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Some Static Spherically Symmetric Perfect Fluid Distribution with Spin

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Abstract – The present paper provides solution of E-C field equations for static fluid sphere by a suitable choice of metric potential in three different cases. Various physical parameters have been found and constants appearing in solution have been fixed using boundary conditions.

Key Words: Sphere, Density, Spin Density, Spin Tensor Torsion Tenor.

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1. INTRODUCTION

Various researchers have shown their keen interest in extension of general theory of relativity. An extension of the Einstein's theory of general relativity has been investigated by many authors like Trautmen [20], Cartan [2, 2(a)] Kerlick [6], Hehl [3,4], Hehl et al. [5]. Kuchowicz [11-4], Prasanna [16-18] Kopczynski [9], Singh and Yadav [20] and Singh and Kumar [20(a)] Banerji [1] has pointed out that E-C spheres must bounce outside the schwarzchild radius if it bowers at all. The problem of static fluid sphere in the framework of Einstein-Cartan theory was considered Prasanna [18] and Yadav et al. [22, 23]. Taking Hehl's approach [3,4] to E-C theory, Prasanna has obtained the solutions similar to solutions obtained by Tolman [19] in general relativity.

In 1978, Singh and Yadav [20] studied the static fluid spheres in E-C theory and obtained a solution in an analytic form by the method of quadratune. Spatially homogenous cosmological models of Bianchi type VI and VII based on Einstein-Cartan theory were considered by Tsoubelies [21]. Som and Bedran [18] got the class of solutions that represent a static incoherent spherical dust distribution in equilibrium under the influence of spin. Mehra and Gokhroo [13] have also given physically meaningful solutions of the field equations for static spherical dust distribution in E-C theory. Krori et al. [8] gave a singularity free solution for a static fluid in Einstein-Cartan theory.

Here in this paper we have solved E-C field equations for static fluid sphere by taking a suitable choice of metric potential in three different cases Constants appearing in the solution have been fixed using boundary conditions. Various physical parameters have been also evaluated for the distribution.

2. THE FIELD EQUATIONS

The Einstein-Cartan field equation are

$$(2.1)\ R^i_j - \frac{1}{2}R\delta^i_j = -kt^i_j$$

$$(2.2)\ Q^i_{jk} - \delta^i_j Q^l_{lk} - \delta^i_k Q^l_{jl} = -kS^i_{jk}$$

where Q^i_j is torsion tensor, t^i_j is the canonical asymmetric energy momentum tensor, S^i_{jk} is the spin tensor and k = 8π.

For a static spherically symmetric system an appropriate metric is

$$(2.3)\ ds^2 = e^B dt^2 - e^A adr^2 - r^2(d\theta^2 + \sin^2\theta d\theta^2)$$

A and B being functions of r. We use comoving co-ordinates with 4-velocity $u^i = \delta^i_4$. The orthonormal coframe is chosen as $\theta^1 = e^{A/2}dr, \theta^2 = rd\theta, \theta^3 = r\sin\theta d\phi, \theta^4 = e^{B/2}dt$.

When we assume a classical description of spin (Weyssenhoff and Raabe 1947, Trautman 1973), we have

$$(2.4)\ S^i_{jk} = S_{jk}u^i \ \text{ with } \ S_{ij}\ u^j = 0$$

Where u^i is the velocity four vector and S_{ij} is the intrinsic angular momentum tensor. In the case of spherical symmetry the tensor S_{ij} has the only nonvanishing independent component S_{23} = K (say) and the non-zero components of S^{i}_{jk} are

$$(2.5)\quad S^4_{23} = -S^4_{32} = k$$

Hence from the E-C field equation (2.2) the non-zero components of Q^{i}_{jk} are

$$(2.6)\quad Q^4_{23} = -Q^4_{32} = -kK$$

Thus for a perfect fluid distribution with pressure p and metter density ρ the field equations (2.1) finally reduce to (Prasanna [16])

$$(2.7)\quad 8\pi p = 16\pi^2 k^2 - \frac{1}{r^2} + e^{-A}\left(\frac{1}{r^2} + \frac{B'}{r}\right)$$

$$(22.8)\quad 8\pi\rho = 16\pi k^2 + \frac{1}{r^2} - e^{-A}\left(\frac{1}{r^2} - \frac{A'}{r}\right)$$

$$(2.9)\quad \frac{c^A}{r^2} = \frac{1}{r^2} = \frac{B'^2}{4} + \frac{B''}{2} + \frac{B'A'}{4} + \frac{B'+A'}{2r}$$

where dashes denote differentiation with respect to r.

