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A Study of Stochastic Model with Queue Network

Mamta[1]* Dr. Naveen Kumar[2]

[1] Research Scholar, Department of Mathematics, BMU, Rohtak

[2] Associate Professor, Department of Mathematics, BMU, Rohtak

Abstract – Mathematicians and operational researchers have done significant work in developing stochastic and simulation models in the analysis of feedback and cyclic queue network. Different authors have introduced different models to encourage theory in realistic circumstances, using various hypothesis situations as their basis. One can observe that, while many such models lack practicability in real life circumstances, the mathematical methods used in the study are truly elegant. Here is a short description of some of the work they choose in a series they have created in order to deal with specific circumstances in their lives. Feedback queues are those in which a customer once served if his service is failing and is served over and over before his service is successful. For instance a packet transmitted from the source to the destination can be returned in certain real life scenarios as for example a feedback queue and it can proceed as it is until the packet is eventually transmitted. In the last few decades, the university has carried out extensive research in the field of feedback and cyclic queue model. In the field of queue networking theory Jackson did a remarkable job. Let us take it for granted that units (customer/message/packages) individually arrive in Poisson at rate β and that the service time is general. Once the service is complete, the device is cycled back into the system with a chance to successfully perform a service apart from the p system or if service is unsuccessful. This is called input from Bernoulli. It is not likely that all customers, who necessarily join and leave at the same node, or who follow the same route once they enter the customer of the system, can return to the previously visited nodes, miss any of the nodes entirely and choose to stay in the system forever.

Key Words – Stochastic Model, Queue Network, Feedback Queues, Queue Model

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INTRODUCTION

Human history reveals that great inventions have led us much of the way in which we live our lives. Novel ways have always been built to satisfy all coming demands, whether it's the invention of the wheel or the machine or IT. The use of these instruments of revolutionization has deeply affected human culture. The hunger for information and the enhancement of life quality have led us to understand the world in which we live better. Man has colonised a venture for many reasons, particularly patience of waiting for others, which is unsuccessful if he had not learnt tolerance. One of them paused and waited somewhere to join others. It was this tolerance and patience to wait for others that gave rise to the colonies. With the rise of the population of the planet, people's settlements became urban cities of different sizes. Had man been pursuing an ideology of 'immediate service on demand,' it would have led his efforts to be used unéconomically, the unity would never have formed between individual citizens, and his problems would have been numerous. He started to lead an organised life as he needed to rely on others' services to a large extent to meet his own demands. Waiting thus became a living part and waiting lines is typically made up of units, customers and messages which wait for service while it becomes obvious and predictable to have waiting lines.

Gordan and Newell[1967] consider a network of Markov queues where a fixed and limited number of customers say (K) circulates across the network. There is no outside feedback or departure from the network, unlike Jackson's open network. Considering a closely connected network of "k" nodes, the contribution of the I nodes will be at the next node i+1 (1 alternatives to i-1), and the last node "k" input will be from the I node and so forth. A cyclic queue such as this is called. A cyclic queue structure consists of several series of service channels, with a closed circle at the centre.

T.P. Singh Singh. [1994] broadened the research carried out by various cyclic queue network researchers. Further Singh T.P., Kusum (2010) carried out detailed feedback queue model empirical studies assuming that the service rate was proportional to the queue numbers and heterogeneous feedback channels. This study is another widespread work done by previous researchers in the field of feedback.

TRANSIENT BEHAVIOR OF THREE STAGE FEEDBACK QUEUE SYSTEM

This segment explores a network of three queues in sequence, input from the third server, both arriving and departing poisonously. We assume that the service rate is proportional to the amount of your queue. The equation of differential differences was tested transiently. The mean queue size and other parameters were obtained by means of statistical tools for the queue model. The model is implemented in the banking service structure, administrative installation and decision-making. The transient behaviour of the system was examined and the differential equation, thus developed in the model, solved by using the generating function technique and Lagrange's partial differential equation solution process. The aim of this section is to discuss different queue characteristics such as medium length of the queue system, variance of it, etc., and useful in the decision-making, produce, banking service, etc.

