Journal of Advances and Scholarly Researches in Allied Education
Vol. 16, Issue No. 4, March-2019, ISSN 2230-7540

Calculus of Infinitesimals and Ultra Real Numbers

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Abstract – Leibnitz proposed infinitely small differentiable i.e., infinitesimal of the first and second orders. He regarded his theory of infinitesimal as foundation for the theory of limits. eohen argued the existence of infinitesimal as the reciprocals of transfinite number. Archimedian property for real numbers system R, existence of infinitely small magnitude (infinitesimal) in comparison with other real number is not possible.

The case of Horn angle is the angle between a curve and its tangent which intersect at origin the proper measurement of horn angle we have to measure beyond the domain of real numbers. The magnitude of the angle C_1 OC_2 may be called ultra-real numbers.

Key Word – Infinitesimals, Horn Angle, Weight of Functions, Curvilinear Angle, Ultra Real Numbers

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1. INFINITESIMALS:

Infinitely great as also infinitely small numbers have caught the attention of the mathematicians as early as the 17th century. Leibnitz calculus was based on infinitely small differentials i.e., infinitesimals, of the first and second order. He used infinitesimals as ideal elements like imaginary numbers but held that they obey the ordinary laws of real numbers , infinitely small quantities are fundamental in Cauchy's approach to analysis also but there the infinitesimal are conceived not as numbers but as 'state of variables' whose limit is zero. He regarded his theory of infinitesimals as foundation for the theory of limits.

But the use of infinitesimals was not acceptable to the mathematicians like Newton, D'Alembert, Weierstrass, Berkley, Cantor etc.

Cantor (1887) founded the theory of infinite set, but claimed that infinitemely small numbers were impossible. Thus the use of infinitesimals was given up by the later mathematicians of Europe.

But again towards the beginning of present century philosophers like Cohen argued the existence of infinitesimals as the reciprocals transfinite numbers which are used by canter himself. Schmiedeh (1958) and Langwitz (1961) identified infinitely small numbers with functions having asymptotic behavior as from the natural numbers to real numbers i.e., as sequence. Infinitesimals used in analysis do not have a constant positive value but they refer to an infinite process of reducing magnitude. We propose to consider the desirability, possibility and use of such infinitesimals in mathematics. infinitesimal numbers are quantities that are closure to zero than any standard real numbers, but are not zero. They were introduced in the department of calculus, where the derivatives was originally thought of as ratio of two infinitesimal quantities. The word infinitesimal comes from a 17th century modern Latin coinage infinitesimus calculus as development by Leibnitz. In mathematical use "infinitesimal" means infinitely small or smaller than any standard real numbers. The Cantor, Dedekind and Weierstrass sought to rid analysis of infinitesimal and like Bertrand Russell and Rudolf Carnap declared that infinitesimal are pseudo concept. Throughout the Late nineteenth and twentith centuries Philip Enrich(2006). In the 20th century, It was found that infinitesimals could serve as a basis for calculus and analysis the German mathematician K. Weierstrass introduce the epsilon-delta process which provided rigorous basis for the calculus. The insight with exploiting infinitesimal was that the entities could still retain certain specific properties such as angle and slope, even though these entities were infinitely small. The conceptual origin of the concept of infinitesimal $\frac{1}{\infty}$ can be traced as far back as the Greek Philosopher Zeno of Elea. Consider the relationship between a finite an interval approaching that of an infinitesimal size interval.

