Journal of Advances and Scholarly Researches in Allied Education
Vol. 16, Issue No. 4, March-2019, ISSN 2230-7540

Study of Basic Theory of Nuclear Models

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Abstract – In this paper, we will discuss about various nuclear models, their basic postulates as well as their limitations. The small size of nuclear and the fact that the forces concerned do not appear elsewhere, make the theoretical approach to nuclear structure more difficult than the theoretical approach to electronic structure. There are many isolated facts, which require explanation when we adopt any nuclear model. The characteristics of nuclear–nuclear forces in conjunction with Pauli exclusion principle cause nuclear matter to exhibit apparently contradictory behavior. The macroscopic properties, such as constant density, constant Binding energy per nucleon, resemble those of drop of a liquid. On the other hand the macroscopic properties such as nuclear wave functions and particle motions resemble those of weakly interacting gas. The resemblance to a drop of liquid serves as the basis for liquid drop model and collective model whereas resemblance to a weakly interacting gas serves as the basis for Fermi gas model and shell model.

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FERMI GAS MODEL

In this model

- The nucleus is taken to be composed of a degenerate Fermi gas of neutrons & protons.
- The gas is considered degenerate because all the particles are crowded into the lowest possible states in a manner consistent with the requirements of Pauli principle.
- For each type of particle, the gas may be characterized by K.E. of highest filled state, the Fermi energy.
- The nucleons move freely within a spherical potential well of the proper diameter with depth adjusted so that the Fermi energy raises the highest lying nucleons upto the observed binding energies.
- For stable nuclei, there will be excess neutrons because of greater depth of neutron well.

LIMITATIONS

- The Fermi gas model is not useful for the prediction of the detailed properties of low-lying state of nucleus observed in radioactive decay process.
- A nucleon in excited state is no longer embedded in the Fermi gas. Its interaction with remaining nucleons becomes more & more important with increasing excitation energy.
- Highly excited states of nuclei are many body states in which energy is shared by many particles.
- An excited nucleon is more like a normal condensed material than a nucleus in ground state.
- For a nucleus composed of many particles, the only practical way of describing nuclear excitation is in the statistical approach. This statistical approach is applicable even to unbound states for medium and heavy nuclei.

LIQUID DROP MODEL

According to this model, the individual nucleons move about within the nucleus much as does an atom of a liquid and one might, therefore, think of a nucleus as being like a small drop of liquid.

The idea that the molecules in drop of liquid corresponds to the nucleons in the nucleus in confirmed due to following similarities.

- The nuclear forces are analogous to surface tension force of a liquid.
- The nucleons behave in a manner similar to that of molecules of the liquid.

- The fact that the density of nuclear matter is independent of A shows resemblance to liquid drop where the density of a liquid is independent of the size of the drop.
- The constant B.E./A is analogous to the latent heat of vaporization.
- The disintegration of nuclei by emission of particles is analogous to evaporations of molecules from the surface of a liquid.
- The energy of nuclei corresponds to internal thermal vibrations of drop molecules.
- The formation of compound nucleus and absorption of bombarding particles correspond to the condensation of drops.

DIFFERENCES

- Molecular forces are long-range forces but nuclear forces are short-range forces.
- The average Kinetic energy (K.E.) of molecules in the liquid is 0.1eV but that of nucleons in nuclei is of the order of 10 MeV.
- De-broglie wavelength of molecules in the liquid is 5x10-11m but that of a nucleon in nuclei is 6x10-15m.
- Motion of molecules in liquid is of classical character whereas in nuclei, is of quantum character.

ADVANTAGES

- Not only gives atomic masses & B.E. accurately but also predicts and emission properties.
- Act as a base line from which shell effects can be calculated.
- Able to explain certain features of nuclear fission but is not very successful in describing actual excited state as it gives too large distances.
- Fore runner of collective model of nuclear structure.

SHELL MODEL

For certain number of p or n, called magic number the nuclei exhibit special characteristics of stability suggestive of the properties shown by noble gases among atoms. Nuclei in which either N or Z is equal to one of these magic numbers (2, 8, 20, 28, 50, 82, 126) show certain particulars properties that are not understandable in terms of liquid drop model. But these can be explained on the basis of shell model of nucleus. Protons & neutrons in the nuclei are not all equivalent as had been assumed in liquid drop model. Shell model is able to explain not only magic numbers but also many other nuclear properties such as spin, magnetic moment and energy levels.