MATHEMATICAL MODELING:

Define $P_{n_1,n_2,n_3}\ t$ the probability at time (t), there are n_1, n_2, n_3 items in system in front of nodes S_1, S_2, S_3 (n_1 n_2, n_3>0),

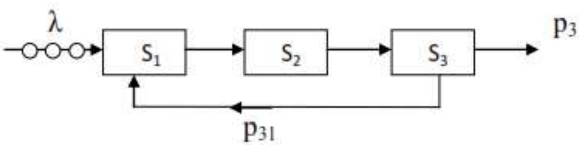

Figure - 1

Connecting the various state probabilities at time '$t+\delta t$' time dependent differential difference equations for the model are obtained as:

$$P'_{n_1,n_2,n_3}(t) = -\ \lambda + \mu_1 n_1 + \mu_2 n_2 + \mu_3 n_3\ P_{n_1,n_2,n_3}(t) + \lambda P_{n_1-1,n_2,n_3}(t)$$
$$+\mu_1(n_1+1)P_{n_1+1,n_2-1,n_3}(t) + \mu_2(n_2+1)P_{n_1,n_2+1,n_3-1}(t) + \mu_3(n_3+1)\,p_3 P_{n_1,n_2,n_3+1}(t) + \mu_3(n_3+1)p_{31}P_{n_1-1,n_2,n_3+1}(t)$$
$$n_1, n_2, n_3 > 0 \qquad (1)$$

$$P'_{0,n_2,n_3}(t) = -\ \lambda + \mu_2 n_2 + \mu_3 n_3\ P_{0,n_2,n_3}(t) + \mu_1 P_{1,n_2-1,n_3}(t) + \mu_2\ n_2 + 1\ P_{0,n_2+1,n_3-1}(t) + \mu_3(n_3+1)p_3 P_{0,n_2,n_3+1}(t)$$
$$n_1 = 0,\ n_2, n_3 > 0 \qquad (2)$$

$$P'_{n_1,0,n_3}(t) = -\ \lambda + \mu_1 n_1 + \mu_3 n_3\ P_{n_1,0,n_3}(t) + \lambda P_{n_1-1,0,n_3}(t) + \mu_2 P_{n_1,1,n_3-1}(t) + \mu_3(n_3+1)p_3 P_{n_1,0,n_3+1}(t) + \mu_3(n_3+1)p_{31}P_{n_1-1,0,n_3+1}(t)$$
$$n_2 = 0,\ n_1, n_3 > 0 \qquad (3)$$

$$P'_{n_1,n_2,0}(t) = -\ \lambda + \mu_1 n_1 + \mu_2 n_2\ P_{n_1,n_2,0}(t) + \lambda P_{n_1-1,n_2,0}(t) + \mu_1\ n_1 + 1\ P_{n_1+1,n_2-1,0}(t) + \mu_3 p_3 P_{n_1,n_2,1}(t) + \mu_3 p_{31} P_{n_1-1,n_2,n_3}(t)$$
$$n_3 = 0,\ n_1, n_2 > 0 \qquad (4)$$

$$P'_{0,0,n_3}(t) = -\ \lambda + \mu_3 n_3\ P_{0,0,n_3}(t) + \mu_2 P_{0,1,n_3-1}(t) + \mu_3(n_3+1)p_3 P_{0,0,n_3+1}(t)$$
$$n_1, n_2 = 0,\ n_3 > 0 \qquad (5)$$

$$P'_{n_1,0,0}(t) = -\ \lambda + \mu_1 n_1\ P_{n_1,0,0}(t) + \lambda P_{n_1-1,0,0}(t) + \mu_3 p_3 P_{n_1,0,1}(t) + \mu_3 p_{31} P_{n_1-1,0,n_3}(t)$$
$$n_2, n_3 = 0,\ n_1 > 0 \qquad (6)$$

$$P'_{0,n_2,0}(t) = -\ \lambda + \mu_2 n_2\ P_{0,n_2,0}(t) + \mu_1 P_{1,n_2-1,0}(t) + \mu_3 p_3 P_{0,n_2,1}(t)$$
$$n_1, n_3 = 0,\ n_2 > 0 \qquad (7)$$

$$P'_{0,0,0}(t) = -\ \lambda\ P_{0,0,0}(t) + \mu_3 p_3 P_{0,0,1}(t)$$
$$n_1, n_2, n_3 = 0 \qquad (8)$$

