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A Static Fluid Sphere with Non Zero Spin Density

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Abstract – This paper provides solutions of E - C. field equations for a static conformlly that fluid sphere by taking a suitable relations between Metric potentials pressure, density and spin density have been also calculated.

Key Words – Spin, Metric, Torsion, Commoving Coordinates, Isotropic.

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1. INTRODUCTION:

Many workers have considered the problem of static fluid spheres in E-C theory (Prasanna [16 -18], Kerlick [8], Kuchowicz [11-14], Skinner and Webb [21] and Singh and Yadav [20].Hehl et.al. [7]and Kopszynski [9] Hehl, Heyde and Kerlick [7] have considered the field equations of general relativity with spin and Torsion U4 theory to describe correctly the gravitational properties of matter on a macro physical level. They have shown how to singularities theorems of Penrose [15] and Hawking [4] must be modified to apply in E-C theory. Prasanna [18] has solved Einstein-Cartan field equations for a perfect fluid distribution and adopting Hehl's [5], [6] approach, and Tolman's technique [25] obtained a number of solutions. Arkuszewski et. al. [1] discussed the junction conditions in Einstein-Cartan theory. Raychaudhuri and Banerji [19], Singh and Yadav [20] studied the static fluid spheres in E-C theory and obtained a solution in an analytic form by the method of quadrature.Som and Bedran [22] got the class of solutions that represent a static incoherent spherical dust distribution in equilibrium under the influence of spin. Krori et. at.[10] gave a singularity free solution for a static fluid sphere in Einstein-Cartan theory. Suh [23] Considering the static spherically symmetric interior solution in Einstein-Cartan theory closely compared with those in the Einstein theory of gravitation. Mehra and Gokhroo [15a] have also given physically meaningful solution of the field equations for static spherical dust distribution in E-C theory.

In this paper we have solved the Einstein-Cartan equations for a static conformally flat fluid sphere by assuming a suitable relation between metric potentials. Pressure, density and spin density have been found for the model.

2. THE FIELD EQUATIONS

The field equations of Einstein-Cartan theory are given by

(2.1) $$R^{\alpha}_{\beta}-\frac{1}{2}R.\delta^{\alpha}_{\beta}=-Xt^{\alpha}_{\beta}$$

(2.2) $$Q^{\alpha}_{\beta\gamma}-\delta^{\alpha}_{\beta}Q^{1}_{1\gamma}-\delta^{\alpha}_{\gamma}Q^{1}_{\beta 1}=Xs^{\alpha}_{\beta\gamma}$$

where $Q^{\alpha}_{\beta\gamma}$ is torsion tensor, t^{α}_{β} is the canonical asymmetric energy momentum tensor, $S^{\alpha}_{\beta\gamma}$ is the spin tensor X = 8π.We take a static spherically symmetric matter distribution given by the metric

(2.3) $$ds^2=\rho^{v}dt^2-\rho^{\lambda}dr^2-r^2(d\theta^2+\sin^2\theta d\phi^2)$$

where λ and v are functions of r only. We use comovring co-ordinates with u4 velocity $u^i=\delta^i_4$.

The orthonormal co frame is chosen as

(2.4) $$\theta^1=e^{\lambda/2}dr,\ \theta^2=rd\theta,\ \theta^3=r\sin\theta d\phi,\ \theta^4=e^{v/2}dt$$

So that

$$g_{\alpha\beta} = \mathrm{diag}(1,-1,-1,-1).$$

When we assume a classical description of spin, we have

(2.5) $$S^{\gamma}_{\alpha\beta} = S_{\alpha\beta} u^{\gamma}$$

With

$$S_{\alpha\beta} u^{\beta} = 0$$

Where uαβ is the antisymmetric tensor of spin density. In the case of spherical symmetry, the tensor Sij has the only non vanishing independent component

S23 = K(say) and the non – zero components of $S^{\alpha}_{\beta\gamma}$ are

(2.6) $$S^{4}_{23} = -S^{4}_{32} = K$$

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Hence from E-C field equations (2.2), the non-zero components of Q^{i}_{ij} are

(2.7) $$Q^{4}_{23} = -Q^{4}_{32} = -XK$$

Thus for a perfect fluid distribution with isotropic pressure p and matter density the field equations (2.1) finally reduce to (Prasanna [18])

(2.8) $$8\pi\rho = 16\pi^2 K^2 - \frac{1}{r^2} + \rho^{-\lambda}\left(\frac{1}{r^2} + \frac{v'}{r}\right)$$

(2.9) $$8\pi\rho = 16\pi^2 K^2 + \frac{1}{r^2} - \rho^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right)$$

(2.10) $$\frac{e^{\lambda}}{r^2} = \frac{1}{r^2} - \frac{v'^2}{4} - \frac{v''}{2} + \frac{v'\lambda'}{4} + \frac{v'+\lambda'}{2r}$$

where dashes denote differentiation w.r.t. r.

