M 2 × N 2 × P2 matrix C , much like matrix Y and listed in exactly the identical way. That is the following.

$$
C = \left[c_{ijpknq} \right]_{M^2 \times N^2 \times P^2} = \left[C_{[ijp]} \right]_{M \times N \times P}
$$

$$
= \left\{
\begin{array}{c}
\left[\begin{array}{ccc} C_{[111]} & \cdots & C_{[11P]} \\ \vdots & \ddots & \vdots \\ C_{[1N1]} & \cdots & C_{[1NP]} \end{array} \right]_{i=1} \\
\vdots \\
\left[\begin{array}{ccc} C_{[M11]} & \cdots & C_{[M1P]} \\ \vdots & \ddots & \vdots \\ C_{[MN1]} & \cdots & C_{[MNP]} \end{array} \right]_{i=M}
\end{array}
\right. \tag{5-20}
$$

Since Y contains M × N × P copies of 3-dimensioal project answer matrix X , it is straightforward to see that the three-dimensional sub matrices C[ijp] of C would ought to each meet the restrictions on X . Thus, a 3-dimensional submatrix of C can be idea of as a generalized 3-dimensional undertaking sub problem of the GQ3AP, much like the idea of a submatrix of the QAP price matrix may be dealt with as a LAP (Hahn and Grant, 1998). The generalized 3-dimensional challenge hassle (G3AP) is an project problem over the identical constraints (5-12b)-(5-12e) on X with simplest linear fee taken into consideration inside the goal function.

The life of complementary pairs opens the door to drastically flexibility in the C matrix. If one detail of a complementary pair is worried in an venture, so is the alternative. Therefore, for every complementary pair, price can be shifted at will among the two elements. For instance, the fee of each elements may be brought and placed at the 1st element whilst the fee of the second becomes zero or vice versa.

It may be shown that positive operations can be executed on matrix $C = \left[c_{ijpknq} \right]$ With a purpose to change the fee Z (X) of task X in such a manner that everyone task expenses are shifted through an same quantity, for that reason keeping their order with admire to fee. These operations are divided into two training (see Grant (1989) for proofs):

Class 1: Subtraction of a regular from all non-disallowed elements from a okay − plane of a three-dimensional sub matrix C[ijp] and the corresponding addition of this constant to both every other okay − plane of the sub matrix or to the linear cost element bijp of the sub matrix. The term "disallowed" turned into defined in (5-19d).

Class 2: Addition or subtraction of a constant to all non-disallowed elements of any i − aircraft in matrix C.

Class 1 operations maintain the fee of all assignments, however allow redistribution of element costs within a given 3-dimensional

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submatrix. This follows because any submatrix worried in a three-dimensional project should itself have the form of a 3-dimensional undertaking matrix. Consequently, the transfer of fee from one submatrix aircraft to any other will keep the price of all assignments that skip via the 3-dimensional submatrix. Furthermore, because the linear price of a concerned 3-dimensional submatrix should also be concerned inside the 3-dimensional assignment, the switch of price from a submatrix plane to the linear fee, and vice versa, may also go away the fee of all assignments that intersect the 3-dimensional sub matrix unchanged.

In assessment, Class 2 operations paintings on the M 2 × N 2 × P2 matrix level, and change the value of ALL 3-dimensional assignments by the amount introduced to or subtracted from the i − aircraft of the matrix. This is proper due to the fact one and handiest one fee element in an i − aircraft of C can be worried within the normal fee of a 3-dimensional venture.

A DUAL-ASCENT PROCEDURE LOWER BOUND CALCULATION

Linear programming considerations-

In order to use the Lagrangian dual technique to clear up the G3RLT, it is beneficial first to acquire a smaller version via the substitution of variables based on constraints (5-13e), thereby reducing the wide variety of variables and constraints, similarly to halving the variety of non-negativity regulations in (5-13i). Then, one makes use of the truth that $xijp2 = xijp$ and replace $xijp$ by y_{ijpijp}, so the y_{ijpijp}'s turn out to be 0-1 variables inside the new model. Finally, it's miles important to replace the binary restrictions (5-13j) on $xijp = y_{ijpijp}$ via $0 \le xijp = y_{ijpijp} \le 1$, in order that opposite to G3RLT being a blended-integer programming hassle, the new G3RLT1 is a non-stop optimization problem. One obtains the subsequent version.