Taking initial condition at t=0 as,

$$\left.\begin{aligned} P_{n_1,n_2,n_3}\ 0 &= 1 \\ &= 0 \end{aligned}\right\} \qquad \begin{aligned} &n_1, n_2, n_3 = 0 \\ &n_1, n_2, n_3 > 0 \qquad (9) \end{aligned}$$

MEAN QUEUE SIZE:

Let the mean queue size be denoted by L. Then L as per statistical formula is defined as

$$L = \sum_{n_1=0}^{\infty}\ \sum_{n_2=0}^{\infty}\ \sum_{n_3=0}^{\infty} (n_1 + n_2 + n_3)\, P_{n_1,n_2,n_3}(t)$$

Now putting $P_{n_1,n_2,n_3}(t)$

$$L = \sum_{n_1=0}^{\infty}\ \sum_{n_2=0}^{\infty}\ \sum_{n_3=0}^{\infty} (n_1 + n_2 + n_3)\, e^{-(A+B+C)} \frac{A^{n_1}}{n_1!}\cdot\frac{B^{n_2}}{n_2!}\cdot\frac{C^{n_3}}{n_3!}$$

$$L = e^{-(A+B+C)} \sum_{n_2=0}^{\infty}\ \sum_{n_3=0}^{\infty} \frac{B^{n_2}}{n_2!}\cdot\frac{C^{n_3}}{n_3!}\ \sum_{n_1=0}^{\infty} n_1 \frac{A^{n_1}}{n_1!} + (n_2 + n_3)\frac{A^{n_1}}{n_1!}$$

$$= e^{-(A+B+C)} \sum_{n_2=0}^{\infty}\ \sum_{n_3=0}^{\infty} \frac{B^{n_2}}{n_2!}\cdot\frac{C^{n_3}}{n_3!}\ Ae^{A} + (n_2 + n_3)\, e^{A}$$

$$= e^{-(B+C)} \sum_{n_3=0}^{\infty} \frac{C^{n_3}}{n_3!}\ \sum_{n_2=0}^{\infty} n_2 \frac{B^{n_2}}{n_2!} + (A + n_2)\frac{B^{n_2}}{n_2!}$$

$$= e^{-(B+C)} \sum_{n_3=0}^{\infty} \frac{C^{n_3}}{n_3!}\ Be^{B} + (A + n_2)\, e^{B}$$

$$= e^{-C} \sum_{n_3=0}^{\infty} n_3 \frac{C^{n_3}}{n_3!} + (A + B)\frac{C^{n_3}}{n_3!}$$

$$= e^{-C}\,[Ce^{C} + (A+B)\, e^{C}]$$

$$= A+B+C \qquad (10)$$

STEADY STATE SOLUTION:

The steady state solution of the model can be found in the usual manner by letting $t \rightarrow \infty$..

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Hence when $t \to \infty$, equation gives P_{n_1,n_2,n_3} the corresponding steady state probability, as"

$$P_{n_1,n_2,n_3} = \frac{1}{n_1!}\left(\frac{\lambda}{p_1}\frac{1}{\mu_1}-\frac{p_{21}}{\mu_1}\right)^{n_1}\frac{1}{n_2!}\left(\frac{\lambda}{p_2}\frac{1}{\mu_2}-\frac{1}{\mu_1}\right)^{n_2}\frac{1}{n_3!}\left(\frac{\lambda}{p_3}\frac{1}{\mu_3}-\frac{1}{\mu_2}\right)^{n_3}\exp -\frac{\lambda}{\mu_3} \qquad (11)$$

Also the mean queue size L of the system in the steady state case is obtained by letting $t \to \infty$ in which is as

$$L = \frac{\lambda}{\mu_3} \qquad (12)$$

PARTICULAR CASE:

On letting $\mu_2 \,\&\, \mu_3 \to \infty$ and no feedback from third server, then one can easily obtain the result of the model that corresponds to the M|M|1|∞ queueing model discussed by Gross and Harries. For this if we identify

$$P_{n_1,n_2,n_3}(t) = P_n(t)$$

$$(\text{with } n_1 = n,\ \mu_1 = \mu \ )$$

Then the result (3.1.37) corresponds to

$$P_n(t) = \frac{1}{n!}\ \frac{\lambda}{p_3\mu}\ (1 - e^{-\mu t})^{n}$$

$$n \geq 0 \qquad (13)$$

On the similar line one can also easily identify the study state results corresponds as

$$P_n = \frac{1}{n!}\left(\frac{\lambda}{p_3\mu}\right)^{n} \qquad n \geq 0 \qquad (14)$$

There will be no length of queue and the balance between arrival and operation pattern will be preserved. Another parameter can be determined using normal statistical formules, such as variances, distribution of busy times etc.

