

Study on Intuitionistic Fuzzy Graphs (IFGS)

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Abstract – The motivation for introducing IFGs is due to The first definition and concept of Intuitionistic Fuzzy Graph (IFG) was introduced. Karunambigai analyzed the properties of minmax IFGs in Shannon and Atanassov defined intuitionistic fuzzy graphs using five types of Cartesian products. In this paper IFGs so defined are named. Some isomorphic properties on IFGs are discussed. et al., discussed the properties of isomorphism on fuzzy graphs and properties of isomorphism on strong fuzzy graphs in which motivated us to develop the same on IFGs and on strong IFGs. The main aim of this study is to build basic definitions of an IFG which will be useful for the researchers for their future study in IFGs. Since Atanassov is the pioneer of our study, the title of the paper is given so. For graph theoretical definitions and throughout this paper all the properties are analyzed for simple minmax IFG.

Keywords – Intuitionistic, Fuzzy Graphs

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INTRODUCTION

The majority of our conventional formal modelling, reasoning, and computing tools are crisp, deterministic, and exact. Precision requires that a model's parameters accurately match either our perception of the phenomena being represented or the characteristics of the actual system being modelled. As a system's complexity grows, our capacity to make accurate but meaningful claims about its behaviour decreases until a point is reached beyond which accuracy and significance are practically mutually incompatible properties. Furthermore, while building a model, we constantly try to make it as helpful as possible. The link between three fundamental properties of any system model: complexity, credibility, and uncertainty, is crucial to this goal. Uncertainty plays a critical role in maximising the utility of system models. Almost all conventional logic implies that exact symbols are being used. "Vagueness" is one of the definitions given to the phrase "uncertainty." That instance, the difficulty in distinguishing between acute and precise distinctions. This is true for many everyday phrases, such as X is 'tall,' Y is 'beautiful,' the sky is 'cloudy,' and so on. It's crucial to remember that the imprecision or ambiguity that characterises natural language doesn't always indicate a loss of precision or meaning.

Lotfi. A. Zadeh proposed a mathematical framework to characterise this phenomenon in his important article "Fuzzy Sets". The crisp set is designed in such a manner that it divides persons in a discourse universe into two groups: members and non-members, whose logic is fully based on Aristotle's "A

or not A." There is a clear and unmistakable separation between members and non-members of the crisp set's class. However, many of the terms we often employ, such as 'height,' 'beautiful,' and other 'linguistic variables,' do not have this property. In his work, Kosko refers to this as the Mismatch problem: While the rest of the world is murky, science is black and white. "Everything is a question of degree," says the fuzzy principle. As a result, membership in a fuzzy set is a question of degree rather than affirmation or denial. As a result, the fuzzy logic: A and Not A, is the basic logic.

A fuzzy set may be mathematically defined by assigning a value to each potential person in the universe of discourse that represents their fuzzy set membership grade. This grade refers to how comparable or compatible that person is with the notion represented by the fuzzy set. A fuzzy subset of a set S is a map termed the membership function with a smooth rather than abrupt transition from membership to non-membership. As a result, using fuzzy sets as a kind of continuously valued logic seems natural.

The following question was in the air from the moment Zadeh's major work appeared. Isn't fuzzy set theory just another name for probability theory? The response has always been a resounding 'no.' the example of swamp water in perfectly demonstrates the spirit. Another obvious distinction is that the sum of probability on a finite universal set must equal 1, but membership grades have no such condition. In probability theory, Aristotle's rule always holds. While a probability density function

may be used to create a membership function, the opposite is not always true. As a result, combining probability theory with fuzzy set theory may result in a 'generalised information theory.'

Fuzzy sets' capacity to convey progressive transitions from membership to non-membership and vice versa has a wide range of applications. It not only gives us a useful and strong representation of uncertainty measurement, but it also gives us a useful and powerful representation of hazy notions articulated in plain language. The mathematical embedding of conventional set theory into fuzzy sets is as natural as the concept of embedding real numbers into the complex plane since every crisp set is fuzzy but not the other way around. As a result, the concept of fuzziness is one of richness rather than replacement. Because subjective sentiments are difficult to describe, the degree of membership is often read as the percentage of a sufficiently large number of referees who agree with the assertion that x belongs to S .