[G3RLT1]

$$\min \ \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P} b_{ijp} y_{ijpijp} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{\substack{k=1\\k>i}}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P} \tilde{c}_{ijpknq} y_{ijpknq} \qquad (5\text{-}21\text{a})$$

where $\tilde{c}_{ijpknq} = c_{ijpknq} + c_{knqijp}$

s.t. $\quad \sum_{n=1}^{N}\sum_{q=1}^{P} y_{ijpknq} = y_{ijpijp} \qquad i,k=1,...,M; j=1,...,N; p=1,...,P; k>i, \qquad (5\text{-}21\text{b})$

$\sum_{n=1}^{N}\sum_{q=1}^{P} y_{knqijp} = y_{ijpijp} \qquad i,k=1,...,M; j=1,...,N; p=1,...,P; k<i, \qquad (5\text{-}21\text{c})$

$\sum_{\substack{k=1\\k>i}}^{M}\sum_{q=1}^{P} s_{kn} y_{ijpknq} + \sum_{\substack{k=1\\k<i}}^{M}\sum_{q=1}^{P} s_{kn} y_{knqijp} \le S_n$

$\qquad\qquad\qquad i=1,...,M; j,n=1,...,N; p=1,...,P, \qquad (5\text{-}21\text{d})$

$$\sum_{\substack{k=1\\k>i}}^{M}\sum_{n=1}^{N} t_{kq} y_{ijpknq} + \sum_{\substack{k=1\\k<i}}^{M}\sum_{n=1}^{N} t_{kq} y_{knqijp} \le T_q$$

$\qquad\qquad i=1,...,M; j=1,...,N; p,q=1,...,P, \qquad (5\text{-}21\text{e})$

$\sum_{j=1}^{N}\sum_{p=1}^{P} y_{ijpijp} = 1 \qquad i=1,...,M, \qquad (5\text{-}21\text{f})$

$\sum_{i=1}^{M}\sum_{p=1}^{P} s_{ij} y_{ijpijp} \le S_j \qquad j=1,...,N, \qquad (5\text{-}21\text{g})$

$\sum_{i=1}^{M}\sum_{j=1}^{N} t_{ip} y_{ijpijp} \le T_p \qquad p=1,...,P, \qquad (5\text{-}21\text{h})$

$0 \le y_{ijpknq} \le 1 \qquad i,k=1,...,M; j,n=1,...,N; p,q=1,...,P; k>i, (5\text{-}21\text{i})$

$0 \le y_{ijpijp} \le 1 \qquad i=1,...,M; j=1,...,N; p=1,...,P. \qquad (5\text{-}21\text{j})$

Notice that a weakened form is used in each equations (5-21d) and (5-21e), due to the fact $xijp = y_{ijpijp} \le 1$, as can be visible from the right-hand-side (RHS) of equations (5-13c) and (5-13d).

The reduction in the number of variables isn't always executed just for the reason of decreasing computational effort. Tied with the gain received from the complementary pairs, you'll be able to shift fee at will among the two factors. It is critical to achieving a whole lot tighter decrease bounds than might be reached without this considerably critical step. The earlier successful stories with developing tight lower bounds for the QAP (Hahn and Grant, 1998), GQAP (Hahn et al., 2006a), and Q3AP (Hahn et al., 2006b) ensure of the effectiveness and efficacy of this system.

Now consider the following Lagrangian relaxation trouble LR (ρ,ρ') at the G3RLT1, wherein constraints (5-21b) and (5-21c) are placed into the objective function the use of multipliers ρ and ρ' respectively.

$\left[\text{LR}(\rho,\rho') \right]$

minimize:

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P} b_{ijp} y_{ijpijp} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{\substack{k=1\\k>i}}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P} \tilde{c}_{ijpknq} y_{ijpknq}$$

$$+ \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{\substack{k=1\\k>i}}^{M} \rho_{ijpk}\left(y_{ijpijp} - \sum_{n=1}^{N}\sum_{q=1}^{P} y_{ijpknq} \right) + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{\substack{k=1\\k<i}}^{M} \rho'_{ijpk}\left(y_{ijpijp} - \sum_{n=1}^{N}\sum_{q=1}^{P} y_{knqijp} \right)$$

$$(5\text{-}22)$$

s.t.(5-21d)-(5-21j).

