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Constraint Satisfaction Problem in Matrices with COP

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Abstract – We consider the accompanying imperative fulfillment issue: Given a set F of subsets of a limited set S of cardinality n, and an assignment of intervals of the discrete set {1, . . . , n} to each of the subsets, does there an exist a bijection f : s →{1,....n} to such an extent that for every component of F, its picture under f is same as the interval alloted to it. An interval assignment to a given arrangement of subsets is called plausible if there exists such a bijection. In this paper, we describe doable interval assignments to a given arrangement of subsets. We at that point utilize this outcome to describe matrices with the Consecutive Ones Property(COP), and to portray matrices for which there is a stage of the rows with the end goal that the columns are altogether arranged in rising request. We additionally present a portrayal of set frameworks which have a possible interval assignment.

Keywords: Constraint, Satisfaction, Matrices, Cop.

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INTRODUCTION

The COP is an intriguing and basic combinatorial property of binary matrices. The COP shows up in numerous applications; information recovery, DNA physical mapping, grouping gathering, interval graph acknowledgment, also, perceiving Hamiltonian cubic graphs. Testing if a given graph is an interval graph, and testing if a given cubic graph is Hamiltonian are uses of algorithms for testing if a given 0-1 matrix has COP. The maximal coterie vertex rate matrix is tried for COP to check if a given graph is an interval graph . Thus, from a cubic graph is Hamiltonian if the matrix A + I has a change of rows that leaves at most two squares of consecutive ones in every column. An is the nearness matrix of the given graph what's more, I is the personality matrix. Testing if a matrix has COP is additionally connected for building physical maps by hybridization and testing if a database has the consecutive recovery property To request a stage of the rows with the end goal that every column is arranged is a characteristic augmentation of the COP. For 0-1 matrices this inquiry is contemplated as the idea of 1-drop matrices. Past work. The principal notice of COP, as indicated by D.G. Kendall [8], was made by Petrie, a prehistorian, in 1899. A few heuristics were proposed for testing the COP in before crafted by Fulkerson and Net who introduced the main polynomial time algorithm. In this manner Tucker displayed a portrayal of matrices with the COP dependent on certain illegal matrix arrangements. Corner and Lueker proposed the principal straight time algorithm for the issue utilizing a ground-breaking information structure called the PQ-Tree. This information structure exists if a just if the given matrix has the COP. Hsu displayed another straight time algorithm for testing COP without utilizing PQ-trees. All the more as of late in 2001, he presented another information structure called PC tree as a speculation of PQ-Tree. This was utilized to test if a binary matrix has the circle Ones Property (CROP). Another speculation of the PQ-tree is the PQR tree presented by Meidanis and Munuera. This speculation was a decent expansion of the methodology of Booth and Leuker so that PQR trees are characterized notwithstanding for matrices that don't have the COP. Further, for matrices that do not have the COP, the PQR tree calls attention to explicit subcollections of columns in charge of the nonappearance of the COP. In 2003, a relatively direct time algorithm has been proposed to develop a PQR-tree. Our Work. Our inspiration in this work was to understandl the Consecutive Ones Testing (COT) algorithm due to and to extend it to finding a change of the rows of matrix with the end goal that the columns are all arranged. Obviously, to sort only one column, we can without much of a stretch recognize a group of row stages that accomplishes the arranging. So for every column in a given matrix we can relate a lot of arranging changes. The inquiry presently is whether the intersection of these sets, one for every column, is vacant or not? In this paper we recognize a characteristic concise portrayal of the arranging changes of a column. This prompts the inquiry that we present in the abstract: Given an interval assignment to a set framework, is it doable? We at that point present a fundamental and adequate condition for an interval assignment to be practical. Specifically, we demonstrate that an interval assignment to a set framework is doable if and just on the off chance that it saves the cardinality of the intersection of each combine of sets. While a plausible interval assignment should fundamentally fulfill this property, shockingly we don't discover this portrayal in the writing, certainly not expressly to the best of our insight. We utilize this portrayal to describe matrices with the COP, and portray matrices that whose columns can be arranged by a row stage. We additionally demonstrate a vital and adequate condition for a plausible interval assignment to exist. Our verifications are largely useful and can be effortlessly changed over into algorithms that keep running in polynomial time in the information measure.