2. WEIGHT OF FUNCTION AND MEASUREMENT OF HORN ANGLE

Due to the Archimedean property possessed by the real number system R, existence of infinitely small magnitudes i.e infinitesimals (as also infinitely large magnitudes) in comparison with other real numbers is not possible in R, but there exist situation which cry for numbers beyond the scope of the real numbers system and the existence of infinitesimals is possible there. For the sake of illustration we describe the following two situations which demand the creation of a more extensive ordered field then R,

(I) Weight of Functions: We first take up the problem of 'weight of functions' as described by Hardy, suppose for real valued functions we define $\propto$(less weighty than) and $\approx$(equally weighty as) by the following scheme:

$$f \propto g \text{ iff } \lim_{x\to\infty}\frac{f(x)}{g(x)} = 0 \text{ or } \lim_{x\to\infty}\frac{g(x)}{f(x)} = \infty \text{ ; and } f \approx g \text{ iff } \lim_{x\to\infty}\frac{f(x)}{g(x)}$$

is finite non-zero,

The functions f and g will be said to be comparable iff (i) $f \propto g$ or (ii) $g \propto f$ or (iii) $f \approx g$, otherwise they are said to be incomparable. Now consider the family of functions consisting of $\log x$ together with all functions of the form x^α for $\alpha \geq 0$ defined for $x > 0$. Again since $x^\alpha < x^\beta$ iff $\alpha < \beta$, we can attach a real number α as the weight of function x^α(and hence it turns out x^α is less weighty than x^β).

Now let $\log x$ stand for $f(x)$ and x^α for $g(x)$ we want to examine if $f(x)$and $g(x)$ ie $\log x$ and x^α are comparable .Since

$$\lim_{x\to\infty}\frac{\log x}{x^\alpha} = 0 \text{ if } \alpha > 0 \text{ and } = \infty \text{ if } \alpha = 0$$

it follows that if we attach weight θ to $\log x$, We have $0 < \theta < \alpha$ for every positive number α, but inforetunately there is no number θ of this description in the real number system R.

(II) Horn Angles: Next we consider the case if a horn angle ie, the angle between a curve and its tangent or more generally an angle between two curves C_1 and C_2 which intersect at O.

Let OT_1 and OT_2 be their tangents at origin O.

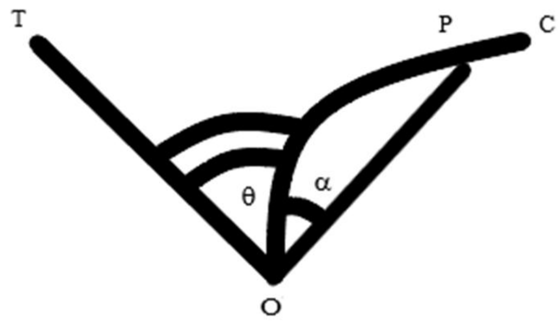

Figure (i)

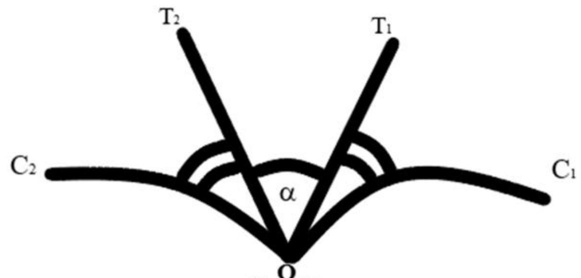

Figure (ii)

Usually the angle between C_1 and C_2 is measured by the angle between their tangent O, viz T_1OT_2 but by measuring the angle in this way we ignore the angle between the curve C_1 and the tangent OT_1 also that between C_2 and OT_2, as we usually accept that the angle between a curve and its tangent is zero. But this does not seem to be an appropriate way of measurement for obviously same space is included between a curve and its tangent and so we need introducing a measurement for such an angle.

Let the curvilinear angle between a curve C and its tangent OT at O be $\theta(\theta \neq 0)$ this number θ(if it exists) is less than α the angle between OT and any second OP, but α canbe made as small as possible so that we have $0 < \theta < \alpha$(arbitrary) and there is no such number θ in the real number system R. for proper measurement of horn angles we have to go beyond the domain of the real number system. Angle between the curve C_1 and C_2 is a combination of three angles viz the anglesT$_1$OT$_2$ and two infinitely small angles C_1OT_1 and C_2OT_2. The magnitude of an angle of the form C_1OC_2 may be called an "ultra real number'' and that of the type C_1OT_1 an infinitesimal''

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