Here, the objective feature of Lagrangian relaxation LR (ρ,ρ') can be reorganized as

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$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\tilde{b}_{ijp}y_{ijpijp} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{\substack{k=1\\k>i}}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P}\bar{c}_{ijpknq}y_{ijpknq}, \qquad (5\text{-}23a)$$

where

$$\tilde{b}_{ijp} = b_{ijp} + \sum_{\substack{k=1\\k>i}}^{M}\rho_{ijpk} + \sum_{\substack{k=1\\k<i}}^{M}\rho'_{ijpk} \quad i=1,...,M; j=1,...,N; p=1,...,P, \qquad (5\text{-}23b)$$

and

$$\bar{c}_{ijpknq} = \tilde{c}_{ijpknq} - \rho_{ijpk} - \rho'_{knqi} \quad i,k=1,...,M; j,n=1,...,N; p,q=1,...,P; k>i. (5\text{-}23c)$$

Therefore, the Lagrangian dual problem LD1 is

[LD1] maximize $\theta(\boldsymbol{\rho}, \boldsymbol{\rho'})$ where

$$\theta(\boldsymbol{\rho}, \boldsymbol{\rho'}) = \min_{y}\left\{\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\tilde{b}_{ijp}y_{ijpijp} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{\substack{k=1\\k>i}}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P}\bar{c}_{ijpknq}y_{ijpknq}\right\} \qquad (5\text{-}24)$$

s.t.(5-21d)-(5-21j).

Then, remember a similarly Lagrangian rest hassle LR (ρ,ρ ',γ) , whereby constraints (5-21f) also are positioned into the objective feature the usage of multipliers **γ** .

minimize:

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\tilde{b}_{ijp}y_{ijpijp} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{\substack{k=1\\k>i}}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P}\bar{c}_{ijpknq}y_{ijpknq} + \sum_{i=1}^{M}\gamma_i\left(1-\sum_{j=1}^{N}\sum_{p=1}^{P}x_{ijp}\right) \qquad (5\text{-}25)$$

Here, the objective feature of Lagrangian rest LR (ρ,ρ ',γ) may be simplified as

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\bar{b}_{ijp}y_{ijpijp} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{\substack{k=1\\k>i}}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P}\bar{c}_{ijpknq}y_{ijpknq} + \sum_{i=1}^{M}\gamma_i, \qquad (5\text{-}26a)$$

Where

$$\bar{b}_{ijp} = \tilde{b}_{ijp} - \gamma_i = b_{ijp} + \sum_{\substack{k=1\\k>i}}^{M}\rho_{ijpk} + \sum_{\substack{k=1\\k<i}}^{M}\rho'_{ijpk} - \gamma_i$$

$$i=1,...,M; j=1,...,N; p=1,...,P. \qquad (5\text{-}26b)$$

Finally, the Lagrangian dual problem LD2 is as shown

[LD2] maximize $\theta(\boldsymbol{\rho}, \boldsymbol{\rho'},\boldsymbol{\gamma})$ where

$$\theta(\boldsymbol{\rho},\boldsymbol{\rho'},\boldsymbol{\gamma}) = \min_{y}\left\{\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\bar{b}_{ijp}y_{ijpijp} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\sum_{\substack{k=1\\k>i}}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P}\bar{c}_{ijpknq}y_{ijpknq}\right\} + \sum_{i=1}^{M}\gamma_i \quad (5\text{-}27)$$

s.t. (5-21d), (5-21e), (5-21g)-(5-21j).

LD2 quantities to solving the following Lagrangian subproblems for given multipliers ρ, ρ' and **γ.**

$$\sum_{i=1}^{M}\gamma_i + \min\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{P}\left\{\begin{array}{l}\left(b_{ijp} + \sum_{\substack{k=1\\k>i}}^{M}\rho_{ijpk} + \sum_{\substack{k=1\\k<i}}^{M}\rho'_{ijpk} - \gamma_i\right)y_{ijpijp} \\ \\ + \sum_{\substack{k=1\\k>i}}^{M}\sum_{n=1}^{N}\sum_{q=1}^{P}\left\{\begin{array}{l}(\tilde{c}_{ijpknq} - \rho_{ijpk} - \rho'_{knqi})y_{ijpknq} \\ s.t. \\ \sum_{k=1}^{M}\sum_{q=1}^{P}s_{kn}y_{ijpknq} + \sum_{\substack{k=1\\k<i}}^{M}\sum_{q=1}^{P}s_{kn}y_{knqijp} \le S_n, (\forall n) \\ \sum_{k=1}^{M}\sum_{n=1}^{N}t_{kq}y_{ijpknq} + \sum_{\substack{k=1\\k<i}}^{M}\sum_{n=1}^{N}t_{kq}y_{knqijp} \le T_q, (\forall q) \\ 0 \le y_{ijpknq} \le 1, (\forall k,n,q) \\ k=1,...,M; n=1,...,N; q=1,...,P; k>i \end{array}\right\} \\ s.t. \\ \sum_{i=1}^{M}\sum_{p=1}^{P}s_{ij}y_{ijpijp} \le S_j, (\forall j) \\ \sum_{i=1}^{M}\sum_{j=1}^{N}t_{ip}y_{ijpijp} \le T_p, (\forall p) \\ 0 \le y_{ijpijp} \le 1, (\forall i,j,p) \\ i=1,...,M; j=1,...,N; p=1,...,P \end{array}\right\}$$