A critical outcome of this work is the thing that we see as the modularization of COT algorithm due to Hsu. Two fundamental modules in the COT algorithm are to discover an attainable interval assignment to the columns of a 0-1 matrix, and after that to discover a change that is observer to the plausibility of the interval assignment. Our investigation in this paper can likewise be viewed as an alternate point of examine, but along the profession started by Meidanis et al. In their work, they think about the set framework related with the columns of the matrix. Specifically their outcomes discover a conclusion of the set framework which additionally has the COP if the given set framework has the COP. In this paper, we adopt another regular strategy to consider the set framework related with the columns of the matrix. We think about the arrangement of row stages that yield consecutive ones in the columns of a matrix. We at that point ask how this set gets pruned when another column is added to the matrix. During the time spent noting this inquiry, we utilize the deterioration of the given matrix into prime matrices as done. Our work likewise opens up normal speculations of the COP. For instance, given a matrix is there a change of the rows with the end goal that in every column the rows are apportioned into at most two arranged arrangements of consecutive rows?. This would be a fascinating method to order matrices, and the combinatorics of this appears to be extremely fascinating and non-unimportant. This would likewise be a whiz combinatorial speculation of the k-drop property for 0-1 matrices which is considered in and references in that

CHARACTERIZATION OF FEASIBLE INTERVAL ASSIGNMENTS

In this paper $\{A_1, \dots, A_m\}$ is a set of subsets of $\{1, \dots, n\}$. Let $r_i = |A_i|, \le i \le m$. An interval assignment to $\{A_1, \dots, A_m\}$ is the set $\{(A_i, B_i) \mid 1 \le i \le m, B_i \subseteq \{1, \dots, n\}$, and elements of B_i are consecutive}. B_i is used to denote the interval assigned to $A_i, 1 \le i \le m$. Further, an interval here is a set of consecutive integers from the set $\{1, \dots, n\}$. An intersection Cardinality Preserving Interval Assignment (ICPIA) to $\{A_1, \dots, A_m\}$ is a set of ordered pairs $\{(A_i, B_i) \mid 1 \le i \le m\}$ such that for every two sets A_i and A_j $|A_i \cap A_j| = |B_i \cap B_j|$. We also use the ordered pair (P, Q) to denote the assignment of interval Q to theset P. Since in each ordered pair (P, Q), $|P| = |Q|$, we also use (P, Q) to represent all permutations of $\{1, \dots, n\}$ such that the set P is mapped to the interval Q. An interval assignment $\{(A_i, B_i) \mid 1 \le i \le m\}$ is defined to be *feasible* if thereis a permutation of $\{1, \dots, n\}$ such that for each $1 \le i \le m$, the image of A_i under the permutation is the interval B_i. Two intervals are said to be *strictly intersecting* if theirintersection is non-empty and neither is contained in the other.

Theorem 1. *If an interval assignment $\{(A_i, B_i) \mid 1 \le i \le m\}$ is feasible, then it is an ICPIA.*

Proof. Since the interval assignment $\{(A_i, B_i) \mid 1 \le i \le m\}$ is feasible, there is permutation σ such that $\sigma(A_i) = B_i, 1 \le i \le m$. Since σ is a permutation it follows that $|A_i| = |B_i|$. Further, for the same reason for all $1 \le i, j \le m, \sigma(A_i \cup A_j) = B_i \cup B_j$, and therefore $|A_i \cap A_j| = |B_i \cap B_j|$. Consequently, the interval assignment is an ICPIA. Hence our claim.