Both within mathematics and in its applications, research on the topic of fuzzy sets has been growing at an exponential rate. This includes everything from logic, topology, algebra, and analysis to pattern recognition, information theory, artificial intelligence, operations research, neural networks, and planning, among other things. As a result, fuzzy set theory has arisen as a promising multidisciplinary study topic.

The fundamental concepts of fuzzy sets have been examined in this part, as well as the current findings.

Definition 1.1 (Zadeh 1965) Let X be any nonempty set and let $I = [0, 1]$. Then any map $\mu: X \rightarrow I$ is called a fuzzy subset of X .

Definition 1.2 (Zadeh 1965) Let μ and ν be fuzzy subsets of a set X . If $\mu(x) \leq \nu(x), \forall x \in X$, then $\mu \subseteq \nu$ and if $\mu(x) = \nu(x), \forall x \in X$, then μ and ν are said to be equal.

Definition 1.3 (Zadeh 1965) Let μ be a fuzzy subset of a set X and $\alpha \in [0, 1]$. Then the set $\alpha_\mu = \{x \in X \mid \mu(x) \geq \alpha\}$ is called α cut of μ .

Definition 1.4 (Biswas 1990). Let μ be a fuzzy subset of a set X and $\alpha \in [0, 1]$. Now the set $\bar{\alpha}_\mu = \{x \in X \mid \mu(x) \leq \alpha\}$ is called μ lower level subset of μ .

Definition 1.5 (Zadeh 1965) Let μ be a fuzzy subset of X . The set $\{x \in X \mid \mu(x) > 0\}$ is called support of μ and it is denoted by supp .

Definition 1.6 (Wong 1974 b). Let X be any nonempty set. A fuzzy point p of X is a fuzzy subset of X which has singleton support $\{x\}$ and fuzzy value $\mu(x) \in (0, 1)$ at that point.

Definition 1.7 (Rekha et al 1981) Let X be any nonempty set. A fuzzy point p with support $\{x\}$ is said to lie in $\mu, p \in \mu$, if $\mu(x) < \mu(x)$.

Definition 1.8 (Pu and Liu 1980 a) Let X be any nonempty set. A fuzzy singleton p of X is a fuzzy subset of X which has singleton support $\{x\}$ and fuzzy value $\mu(x) \in (0, 1)$ at that point.

Definition 1.9 (Pu and Liu 1980 a) Let X be any nonempty set. A fuzzy singleton p with support $\{x\}$ is said to lie in $\mu, p \in \mu$, if $\mu(x) \leq \mu(x)$.

Definition 1.10 (Zadeh 1965) The union $\cup \mu_\alpha$ and intersection $\cap \mu_\alpha$ of any family $B = \{\mu_\alpha\}_{\alpha \in I}$ of fuzzy sets are given respectively by $(\cup \mu_\alpha)(x) = \sup \mu_\alpha(x)$ and $(\cap \mu_\alpha)(x) = \inf \mu_\alpha(x)$, where supremum and infimum are taken over all members μ_α of B .

Note 1.1 (Wong 1974 b) If p is a fuzzy singleton then $p \in \mu_i$, for some $i \Rightarrow p \in \cup \mu_i$, but $p \in \cup \mu_i$ does not imply $p \in \mu_i$, for some i . However if p is a fuzzy point, then $p \in \cup \mu_i \Leftrightarrow p \in \mu_i$, for some i .

BASIC CONCEPTS OF INTUITIONISTIC FUZZY SETS

The definitions of intuitionistic fuzzy sets, as well as existing findings in the literature, are presented in this section. The non-membership grade of an element to reside in a fuzzy subset equals 1 membership grade of that element to lie in the fuzzy subset in fuzzy set theory. For example, if a person has 0.6 membership grades in the fuzzy subset of a good man, he has 0.4 membership grades in the fuzzy subset of not a good guy, according to fuzzy set theory. However, in real-life situations, this is no longer the case.