The conservation law yields the relation

$$(2.10)\quad \left[p + \frac{1}{2}(p+\rho)\right] + BK\left(K + \frac{1}{2}KB'\right) = 0$$

Librium, viz.,

$$(2.11)\quad p + \frac{1}{2}(\rho + p)B' = 0$$

From (2.12) we have

$$(2.13)\quad K + \frac{1}{2}KB' = 0$$

where H is a constant of integration.

Following Hehl [3, 4] if we define

$$(2.14)\quad \bar{\rho} = \rho - 2\pi K^2, \bar{p} = p - 2\pi K^2$$

We find that the equations (2.7) and (2.8) take the usual general relativistic (2.7) and (2.8) take the usual general relativistic form for a static field sphere as given by

$$(2.15)\quad 8\pi\bar{p} = \frac{1}{r^2} + e^{-A}\left(\frac{1}{r^2} + \frac{B'}{r}\right)$$

$$(2.16)\quad 8\pi\bar{p} = \frac{1}{r^2} + e^{-A}\left(-\frac{1}{r^2} + \frac{B'}{r}\right)$$

Along with (2.9). the equation of continuity (2.10) now becomes

$$(2.17)\quad \frac{d\bar{p}}{dr} + \frac{1}{2}(\bar{\rho} + \bar{p})B' = 0$$

In $\bar{P}$ and $\bar{P}$ the aquare term of spin behaves as the effective repulsive force. The repulsion can be important if the expression $2\pi k^2$ is of the same order as the energy density ρ. It is clear from these equations that it is the $\bar{P}$ and not p which is continuous across that boundary $r = r_0$ of the fluid sphere. The continuity of $\bar{P}$ across the boundary ensures that of (e^B). Further with $\bar{P}$ and $\bar{P}$ replacing p and ρ respectively, we are assured that the metric coefficients are continuous across the boundry. Hence we shall apply the usual boundary conditions to the solutions of equations (2.9), (2.15) and (2.16).

The boundary conditions are

$$(2.18)\quad \left[e^{-A}\right]_{r=r_0} = [e^B]_{r=r_0} = \left(1 - \frac{2M}{r_0}\right), \bar{p} = 0 \text{ at } r = r_0$$

where r_0 is the radius and M the mass of the fluid sphere.

3. SOLUTION OF THE FIELD EQUATIONS

It is well known that the equation (2.9) may be solved by quadrature in a number of ways, by specifying various conditions on the function A and B that simplify the equations and allow immediate integration. One a and B are obtained $\bar{P}$ and $\bar{P}$ follow directly from (2.15) and (2.16). We define

$$(3.1)\quad y(r) = e^{B/2}, \tau(r) = e^{-A}$$

Then (2.9) may be written as

$$(3.2)\quad \tau' - \left[\frac{2(y + ry' - r^2y'')}{r(y+ry)}\right]\tau = \frac{-2y}{r(y+ry')}$$

It has the solution

$$(3.3)\quad \tau(r) = \exp[-\psi(r)][\int^r \exp[\psi(u)h(u)du + L]$$

Where

$$\psi^{(r)} = \int^r \phi(u)du, \phi(r) = \frac{-2(y + ry' - r^2y')}{r(y+ry')}$$

$$h(r) = \frac{-2y}{r(y + ry^1)}$$

L being a constant of integration to be fixed by the boundary conditions.

The remaining equation (2.15) and (2.16) give $\overline{P}$ and $\overline{P}$ as

$$(3.4)\quad 8\pi\overline{p}r^2 = -1 + \tau\left(1 + \frac{ry}{2y}\right)$$

$$(3.5)\quad 8\pi\overline{\rho} = -1\tau\left(1 + \frac{r\tau}{\tau}\right)$$

Exact solutions in terms of know functions are most easily obtained by requiring one of the field variables to satisfy some subsidiary condition which simplifies the full set of equations. Once the field equations are solved in this manner an equation of state can then be found. Such solutions may be useful in under stating a system in the extreme relativistic limit where we cannot specify a priori what the equation of state might be.

Further there is no reason to expect that all solutions will be physically reasonable. Only a subclass of these solutions, corresponding to certain choices of the function will be physically realistic and a still smaller subclass will correspond to physically reasonable equation of state. Thus judicious choice of the function B(r) is necessary for a physically interesting solution.

We see that equation (3.2) is linear in π if y is a known function. For this we choose y as

$$(3.6)\quad y = (\alpha + \beta r^{\mu})^{\zeta}$$

where α and β are constants and μ and ζ are +ve integers.

We consider the solution for different values of μ and ζ.