The conservation laws give us the relation

(2.11) $$\left[p' + \frac{1}{2}(\rho+p)v'\right] + \frac{1}{2}XK\left[K' + \frac{1}{2}Kv'\right] = 0$$

If we use the equation of hydrostatic equilibrium viz

(2.12) $$p' + \frac{1}{2}(\rho+p)v' = 0$$

We get

(2.13) $$K' + \frac{1}{2}Kv' = 0$$

From (2.13) we have

(2.14) $$K = He^{-v/2}$$

where H is a constant of integration.

Following Hehl [6,7], if we define

(2.15) $$\bar{\rho} = \rho - 2\pi K^2, \bar{p} = p - 2\pi K^2$$

we find that the equations (2.8) and (2.9) take the usual general relativistic form for a static fluid sphere as given by

(2.16) $$8\pi\bar{\rho} = -\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} + \frac{v'}{r}\right)$$

(2.17) $$8\pi\bar{\rho} = -\frac{1}{r^2} + e^{-\lambda}\left(-\frac{1}{r^2} + \frac{v'}{r}\right)$$

equation (2.10) remaining the same.

The equation of continuity (2.11) now becomes

(2.18) $$\frac{d\bar{p}}{dr} + \frac{1}{2}(\bar{\rho} + \bar{p})v' = 0$$

It is clear from these equations that it is the $\bar{\rho}$ not the p which is continuous across the boundary $r = r_0$ of the fluid sphere. The continuity of p across the boundary ensures that v' (exp.v). Further with $\bar{\rho}$ and $\bar{\rho}$ replacing p and ρ respectively we are assured that the metric coefficients are continuous across the boundary. Hence we shall apply the usual boundary conditions to the solution of equations (2.10), (2.16) and (2.17).

We use the boundary conditions

(2.19) $$\left[e^{-\lambda}\right]_{r=r_0} = \left[e^{v}\right]_{r=r_0} = \left[1 - \frac{2m}{r_0}\right]$$

$\bar{p} = 0$ at r = r0 where r0 is the radius of fluid sphere and m is the mass of the fluid sphere.

The total mass, as measured by an external observer, inside the fluid sphere of radius r0 is given by

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(2.20) $$m = 4\pi\int_0^{r_0} \bar{\rho} r^2 dr$$

$$= 4\pi\int_0^{r_0} \rho r^2 dr - 8\pi^2\int_0^{r_0} K^2(r) r^2 dr$$

Thus the total mass of the fluid sphere is modified by the correction

$$8\pi^2\int_0^{r_0} K^2(r) r^2 dr$$

For conformal flatness, vanishing of the Weyl tensor [24] yields:

(2.21) $$4(e^{\lambda} - 1) = r^2(2v'' + v'^2 - \lambda' v') + 2r(\lambda' - v')$$

Now we choose

(2.22) $\lambda = \log[1 + \exp(2v)]$ and also $\lambda' = 2v'$

Equations (2.21) and (2.22) yield

(2.23) $$v'' - \frac{1}{2}v'^2 + \frac{v'}{r} - \frac{2e^{2v}}{r^2} = 0$$

It's solutions is

(2.24) $$\exp(v) = \left[C(\log Dr)^2 - \frac{1}{C}\right]^{-1}$$

where C and D are constants of integration.

Hence equation (2.22) gives λ as

(2.25) $$e^{\lambda} = 1 + \left[\frac{C}{C^2(\log Dr)^2 - 1}\right]^2$$

Now from equations (2.10), (2.22) and (2.17)

(2.26) $$8\pi\bar{\rho} = \frac{3\lambda'}{2r}$$

which by use of equation (2.25) gives

(2.27) $$8\pi\bar{\rho} = \frac{6C^2}{r^2} - \frac{\log Dr}{C^2(\log Dr)^2 - 1}$$

Equation (2.27) with (2.24), (2.15) yields

(2.28) $$8\pi\rho = \frac{6C^2}{r^2} - \frac{\log Dr}{C^2(\log Dr)^2 - 1} + 16\pi^2 K^2$$

$$= \frac{6C^2}{r^2} - \frac{\log Dr}{C^2(\log Dr)^2 - 1} + 16\pi^2 H^2\left[C(\log Dr)^2 - \frac{1}{C}\right]$$

Now using equations (2.10), (2.21) and (2.22) we obtain

(2.29) $$8\pi\bar{p} = 0$$

which means effective pressure is zero.

Next using equations (2.29), (2.24) and (2.15) we get

(2.30) $$8\pi p = 16\pi^2 H^2\left[C(\log Dr)^2 - \frac{1}{C}\right]$$

Also spin density K is given by

(2.31) $$K = H\left[C(\log Dr)^2 - \frac{1}{C}\right]^{1/2}$$

The constants appearing in the solution can be found by matching the solution at the boundary = r0 the Schwarzschild exterior solution. Also here we see that metric is regular.

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