QUEUE MODEL WITH APPLICATIONS

This segment discusses the constant status conduct of a feedback queue network in which the consumer has two options to move to the next or to obtain feedback on a previous channel after being served on a serial channel. Arrivals and departures obey each channel's Poisson law. A solution technique was introduced to define various queue parameters by generating function (i.e. f. t), L-hospital rule, laws on calculus, and statistical formulas for marginal probabilities distribution. The goal of the analysis is to detect each channel and the total system's mean queue size. A numerical example supports the model.

MODEL DESCRIPTION:

There are three service channels in accordance with the model such that one unit or customer exits the unit or joins the next channel for further activity after service at the initial channel. The products found faulty are returned to the third channel after the service has been obtained on the second service channel in a similar way. The system's constant state behaviour was investigated with the application of the generation function technique, L-hose rules and statistical formulae for the distribution of the marginal likelihood, to find different queue parameters. Poisson is believed to be distributed on arrival and departure. For a decision-making phase, the medium queue size for each channel was achieved.

PROBLEM FORMULATION:

The Poisson units/customers arrive at S_1 in a medium μ, with a three-phase demand. The first service is completed in phase 1, and in phase 2 in phase 2 in phase 2 and in phase 3 in serial order in phase 3 in phase 3. After the first step, the units either exit S_1 with probability p_1 or pass through second channel S_2 with probability P_{12} so that the units either depart with probability p_2 after the service S_2 or pass through the second channel p_{23} or feedback on the previous channel p_{21} with probability (where items are detected defective), so that $p_2+p_{21}+p_{23}=1$. Similar feedback is obtained.

The service parameters are assumed to be μ_1, μ_2 , μ_3.

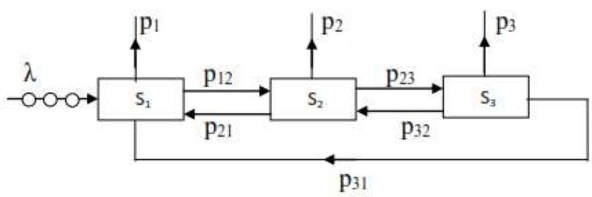

Figure-2

MATHEMATICAL ANALYSIS OF QUEUE MODEL:-

Define P_{n_1,n_2,n_3} There are possibly n1 in the system before the S_1 units, n_2 units before the S2, and n3 units before the S_2 units. of $S_1,\ (n_1, n_2, n_3 > 0)$ The standard argument leads to the following differential difference equations in steady state –

$$(\lambda+\mu_1+\mu_2+\mu_3)P_{n_1,n_2,n_3} = \lambda P_{n_1-1,n_2,n_3} + \mu_1 p_1 P_{n_1+1,n_2,n_3} + \mu_1 p_{12} P_{n_1+1,n_2-1,n_3} + \mu_2 p_2 P_{n_1,n_2+1,n_3} + \mu_2 p_{21} P_{n_1-1,n_2+1,n_3} + \mu_2 p_{23} P_{n_1,n_2+1,n_3-1} + p_3 \mu_3 P_{n_1,n_2,n_3+1} + \mu_3 p_{32} P_{n_1,n_2-1,n_3+1} + \mu_3 p_{31} P_{n_1-1,n_2,n_3+1} \quad n_1,n_2,n_3 > 0 \tag{15}$$

$$\lambda+\mu_1+\mu_2+\mu_3 \; P_{0,n_2,n_3} = \mu_1 p_1 P_{1,n_2,n_3} + \mu_1 p_{12} P_{1,n_2-1,n_3} + \mu_2 p_2 P_{0,n_2+1,n_3} + \mu_2 p_{23} P_{0,n_2+1,n_3-1} + p_3 \mu_3 P_{0,n_2,n_3+1} + \mu_3 p_{32} P_{0,n_2-1,n_3+1} \quad n_1 = 0, n_2, n_3 > 0 \tag{16}$$