$$(5\text{-}28)$$

Observe first that during formula (5-28), if the multipliers ρ and ρ' are high quality portions, then these quantities are subtracted from okay − planes within the 3-dimensional modified fee sub matrices whose factors turn out to be $c_{ijpknq} = (c_{ijpknq} - \rho_{ijpk} - \rho_{knqi'})$ and introduced to the corresponding linear cost detail b_{ijp} , which might be without difficulty identified to be Class 1 operations described in advance. Observe subsequent that, if the multipliers γ are also high quality quantities, then those amounts are subtracted from i-planes within the three-dimensional matrix of changed linear fee $\bar{b}_{ijp} = \left(b_{ijp} + \sum_{\substack{k=1\\k>i}}^{M}\rho_{ijpk} + \sum_{\substack{k=1\\k<i}}^{M}\rho'_{ijpk}\right)$ and added to the price of θ (ρ , ρ ,γ) , which might be diagnosed to be Class 2 operations. Therefore, the operations implied by LD2 constituting both Class 1 and Class 2 operations to obtain the level-1 RLT dual-ascent process certain.

If operations on C decrease the price through an amount Z ' and are executed in a manner that continues the elements of C nonnegative, then no task cost can become bad and the subsequent relationship holds.

$$Z' \le \min_{X} Z(X) \qquad (5\text{-}29)$$

Thus the twin-ascent procedure bound calculation for the GQ3AP may be addressed as maximizing the sum of downward cost shifts Z ' authorized with the aid of Class 1 and Class 2 operations below the restrictions that no fee detail in C is pushed poor.

The level-1 RLT dual-ascent procedure-

In this subsection, I describe the extent-1 RLT twin-ascent manner for the GQ3AP programmed with the aid of Professor Hahn.

The stage-1 RLT twin-ascent procedure set of rules is based totally on the concept that regular amounts are subtracted from the k − planes of every and each 3-dimensional changed cost

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coefficient submatrix $\left[\tilde{c}_{ijpknq}\right]$ And brought to the corresponding linear value element bijp of the submatrix (Class 1 operations). Then, consistent quantities are subtracted from the i-planes of the modified linear value matrix $\left[\tilde{b}_{ijp}\right]$ And introduced to θ (ρ, ρ ',γ) ,which contains a decrease bound value (Class 2 operations). Here are the steps,

1. For each of the M × N × P 3-dimentsional modified cost submatrices, first accumulate into the submatrix those expenses similar to complementary variables y_{ijpknq} and y_{knqijp} . Then, inside the submatrix, subtract the minimal changed fee from each k-aircraft and add it to the corresponding linear fee detail. Adding complementary costs first assures the biggest viable transfer of quadratic to linear prices.

2. After Step 1 has been completed for all the submatrices, subtract the minimum value from each i-aircraft of the linear price matrix and upload it to a discount steady which constitutes a decrease certain to the problem. If the ensuing sample of zeros satisfies the problem constraints, the procedure terminates with an best option to the GQ3AP. If now not, maintain to Step three. Stop if the certain boom is terrible.