FEASIBLE PERMUTATION FROM AN ICPIA

We now show that given an ICPIA $\{(A_i, B_i) \mid 1 \le i \le m\}$, there is a permutation a of

($1,\dots,n$} such that $\sigma(A_i) = B_i, 1 \le i \le m$. Without loss of generality, we assume that the ordered pairs in the ICPIA are indexed according to the order obtained by sorting the left end point of the intervals ($B_i s$) in the ICPIA, and ties are broken by sorting in ascending order of right end points. In other words, the interval B_1 has the smallest left end point among all intervals and the interval (B_m) has the largest left end point. Before we outline the algorithm for constructing a feasible permutation from the ICPIA, we prove the following two crucial lemmas.

Lemma 1.*Let* (A_1, B_1), (A_2, B_2), (A_3, B_3) *beelements of an* ICPIA. Then, $|A_1 \cap A_2 \cap A_3||B_1 \cap B_2 \cap B_3|$.

Proof. If for any two intervals the intersections areempty, then the corresponding sets haveempty intersection, and therefore, it follows that the intersection of the 3 intervals is empty, and so is the intersection of the 3 sets. The claim is true in this case. Therefore, we consider the case when the pairwise intersection of the intervals is non-empty. By the Belly Property, if a set of intervals are such that the pairwise intersection is non-empty, then the intersection of all the intervals in the set is also non-empty. Further, it is also clear that if three intervals have a non-empty intersection, then one of the intervals is contained in the union of the other two.

Without loss of generality, let $B_3 \subseteq B_1 \cup B_2$, therefore $|B_1 \cup B_2 \cup B_3| = |B_1 \cup B_2| = |A_1 \cup A_2| \le |A_1 \cup A_2 \cup A_3|$. Further it is also clear that $|B_1 \cap B_2 \cap B_3| = \min\{|B_1 \cap B_2|, |B_1 \cap B_3|, |B_2 \cap B_3|\}$. Without loss of generality, let us assumethat $|B_1 \cap B_2 \cap B_3| = |B_2 \cap B_3|$. Applying this to the Inclusion-Exclusion formula for $|B_1 \cup B_2 \cup B_3|$, we get $|B_1 \cup B_2 \cup B_3| = |B_1| + |B_2| + |B_3| - |B_1 \cap B_2| - |B_1 \cap B_3|$. The r.h.s is in turn equal to $|A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| \ge |\{A_2 \cup A_3\} \backslash A_1| + |A_1| = |A_1 \cup A_2 \cup A_3|$. Therefore, it follows that $|A_1 \cup A_2 \cup A_3| = |B_1 \cup B_2 \cup B_3|$ From, the given hypothesis and the Inclusion-Exclusion formula it now follows that $|A_1 \cap A_2 \cap A_3| = |B_1 \cap B_2 \cap B_3|$ Hence the proof.

Corollary 1.*Let* (A_1, B_1), (A_2, B_2), (A_3, B_3) *beelements of an ICPIA. Then,* $|(A_1 \backslash A_2) \cap A_3| = |(B_1 \backslash B_2) \cap B_3|$.

Proof.
Clearly, $|(A_1 \backslash A_2) \cap A_3| = |(A_1 \backslash (A_1 \cap A_2))| = |A_1 \cap A_3| - |(A_1 \cap A_2) \cap A_3|$. From lemma 1 we know that $|(A_1 \cap A_2) \cap A_3| = |(B_1 \cap B_2) \cap B_3|$, and that $|A_1 \cap A_3| = |B_1 \cap B_3|$ follows from the fact that we have an ICPIA. Therefore, it follows that $|A_1 \cap A_3| - |(A_1 \cap A_2) \cap A_3| = |B_1 \cap B_3| - |(B_1 \cap B_2) \cap B_3| = |(B_1 \backslash B_2) \cap B_3|$. Hence the corollary.