Extreme Single particle model –

In this model, it is assumed that nucleons in the nucleus move independently in a common potential determined by average motion of all the other nucleons. Most of the nucleons are paired so that a pair of nucleons contributes to zero spin and zero magnetic moment. The paired nucleons thus form an inert core. The properties of odd a nuclei are characterized by unpaired nucleon and of odd-odd nuclei by unpaired proton and neutron.

Single particle model –

In this model the nucleus with mass number A is considered to consist of filled shells containing maximum number of n and p allowed by Pauli's principle and unfilled shells containing "loose particles". In the extreme single particle model, all the K-particles with same (n, l, j) have same energy. In the single particle model, the particle state is the superposition of the wave function of states whose energies are close to one another; such a procedure is called configuration mixing. For unfilled shells containing only proton, the angular momentum wave function must be anti-symmetric in all pairs. For the shell contain & P both, the angular momentum function must be symmetric or anti-symmetric for exchange of a given pair, depending on the symmetry of ISO spin function for the same exchange.

Individual particle model–

In this model, all the A particles in the nucleus are taken into account, hence is also named as many particle shell model. Each nucleon moves independently of all the other nucleons in a common potential field. Hence is considered as individual particle. The individual particle model has been successfully applied for predicting energy level configuration & binding energies of light nuclei.

Predictions of shell Model –

- **Stability of closed shell nuclei**

This scheme clearly reproduced all the magic numbers.

- **Spins & parities of nuclear ground states**

It has been very successful in predicting the ground state spin of a large number of nuclei. According to this model, the neutron & proton levels fill independently.

i) Even-Even nuclei have total ground state angular momentum I=0+.

ii) With an odd number of nucleons i.e. nucleus with odd Z or odd N, the nucleons pair off as far as possible so that resulting orbital angular momentum and spin direction are just that of single odd particle.

iii) An odd-old nucleus will have a total angular momentum, which is vector sum of odd neutron and odd protons j-values. The parity will be product of the proton and neutron parity i.e.

$$(-1)^{\ln+\mathrm{lp}}$$

- **Magnetic moments of the nuclei**

In an odd nucleus, the total angular momentum I of the nucleus is equal to the angular momentum of the last unpaired nucleon. Therefore magnetic moment is of the nucleus is produced by the odd nucleon only. The orbital angular momentum with value √ l(l+1) and the spin 'S' with numerical value √s(s+1) couple to a total angular momentum of with numerical value √ j(j+1) in units of ħ.

- **Electric Quadrupole Moment**

According to shell model, the electric quadrupole moment of a odd A –odd Z nuclide is due is the unpaired proton. The quadrupole moment for a given I is given by

$$Q = -\frac{(2I-I)}{(2I+I)}(r_{rms})^2$$

Where I = l ± s,

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r_{rms} is root mean square radius of the orbit.

- **Nuclear Isomerism**

Shell model predict that almost all of the isomeric states with long life are found for nuclei with N or Z near the end of a shell. Their nuclei are found in four distinct groups.

- **β-Decay**

Theory of β-Decay predicts / shows that lifetime can be understood in terms of relative parity and angular moments of the states involved. The nuclear shell model is able to predict the spins and parities of these states of unstable nuclei that decay by β-emission.

Stripping reactions

In stripping reaction the captured nucleon, say neutron, occupies one of the available quantum state in the target nucleus and the other, say the proton, carries information concerning this state. Thus stripping reaction can be used to study unoccupied nuclear state.

Collective Nuclear model -

The shell model has been most successful in explaining a number of nuclear features. The deviations of magnetic moment from the Schmidt curve make this model less acceptable. The measured quadrupole moments are several times larger than can be attributed to the odd nucleon even in nuclei with just one nucleon more or less than a closed shell, where the single particle model should be at its best.