3. Scan the matrix of changed linear fees for positive factors. Record places of the non-fine (i.E., 0) linear fees as this records may be wished in Step four. Divide every advantageous linear cost detail into M −1 approximately equal parts and add each element to a unique (non-linear-price) ok-aircraft of its submatrix. Positive linear charges are divided into M −1 approximately identical parts as follows. First of all, it need to be emphasised that it is vital that spherical-off mistakes should in no way be allowed in manipulating the C matrix. Thus, the C matrix within the set of rules is continually integer. So, to divide the linear cost into M −1 almost identical components, the process is to divide by using M −1 and spherical that right down to the closest integer. That presents M − 2 equal components. Then those parts are delivered up and subtracted from the authentic linear price to calculate the last component. The motivation here is that by changing expenses into submatrices, there's a brand new opportunity to make those linear charges which had low or 0 values after Step 2 large, allowing even greater price to be sooner or later moved to tsthe decrease certain. Step 3 leaves the cost matrix C dramatically rearranged. Because of the rearrangement, the method may be repeated, i.E., extra prices may be

subtracted from within submatrices and moved to the linear fee detail. The result is an iterative manner that produces increase within the decrease sure with each round. This increase, while large, diminishes with each round, so at some point it does not pay to maintain the manner.

4. Repeat Step1, besides this time make certain to do the collection of complementary charges and subtractions first for those submatrices whose linear costs have been zero prior to Step three. Go to Step 2.

The gathering of complementary charges prior to solving each submatrix is defined by means of the substitution of variables that eliminated constraints (5-13e) in G3RLT. Step 1 and Step four are completely explained with the aid of method LD1. Since Step three consists only of Class 1 operations, it's also explained. Step 2 is clarified by method LD2 which constitutes each Class 1 and Class 2 operations, as are all 4 steps of the extent-1 twin-ascent technique.

Adding the resource constraints-

Equality constraints (5-21b), (5-21c) and (5-21f) are dualized and dealt with as it should be within the creation of the extent-1 RLT dual-ascent manner. The useful resource constraints (5-21d)-(5-21e), and (5-21g)-(5-21h) need to be enforced in other methods. For the most component, it's far completed by means of calculating dual-ascent method lower bounds on most effective those assignments that fulfill the 4 units of resource constraints. However, an resourceful technique is implemented that improves the twin-ascent system sure price, through taking into account the resource constraints. Constraints (5-21d)-(5-21e) are enforced throughout the extent-1 RLT lower sure calculation via raising to a very high value price those submatrix elements whose choice would violate these sets of constraints. Thus, it's far feasible to enhance the lower bound beyond that to be had if only the equality constraints (5-21b) and (5-21c) are enforced.

Consider the Class 1 subtraction operations defined before. After collecting complementary charges in a given submatrix C[ijp] , minimal fees are subtracted from submatrix okay-planes and added to the corresponding linear fee element bijp .Prior to these subtractions, an crucial step has been introduced. Each detail in the n-plane containing the linear value detail is examined, right here the n-aircraft is the plane alongside (k , q) in the sub matrix Y[ijp]. If which includes this detail in an answer would render infeasible answer pattern concerning constraints (5-21d) for this sub matrix, then the cost for this detail is raised to an arbitrarily big number referred to as GREAT and dictated only by using the integer length boundaries of the

computer. The identical technique is accomplished in each q-aircraft for analyzing the infeasibility posed by using constraints (5-21e), even as the q-plane is the plane alongside (ok , n) inside the sub matrix Y[ijp] .

Similar operations also are performed within the linear value matrix that allows you to put into effect the useful resource constraints (5-21g) and (5-21h). This method correctly bars infeasible elements from inclusion in a stage-1 RLT twin-ascent system answer, and focuses the discount efforts on the ones elements still eligible for consideration. The ensuing subtractions permit extra value to be extracted from each and each submatrix, leading to a much-stepped forward decrease certain.

Advantages of the level-1 RLT dual-ascent procedure-

The degree-1 RLT twin-ascent procedure has a number of treasured residences that makes it perfect to be used in a branch-and-certain set of rules for fixing the GQ3AP.

1.   The level-1 RLT dual-ascent method is still among competing decrease bounding strategies for the QAP, as validated from experimental consequences. Moreover, it's miles the only method that is able to resolve the GQAP of length 20 ×15 , see Hahn et al. (2006a).

2.   Similar to some of different decrease bounding techniques, the level-1 RLT dual-ascent process generates a series of non-reducing decrease bounds for the GQ3AP. More importantly, with every such lower sure, it modifies the GQ3AP objective characteristic in order that a new GQ3AP is generated whose viable answer set is equal to that of the authentic and whose goal function values are merely lessened by the quantity of the lower bound.