To address this issue, Atanassov (1986) introduced the concept of intuitionistic fuzzy sets. The idea of fuzzy sets is generalised in this concept.

Definition 1.11 (Atanassov 1986) Let X be a nonempty set. An intuitionistic fuzzy set A of X is defined by $A = \{(\mu_A, \nu_A)\}$, where: $\mu_A: X \rightarrow [0, 1]$ and $\nu_A: X \rightarrow [0, 1]$ with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$. The numbers $\mu_A(x), \nu_A(x) \in [0, 1]$ denote the degree of membership and non-membership of x to lie in A respectively.

Definition 1.12 (Atanassov 1986) Assume that X is a nonempty set, and that $A = \{(\mu_A, \nu_A)\}$ and $B = \{(\mu_B, \nu_B)\}$ be two intuitionistic fuzzy sets of X . Then

1. $A \subset B$ iff $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.
2. $A = B$ iff $A \subset B$ and $B \subset A$.

3. $A \cup B = (\mu_A \vee \mu_B, \nu_A \wedge \nu_B)$
 $= (\max(\mu_A, \mu_B), \min(\nu_A, \nu_B)).$
4. $A \cap B = (\mu_A \wedge \mu_B, \nu_A \vee \nu_B)$
 $= (\min(\mu_A, \mu_B), \max(\nu_A, \nu_B)).$
5. $A \times B(x, y) = (\mu_{A \times B}, \nu_{A \times B})(x, y)$
 $= (\min\{\mu_A(x), \mu_B(y)\}, \max\{\nu_A(x), \nu_B(y)\})$
6. $A^c = (\nu_A, \mu_A).$
7. $[]A = (\mu_A, 1 - \mu_A), <> A = (1 - \nu_A, \nu_A).$

Definition 1.13 (Coker 1997) Let $\{A_i = (\mu_{A_i}, \nu_{A_i}) \mid i \in J\}$, If X is an arbitrary family of intuitionistic fuzzy sets, then (a) $(\bigcap_{i \in J} A_i) = (\bigwedge_{i \in J} \mu_{A_i}, \bigvee_{i \in J} \nu_{A_i})$; (b) $(\bigcup_{i \in J} A_i) = (\bigvee_{i \in J} \mu_{A_i}, \bigwedge_{i \in J} \nu_{A_i})$
 If X is an arbitrary family of intuitionistic fuzzy sets, then where \wedge and \vee are defined in usual.

REVIEW OF LITERATURE

Dubois and Prade (2012) published a review of fuzzy set aggregation connectives that provides an in-depth look at fuzzy set-theoretic operations, emphasising the importance of functional equation theory in the axiomatic construction of classes of such operations and the derivation of functional representations. The use of fuzzy set theory to multifactorial assessment is the focus of the second section. The relationship between this method and multiattribute utility theory was investigated.

The idea of 'intuitionistic fuzzy set' (IFS) was established by KrassimirAtanassov (2013) as an extension of the concept of 'fuzzy set.' The characteristics of various operations and relations over sets, as well as modal and topological operators defined over the set of IFS's, are proven. New findings on intuitionistic fuzzy sets were presented by KrassimirAtanassov (2012). On intuitionistic fuzzy sets, two novel operators, the modal operator and the topological operator, are defined and their fundamental characteristics are investigated. Krassimir Atanassov (2013) made another set of statements on intuitionistic fuzzy sets, this time discussing the relationship between some intuitionistic fuzzy set (IFS) A and the universe F, which is a universe of the IFS E, where the latter is a universe of A. Along with the current operations, KrassimirAtanassov (1994) specified four new operations (@, \$, # and *) (U, ∩, + and) over the intuitionistic fuzzy sets and some of their basic properties were discussed. Later KrassimirAtanassov (1996) proved equality between

intuitionistic fuzzy sets. It is proved that for every two intuitionistic fuzzy sets

$$A \text{ and } B : ((A \cap B) + (A \cup B)) @ ((A \cap B) \cdot (A \cup B)) = A @ B.$$

Bustince and Burillo (1996) investigated the link and coincidence between intuitionistic fuzzy sets defined by KrassimirAtanassov and vague sets defined by Gau and Byehrer. Intuitionistic fuzzy sets were defined by KrassimirAtanassov and George Gargov (1998) based on the definition of several types of intuitionistic fuzzy logics. They created two versions of intuitionistic fuzzy propositional calculus (IFPC) and an intuitionistic fuzzy predicate logic version (IFPL). Krassimir Atanassov (2000) established two theorems about the relationships between some of the operators defined over intuitionistic fuzzy sets.