$$\lambda+\mu_1+\mu_2+\mu_3 \; P_{n_1,0,n_3} = \lambda P_{n_1-1,0,n_3} + \mu_1 p_1 P_{n_1+1,0,n_3} + \mu_2 p_2 P_{n_1,1,n_3} + \mu_2 p_{21} P_{n_1-1,1,n_3} + \mu_2 p_{23} P_{n_1,1,n_3-1} + p_3 \mu_3 P_{n_1,0,n_3+1} + \mu_3 p_{31} P_{n_1-1,0,n_3+1} \quad n_2 = 0, n_1, n_3 > 0 \tag{17}$$

$$(\lambda+\mu_1+\mu_2+\mu_3)P_{n_1,n_2,0} = \lambda P_{n_1-1,n_2,0} + \mu_1 p_1 P_{n_1+1,n_2,0} + \mu_1 p_{12} P_{n_1+1,n_2-1,0} + \mu_2 p_2 P_{n_1,n_2+1,0} + \mu_2 p_{21} P_{n_1-1,n_2+1,0} + p_3 \mu_3 P_{n_1,n_2,1} + \mu_3 p_{32} P_{n_1,n_2-1,1} + \mu_3 p_{31} P_{n_1-1,n_2,1} \quad n_3 = 0, n_1, n_2 > 0 \tag{18}$$

$$(\lambda+\mu_1+\mu_2+\mu_3)P_{0,0,n_3} = \mu_1 p_1 P_{1,0,n_3} + \mu_2 p_2 P_{0,1,n_3} + \mu_2 p_{23} P_{0,1,n_3-1} + p_3 \mu_3 P_{0,0,n_3+1} \quad n_1, n_2 = 0, n_3 > 0 \tag{19}$$

$$(\lambda+\mu_1+\mu_2+\mu_3)P_{n_1,0,0} = \lambda P_{n_1-1,0,0} + \mu_1 p_1 P_{n_1+1,0,0} + \mu_2 p_2 P_{n_1,1,0} + \mu_2 p_{21} P_{n_1-1,1,0} + p_3 \mu_3 P_{n_1,0,1} + \mu_3 p_{31} P_{n_1-1,0,1} \quad n_2, n_3 = 0, n_1 > 0 \tag{20}$$

$$\lambda+\mu_1+\mu_2+\mu_3 \; P_{0,n_2,0} = \mu_1 p_1 P_{1,n_2,0} + \mu_1 p_{12} P_{1,n_2-1,0} + \mu_2 p_2 P_{0,n_2+1,0} + p_3 \mu_3 P_{0,n_2,1} + \mu_3 p_{32} P_{0,n_2-1,1} \quad n_1, n_3 = 0, n_2 > 0 \tag{21}$$

$$(\lambda+\mu_1+\mu_2+\mu_3)P_{0,0,0} = \mu_1 p_1 P_{1,0,0} + \mu_2 p_2 P_{0,1,0} + p_3 \mu_3 P_{0,0,1} \quad n_1, n_2, n_3 = 0 \tag{22}$$

CONCLUSION

Queuing Networks is the mathematical approach for the analysis of waiting lines. A trade-off decision is central to any Queueing Network: the manager must weigh up the additional costs of offering a faster service against the inherent costs of waiting. This research introduces a new fluffy approach to Queueing systems in the event of inconsistent knowledge available to remove the drawbacks of the point evaluation and the related paradoxes when measuring the efficiency of a queue. Both the arrival rate of Poisson and the exponential service times are taken into account with fuzzy estimators. To address the main estimation problem in uncertainty, the implementation of versatile estimators in M/M/S Queueing device performance measures is presented. Queuing delay is a common phenomenon and one of the most important fields in the management of operations. It comes when the demand in a system exceeds its ability or its capacity to provide service. The first applications for queuing theory began in the telecommunications field at the beginning of the 20th century. Later several researchers generalised it and today there are various applications in daily life. Traditional problems with the Queueing Network suggest that the rate of arrival at queue and the service delivery time are dependent on Poisson and exponential probability distribution. However, if we are forced to substitute random knowledge by ambiguity in a way if it is inaccurate about the present method. Fuzzy Queueing network methods offer a versatile and appropriate way to solve confusion in these issues. Our approach to queue efficiency avoids uncertainty prediction issues.

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Corresponding Author

Mamta*

Research Scholar, Department of Mathematics, BMU, Rohtak

mamtanagwan@gmail.com