3.   The twin-ascent process for a given partial venture may be stopped as soon as the decrease sure on the assumed partial venture exceeds an top sure at the authentic hassle. It also can be stopped, in favor of making an additional partial venture, whilst it will become obvious that from its sluggish development it's miles not likely to ever attain the higher bound.

4.   In a department-and-certain set of rules, fathoming selections can without problems be recorded by means of arbitrarily growing the fees of these elements within the fee matrix that correspond to partial assignments that have been removed as possibly being superior. This modifies the GQ3AP, however is assured to absolutely enumerate all feasible solutions of the authentic hassle.

The level-1 RLT dual-ascent procedure in branch-and-bound-

The degree-1 RLT twin-ascent method is utilized inside a department-and-bound algorithm because the auxiliary method for computing decreases bounds. This method is just like the department-and-certain algorithm for the GQAP by Hahn et al. (2006a) for the reason that GQ3AP trees seek need to be confined to handiest the ones assignments that meet the resource constraints.

Branching follows the conventional approach of selecting a single 3-dimensional $i − j − p$ undertaking at the 1st (maximum) stage, as well as at subsequent ranges of partial task. In order to put into effect this option, a linear cost is selected to be worried inside the 3-dimensional challenge. Based on the selection of linear value b_{ijp} , the 3-dimensional sub matrix C[ijp] is worried inside the assignment. The remaining sub matrices of cost matrix C whose i-plane containing sub matrix C[ijp] disappear (as they cannot be involved inside the venture) and the hassle is therefore decreased to a GQ3AP of size $M − 1)× N × P$. It turns out that one ok-plane likewise disappears from each submatrix of the remainders in the fee matrix C , which make all of the submatrices to be$(M − 1)× N × P$ in length as well.

It is the utility of the extent-1 RLT twin-ascent manner lower bounding calculation on the $(M − 1)× N × P$ size trouble that tries to fathom a partial venture postulated by the choice of linear value b_{ijp} . By fathoming, one calculates a decrease certain and assessments it towards the first-rate-known top bound. If the quality-recognised upper bound is surpassed, the partial project is eliminated from the problem.

Recall in the previous sections, the dual-ascent technique movements charges out of the matrix right into a decrease certain value, leaving a changed matrix $C′$. For next branch-and-sure operations alongside a given partial mission course, the method is to take advantage of this reality and to apply this reduced price matrix $C′$ for putting in subsequent subproblems deeper into the tree. Thus, lower bounds are calculated no longer from the authentic hassle, however from the sub problems that the level-1 RLT dual-ascent method already processed at in advance (higher) levels of partial undertaking. Using the changed matrix $C′$ of every of these sub problems has the additional benefit that the sub problems is added in the direction of twin answer, making it much more likely that better possible solutions can be observed or that decrease bounds will exceed upper bounds, consequently reducing off a branch. The preference of the variety of dual iterations at a given partial project is a dynamic selection, based totally at the development of the lower bound performed after a set number of iterations. If after a small variety of iterations on the contemporary

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partial task, sufficient development is not reached, the lower bounding try stops and the set of rules proceeds to make an additional task.

Tree search is depth-first and based on single $i - j - p$ undertaking. The order wherein i assignments are made may be very crucial and has profound influence on branch-and-bound runtime. The branching strategy follows the same strategies as those advanced by way of Hahn et al. (2001) for the QAP. Levels within the seek tree are described by way of what number of partial assignments are made. The root is wherein no partial assignments were made and is considered stage 0. A single partial assignment is taken into consideration level 1, and so on. Since the tree seek includes one-to-one-to-one 3-dimensional assignments, at each level of the tree it's far essential simplest to look at a given i and its assignments to all possible (i.E., feasible) j 's and p 's.

In the GQ3AP, more than one i can be are living at a given j or p . Therefore, one ought to exhaust all feasible j 's and p 's at a given stage, earlier than you can decide if a higher solution exists or if a branch of the tree at that level can be reduce off. The exception to this rule is that no in addition consideration need be given to evaluating assignments if the aid constraints notify that the locations can receive no further assignments.

The search approach for deciding on the next node is a simple intensity-first method. If fathoming a given node is unsuccessful, a further partial project is made, therefore growing the intensity into the tree. If a node has been fathomed correctly, the following to be had node is chosen. Partial task at a given level are decided on consistent with a fixed of look-ahead bounding calculations that essentially determine the problem of eliminating the department containing that assignment. Assignments selected in the order of decreasing trouble are made while it's far desired to find appealing viable answers to update the top sure, early in the seek.