Concentration, dilatation, and normalisation of intuitionistic fuzzy sets were established by Supriya Kumar et al. (2014). These definitions will come in handy when dealing with linguistic hedges such as "extremely," "more or less," "very," "very very," and other terms that are used in situations in an intuitionistic fuzzy environment.

Gallo et al. definitions of directed hypergraph subfamilies may be linked to older definitions such as the one offered by Ausilo et al. B-graph, F-graph, and BF-graph are examples of subfamilies. A digraph is an example of a BF-graph.

A hypergraph's visual depiction is just as significant as that of graphs and digraphs. Makinen proposed the subset standard and the edge standard, two hypergraph drawing concepts based on techniques for defining hypergraphs. For directed hypergraphs, the edge standard is the ideal option since the hyperedges may be drawn as two sets linked by lines. In fact, practically every study on the topic has adopted this graphic portrayal.

Because hyperedges naturally give a representation of implication dependencies, directed hypergraphs have a wide range of uses. Among other things, they were used to answer a number of difficulties in propositional logic relating to satisfiability, particularly in relation to Horn formulae. They also show up in issues involving network checking, chemical reaction networks, and, more recently, convex polyhedraalgorithms in tropical algebra. Many algorithmic elements of directed hypergraphs related to optimization have been explored, including calculating shortest pathways, maximum flows, least cardinality cuts, and minimal weighted hyperpaths. None of the directed graph techniques can be extended to directed hypergraphs, unfortunately. The fundamental reason for this is because hypergraphs' reachability relations do not have the same structure.

Karunambigai et al. (2017) used the intuitionistic fuzzy graph approach to investigate the shortest routes in networks. One of the most basic and extensively used notions in networks is shortest routes. Fuzzy networks, as well as generalised techniques for finding optimum pathways within them, have lately emerged as a useful modelling tool for imprecise systems. Fuzzy shortest routes may be used in a number of ways. The authors introduced a dynamic programming-based methodology for finding the shortest pathways in intuitionistic fuzzy graphs.

Thilagavathi et al. (2015) provided intuitionistic fuzzy analogues of various core fuzzy graph theoretic notions, as well as an alternative definition for intuitionistic fuzzy graph.

The notion of complement of an intuitionistic fuzzy graph (IFG) was examined by Parvathi et al. (2009), and several aspects of self-complementary IFGs were shown. Union, join, cartesian product, and composition are some of the operations on intuitionistic fuzzy graphs that are specified, as well as some of its features.

OBJECTIVES

1. To investigate edge dominance in fuzzy graphs and intuitionistic fuzzy graphs in a safe and fair manner.
2. For fuzzy hypergraphs and intuitionistic fuzzy hypergraphs, get line graph construction.

RESEARCH METHODOLOGY

The predominant emphasis reported in this thesis is positioned to have a look at intuitionistic fuzzy hypergraphs. On the inspiration of intuitionistic fuzzy graphs (IFGs) and fuzzy hypergraphs (undirected and directed), this work extends to an intuitionistic fuzzy hypergraphs (undirected and directed)(IFHG). Based on the definition of IFHG, twin of an IFHG, Strength, (α, β) - cut are described. The operations like complement, join, union, intersection, ring-sum, Cartesian product, composition are defined and are illustrated with suitable examples. The component is dedicated to intuitionistic fuzzy directed graphs (IFDGs), intuitionistic fuzzy directed hypergraphs (IFDHGs) and their index illustration. The operations described on IFDGs and IFDHGs using index matrices are very interesting and they're used to observe the structural behaviour of IFDGs and IFDHGs. Final component, isomorphism among IFDHGs is defined and some of their isomorphic houses are studied. A new approach specifically, the score based totally approach for finding shortest hyperpaths in a network with intuitionistic fuzzy weights for hyperedges with out defining similarity degree and Euclidean distance, is likewise established. The rankings of triangular intuitionistic fuzzy numbers and rating the paths primarily based

on lowest accuracy helps the decision maker to pick out the best intuitionistic fuzzy shortest hyperpath.