Case I : For general value of μ and ζ=1, we get

$$(3.7)\quad y(r) = \alpha + \beta r^{\mu}$$

Differentiating

$$(3.8)\quad y'(r) = \mu\beta r^{\mu-1}$$

and

$$(3.9)\quad y''(r) = \mu(\mu-1)\beta r^{\mu-2}$$

Putting these values in (3.2) we get

$$(3.10)\quad \tau' - \left\{\frac{2\{\alpha + \beta r^{\mu}(1 + 2\mu - \mu^2)\}}{r\{\alpha + \beta r^{\mu}(1+\mu)\}}\right\}\tau + \frac{2(\alpha + \beta r^{\mu})}{r[\alpha + \beta r^{\mu}(1+\mu)]}$$

Using (3.3) and by taking μ = 2, we get

$$(3.11)\quad \tau(r) = r^{-\lambda} = 1 + \frac{r^2}{(\alpha + 3\beta r^2)^{2/3}}$$

Using boundary conditions the constants α, β, r are given by

$$(3.12)\quad \alpha = \left(1 - \frac{5}{2}\right) / \sqrt{(1 - 2\in)}$$

$$(3.14)\quad \beta = \frac{\zeta}{2r_b^2\sqrt{(1 - 2\in)}}$$

$$(3.15)\quad r = \frac{-2(1-\in)^{2/3}}{r_b^2\sqrt[3]{(1 - 2\in)}}$$

Where

$$\in = \frac{m}{r_b}$$

The spin density K is given by

$$(3.16)\quad K = H(1 - 2\in)^{1/2} / \left(1 - \frac{5}{2}\in + \frac{\in r^2}{2\, r_b^2}\right)$$

Also we have

$$(3.17)\quad p = \left(\frac{\in}{4\pi r_0^2}\right)\left[e^{-\lambda}\left(1 - \frac{5}{2}\in + \frac{1}{2}\in\frac{r^2}{r_0^2}\right)^{-1} - (1-\in)^{2/3}\left(1 - \frac{5}{2}\in + \frac{3}{2}\in\frac{r^2}{r_b^2}\right)^{-2/3}\right] + \frac{16\pi^2 H^2(1 - 2\in)}{\left(1 - \frac{5}{2}\in + \frac{1}{2}\in\frac{r^2}{r_0^2}\right)^2}$$

$$(3.18)\quad \rho = \frac{16\pi^2 H_1^2(1 - 2\in)}{\left(1 - \frac{5\in}{2} + \frac{\in}{2}\times\frac{r^2}{r_b^2}\right)^2} + \left(\frac{\in}{4\pi r_0^2}\right)\left[\frac{(1-\in)}{\left(1 - \frac{5\in}{2} + \frac{3\in}{2}\frac{r^2}{r_b^2}\right)}\right]^{2/3}$$

The constant H is given by

$$(3.19)\quad H^2 = \left(\frac{1 - 2\in}{16\pi^2}\right)\left[\frac{\in}{4\pi r_b^2}\left(\frac{3 - 5\in}{1 - \zeta}\right) - \rho(r_b)\right]$$

Case II : When $\mu = 2, \zeta = \frac{1}{2}$ then $y = (\alpha + \beta r^2)^{1/2}$. In this case the differential equation (3.2) on integration yields.

$$(3.20)\quad e^{A} = \tau(r) = \frac{(\alpha+\beta r^2)(1+r^2)}{(A+2\beta r^2)}$$

Where r is constant of integration.

The constants α, β, r are determined by matching the solution to the exterior Schwarzohild solution at the boundary $r = r_b$. they are

$$(3.21)\quad \alpha = -\frac{M}{2r_b^3}$$

$$(3.22)\quad \beta = \frac{M}{r_b^3}$$

$$(3.23)\quad r = 2\left(1-\frac{2m}{r_b}\right)$$

Here $\alpha \le 0$ and $\beta > 0$.

The spin density K is given by

$$(3.24)\quad K^2 = \frac{H^2}{(\alpha+\beta r^2)}$$

Also pressure and density are given by

$$(3.25)\quad p = \frac{1}{8\pi}\left[r+\beta\frac{(1+r^2)}{\alpha+2\beta r^2}\right]+\frac{2\pi H^2}{\alpha+\beta r^2}$$

$$(3.26)\quad \rho = \frac{1}{8\pi}\left[\frac{(2\beta^2-7\alpha\beta r)r^2-6\beta^2 rr^4+3\alpha(\beta-r)}{(A+2Br^2)^2}\right]+\frac{2\pi H^2}{\alpha+\beta r^2}$$

The constant H is found to be

$$(3.27)\quad H^2 = \frac{1}{16\pi^2}[8\pi\alpha\rho_0 - 3(\beta-\alpha r)]$$

Case III : When μ = 1, ζ = 1

$$(3.28)\quad y(r) = \alpha+\beta^r = e^{B/2}$$

The π and hence e^{-A} can be found from (3.3) constants appearing in the solution and other parameters like pressure, density etc. can be found as in previous case.

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