When the algorithm accumulates sufficient fathoming information, permanent decisions may be made at the undertaking matrix that certain elements of the assignment matrix $Y = \left[y_{ijpknq} \right]$ Are zero. These are recorded in the changed price matrix C 'by setting the corresponding element charges to be a very large value GREAT. This efficiently bars the "determined-0" elements from inclusion in a stage-1 RLT dual-ascent manner answer, and focuses the reduction efforts on the ones factors still eligible for consideration. The conversation of permanent selections commonly allows extra value to be extracted from the matrix, resulting in an advanced decrease bound.

COMPUTATIONAL EXPERIMENTS

The department-and-bound code for the GQ3AP become programmed in FORTRAN 77 by means of Professor Hahn, using the techniques advanced previously for the department-and-bound algorithms of the Q3AP and the GQAP. Here I gift the outcomes I done the use of this code for a set of MSAP hassle times with varying quantity of departments (M) , floors (N) , stairwells (P) , and evacuation population float traits. I am grateful to Prof. J. Mac Gregor Smith from University of Massachusetts Amherst for imparting the instance data for computational trying out.

Figure 3 illustrates the general arrangement of the stairwells. I even have various the variety of stories in those experiments from seven to eight and the range of departments from ten to thirteen and the number of stairwells from two to a few. Figure 4-four suggests an average facility segment configuration for the stairwells and stories and evacuation goal at the ground level. For the 1st instance, it's far assumed that the occupants of the seven-tale constructing incorporate ten unique departments and that there are two stairwells strategically positioned that may be accessed on every floor.
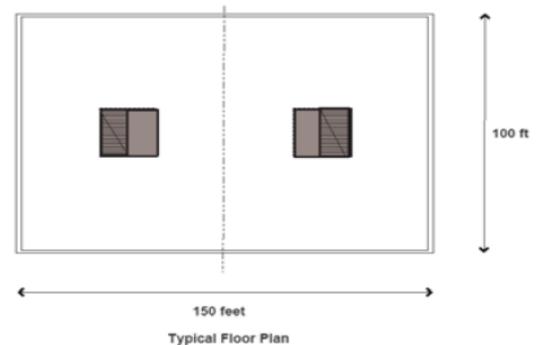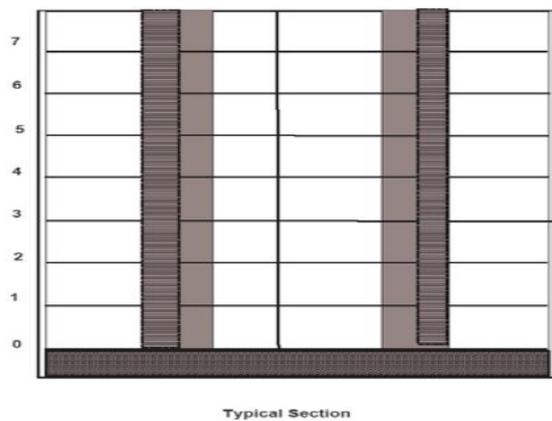

Figure 3. Multi-story plan.

Figure 4. Multi-story facility.

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A typical set of flows for the ten departments are given in the flow matrix F.

$$F = \begin{pmatrix} 0 & 10 & 0 & 3 & 0 & 5 & 4 & 5 & 5 & 2 \\ 0 & 0 & 5 & 4 & 5 & 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 1 & 0 & 3 & 4 & 7 & 4 & 7 \\ 0 & 0 & 0 & 0 & 4 & 6 & 10 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 5 & 3 & 8 & 7 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 & 8 & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In this first example, it is assumed that the following area requirements a for ij the ten departments.

$$[a_{ij}] = \begin{pmatrix} 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 \\ 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 \\ 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 \\ 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 \\ 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 \\ 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 \\ 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 \\ 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 \\ 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 \\ 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 & 7.5 \end{pmatrix}$$

And the following evacuation flows λip for the ten departments.

$$[\lambda_{ip}] = \begin{pmatrix} 100 & 100 \\ 50 & 50 \\ 40 & 40 \\ 120 & 120 \\ 100 & 100 \\ 65 & 65 \\ 70 & 70 \\ 150 & 150 \\ 50 & 50 \\ 70 & 70 \end{pmatrix}$$

To complete the restrictions for the 10-branch, seven-tale, two-stairwell instance, the ground area on every ground is chosen to be 15,000 squarest. Gadgets and every stairwell ability to be 1500 human beings/minute.