DATA ANALYSIS

An ordered pair of disjoint subsets of vertices $E = (X, Y)$, is an intuitionistic fuzzy directed hyperedge; X is the tail of E and Y is its head $T(E)$ and $H(E)$ are the tail and head of the intuitionistic fuzzy directed hyper-edge E, respectively.

A BIFD - edge, or backward intuitionistic fuzzy directed hyper-edge, is a hyper-edge $E = (T(E), H(E))$ with $|H(E)| = 1$. A FIFD - edge, or forward intuitionistic fuzzy hyper-edge, is a hyper-edge $E = (T(E), H(E))$ with $|T(E)| = 1$. A backward intuitionistic fuzzy directed hypergraph (BIFD - graph) is an intuitionistic fuzzy directed hypergraph with BIFD - edges. A forward intuitionistic fuzzy hypergraph (or FIFD - graph) is an intuitionistic fuzzy hypergraph with FIFD - edges as hyper-edges. A BF - graph (also known as a BF - intuitionistic fuzzy directed hypergraph) is an intuitionistic fuzzy directed hypergraph with hyper-edges that are either BIFD or FIFD. A BIFD hypergraph or FIFD hypergraph with BIFD - edges or FIFD - edges is an intuitionistic fuzzy directed hypergraph. Figure 4.1 shows an example of an intuitionistic fuzzy directed hypergraph.

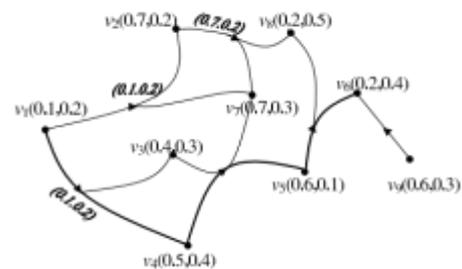


Figure 1.1: Intuitionistic fuzzy directed hypergraph

A route between nodes s and t in an IFDHG G is an alternating sequence of different nodes and intuitionistic fuzzy hyper-edges $s=v_0, e_1, v_1, e_2, \dots, e_k, v_k = t$, such that v_i, e_i for all $i = 1, 2, \dots, k$. A route between any two unique nodes is shown in Figure 4.2.

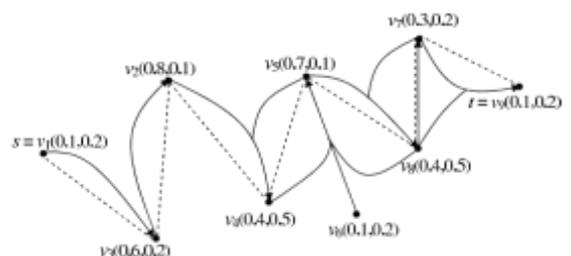


Figure 1.2: A path between s and t in an intuitionist fuzzy directed hypergraph

CONCLUSION

We Defined is the intuitionist fuzzy directed hyper graph (IFDHG). In intuitionist fuzzy hyper graphs, the shortest route problem is one of the most researched issues. The length of pathways in a network issue is represented by triangular intuitionist fuzzy numbers (TriIFNs). In an intuitionist fuzzy environment, a novel approach called the score based method is suggested for determining the shortest hyper path. The decision maker may determine the preferred intuitionist fuzzy shortest hyper path using the scores of TriIFNs and ranking the pathways based on lowest accuracy.

Some operations including addition, multiplication, vertex wise multiplication, and structural subtraction of both IFGs and IFDHGs are investigated with suitable examples, and a novel approach of encoding IFDHG, called index matrix representation, has been proposed. The original graphs undergo structural alterations as a result of these processes.

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