J. Rainwater, the American physicist in 1950, suggested that these discrepancies might be overcome in odd-A nuclei by considering the polarization of even core by the motion of odd nucleon. The nuclear core, consisting of even nucleons, thus has an spheroidal shape instead of spherical shape. This distortion would make an additional contribution to the quadrupole moment and to the quadrupole transition rate. A. Bohr and B. Mottelson have developed the idea of deformed nuclear core. The individual nucleons are imagined to be moving in orbits as before in a potential distribution determined by remaining nucleons. The collective motion of nucleons influences the individual particle orbits because it changes the potential of the region in which these particles move. Because of stability of core, the collective motion is small and independent particle characteristics are prominent, for the nucleus consisting of closed shells.

The total energy is expressed as

$$W = E_{rot}+E_{vib}+E_n.$$

Mathematically this means that the Hamiltonian is composed of three additive parts containing

1. Rotational co-ordinates

2. Vibrational co-ordinates

3. Nucleonic co-ordinates.

The wave function is than the product of three wave functions each containing the respective co-ordinates. The collective motion now becomes a vibration about the equilibrium shape and a rotation of the nuclear orientation, which maintains the deformed shape. In general,

$$\delta Rj = (5/4\pi)^{1/2} \beta R_o \cos(\gamma - 2\pi j/3)$$

For $\gamma = 0$, the nucleus would be prolate spheroids with 3 axis as its symmetry axis. For $\gamma = 2\pi/3$ and $4\pi/3$, the nuclei would be prolate spheroids with 1 and 2 axes as symmetry axes.

For $\gamma = \pi$, $\pi/3$, $5\pi/3$ the nuclei one oblate spheroids. If $\gamma = n\pi/3$, the nuclear shape is that of an ellipsoid with three unequal axes.

Super-conductivity Model

According to this model, the individual nucleons retain their shell model states. The quantum states l, j and + m_j are used to describe the pairing of nucleons. If individual nucleons were non-interacting, as suggested by shell model, the first nucleon or two beyond a closed shell would be expected to be describable by the lowest energy state configuration beyond the closed shell. However, quantum mechanics allows some admixture of levels. A missing nucleon (hole) can create the same effect as an extra nucleon. Thus a nucleus with two nucleons less than a closed shell can be treated in a manner analogous to a nucleus with two extra nucleons.

The formation of super-conductivity model of the nucleus gives approximately the same levels as the other models. This model is more satisfactory than other.

Phenomenological and VMI Model

The variation of moment of Inertia I of deformed nuclei as a function of angular momentum L, has been dealt with semi classically from the early days. Even recently continuation of this interest has brought some interesting results. The concept of nuclear softness was first introduced by Morinaga[29] in 1966. Based on these concepts, Marisotti et. al.[30] developed a model of variable moment of Inertia (VMI) and its latter version- the generalized VMI (GMVI). The softness parameter σ and the stiffness parameter C with have been calculated by making different assumptions about the equilibrium condition. A more quantitative approach for the variation of I with L has been most recently made by dividing the moment of into kinematics and dynamic parts given by

$$\frac{I^{(1)}}{\hbar^2} = I_o [1+bL(L+1)]^{1/2} \quad \text{(Kinematic Part)}$$

$$\frac{I^{(2)}}{\hbar^2} = I_o [1+bL(L+1)]^{3/2} \quad \text{(Dynamic Part)}$$

Where a & b is derived from the energy expression obtained from Bohr- Hamiltonian for a well-deformed nucleus $I_0 = \hbar^2/ab$ is referred to as band of moment of Inertia.

Interacting BoSON Model

IBM - The interacting Boson Model was introduced by Arima & Loachello in 1975 in an attempt to describe the collective properties of nuclei in a unifying manner. It was meant to interpolate between the spherical shell model by Jenser and Mayer, which considers the nucleons to behave like non-interacting particles inside a “well-chosen well” and the geometric collective model of Bohr and Mottelson.

In the IBM, the collective nuclear excitations are described by bosons with certain angular momentum. Originally, this was invented just to describe the phenomenology of low energy rotation and vibration spectra, but there is a very tempting microscopic interpretation of the bosons being formed by valence nucleon pairs coupled to definite angular momentum. Thus analogy of ‘cooper pairs from superconductivity theory would naturally explain the pairing interaction, experimentally found to be very significant in even-even nuclei. But the attempts to derive parameters from microscopic fermion models have not been satisfactory so far.

The IBM and its extensions have been phenomenological very successful in describing low energy collective states. In our work we will concentrate only on the simplest version of the model, the so-called IBM-1 and its U (5) limit.

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