In the subsequent numerical experiments, the price of λ plays an essential function in is described as the appearance charge of persons within the evacuation the very last outcome. $\lambda = \lambda ip$ from branch i to stairwell p. Besides being the constrained parameter for the stairwell capability constraints (5-1d), λ within the goal feature (5-1a) is the size of ways crucial escape prices are as compared to ordinary inter-department travel costs. With large λip values, the MSAP design places greater weight at the

evacuation cost, which makes the runtime longer. I actually have selected 3 one of a kind orders of vector. The first choice is that λip are all among 40 and a hundred and fifty, magnitude of λ = λip resulting in a difficult problem. The 2nd choice is that λip are all among 4 and 15, ensuing in a medium-hard trouble and the 1/3 desire is that λip are all between 0.4 and 1.5, resulting in an clean trouble.

Table 1 offers the set of rules consequences for a small series of experiments. The 1st experiment in this desk refers to the seven-tale constructing, ten-branch and two-stairwell example defined above. The experiments in Table 1 were executed on a unmarried 900MHz CPU of a Sun Blade a thousand workstation or on a unmarried 1.2GHz CPU of a Sun Fire V880 server. Column one presents the wide variety of departments, range of floors and variety of stairwells. Column gives the ratio of the common magnitude of the λip ejp term within the goal function to the average value of the fik d jn term.

Column 3 shows the optimum goal function value of each instance. Column four reports the quantity of optima discovered in the branch-and-sure search. Column 5 offers the range of nodes (partial assignments) evaluated in the search. Column six lists the whole runtime in seconds, normalized to the velocity of the quicker V880 server and column 7 addresses the runtime in seconds to the foremost.

Table 1. Branch-and-bound algorithm results

M,N,P	OF Ratio	Optimum	# Optima	# Nodes	Runtime	Time to Opt.
10, 7, 2	22.4	2,582	408	964,134	730	19
13, 8, 2 easy	3.03	692	2	41,606,506	53,169	29,380
13, 8, 2 hard	30.32	2,788	1	125,090,900	168,100	33,431
13, 8, 3 easy	0.33	351.049	19,683	4,521,819	26,610	13,461
13, 8, 3 medium	3.29	630.5	46,902	32,607,159	170,592	82,317
13, 8, 3 hard	32.92	3,086	59,049	818,696,328	2,330,368	76,791

Figure 5 illustrates the performance of the set of rules as a function of time for decided on hassle times. It demonstrates that the algorithm encounters very good excellent possible answers early inside the branch-and-bound seek manner. Furthermore, the ultimate answer is in the end reached in a completely short quantity of time, for this reason indicating the capacity heuristic price of the department-and-bound set of rules for producing heuristic answers speedy. While a overall performance guarantee of this sort of heuristic isn't always available, it nevertheless represents a viable method to producing suitable solutions quick, because the cutoff-time period is small in assessment to the entire running time for the set of rules.
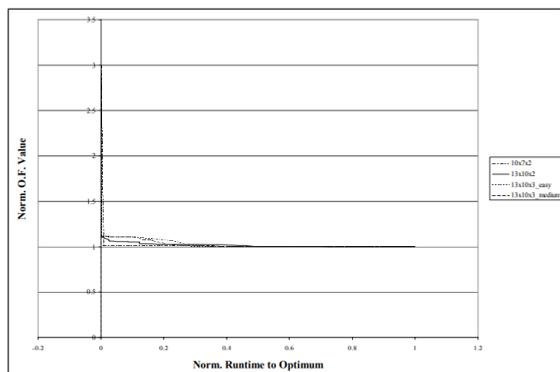
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Figure 5. Algorithm solution graph

CONCLUSION

This section examinations another characterized task issue, supposed the summed up quadratic 3-dimensional task issue (GQ3AP), which is the speculation of both the summed up quadratic task issue (GQAP) and the quadratic 3-dimensional task issue (Q3AP). This GQ3AP emerges in numerous applications, for example, multi-story departure plan and cross dock format structure. I portray a Lagrangian double for the GQ3AP dependent on a level-1 reformulation-linearization method (RLT) double rising system like those effectively utilized for the GQAP and Q3AP. My trial results demonstrate the branch-and-bound calculation installed with the double climb strategy has illuminated a few cases of multi-story space task issue (MSAP) just because.

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