Algorithm 1 Permutations from an ICPIA of M

Let $P_1 = A_1, Q_1 = B_1$
Let $\Pi_0 = \{(A_i, B_i) | 1 \le i \le m\}$
$j = 1$;
while There is $(P_1, Q_1), (P_2, Q_2) \in \Pi_{j-1}$ with Q_1 and Q_2 strictly intersecting **do**
  $\Pi_j = \Pi_{j-1} \setminus \{(P_1, Q_1), (P_2, Q_2\}$;
  $\Pi_j = \Pi_j \cup \{(P_1 \cap P_2, Q_1 \cap Q_2), (P_1 \setminus P_2, Q_1 \setminus Q_2), (P_2 \setminus P_1, Q_2 \setminus Q_1)\}$;
  j = j+1;
end while
$\Pi = \Pi_j$;
Return Π;

In Algorithm 1, Π_j represents the set {$\sigma : \{1,\dots,n\} \to \{1,\dots,n\} | \sigma$ is a permutation, and {$\sigma(A) = B, \forall (A, B) \in \Pi_j$}. We now prove that Π_m represents a set of permutations of the rows such that the ones in each column are consecutive.

Lemma 2.***At theend of the j-th iteration,* $j \ge 0$*, of the while loop of Algorithm 1, the following three are invariant.***

- Invariant I*:* Q *is an interval for each* (P, Q) $\in \Pi_j$.

- Invariant II: $|P| = |Q|$ *for each* (P, Q) $\in \Pi_j$.

- Invariant III: *For any two*(P', Q'),(P'', Q'') $\in \Pi_j$, $|P' \cap P''| = |Q' \cap Q''|$

Proof. The proof of the lemma is by induction on j, which is the number of times the while loop has executed. For $j = 0$, by definition, Π_0 = {(A_i, B_i)$|1 \le i \le m|$}. All the invariants hold because we are dealing with an ICPIA. Therefore the base case is proved. Let us assume that the lemma holds for $j-1$. Now we now show that the lemma holds for j. First, invariant I holds due to the following reason: If $(P,Q) \in \Pi_j$ and Π_{j-1}, then by the induction hypothesis Q is an interval, $|P| = |Q|$, and invariant II also holds. If $(P,Q) \in \Pi_j$, but not in

$\in \Pi_{j-1}$, then it means that (P,Q) is one of the following three pairs for some (P_1,Q_1), $(P_2,Q_2) \in \Pi_{j-1}$ I such that Q_1 and Q_2 are strictly intersecting: $(P_1 \cap P_2, Q_1 \cap Q_2)$, or $(P_1 \backslash P_2, Q_1 \backslash Q_2)$, or $(P_2 \backslash P_1, Q_2 \backslash Q_1)$. By invariant III of the induction hypothesis, it follows that $|P| = |Q|$. Since the Q_1 and Q_2 are strictly intersecting, it follows that Q is an interval. To prove invariant III, let us consider a pair (P', Q'),$(P'', Q'') \in \Pi_j$. If both arein $\in \Pi_{j-1}$, then invariant III holds. If one of them is not in $\in \Pi_{j-1}$, then it is one of the following three pairs for some (P_1,Q_1), $(P_2,Q_2) \in \Pi_{j-1}$ where Q_1 and Q_2 are strictly intersecting: $(P_1 \cap P_2, Q_1 \cap Q_2)$, or $(P_1 \backslash P_2, Q_1 \backslash Q_2)$, or $(P_2 \backslash P_1, Q_2 \backslash Q_1)$. Now applying lemma 1 and corollary 1, it follows that in this case too for each pair (P', Q'),$(P'', Q'') \in \Pi_j$, $|P' \cap P''| = |Q' \cap Q''|$ Therefore the induction hypothesis is proved. Hence thelemma.

Theorem 2. *Let* $\{(A_i, B_i) | 1 \leq i \leq m\}$ *be an ICP1A. Then, there is a permutation* $\sigma: \{1, \ldots, n\} \to \{1, \ldots, n\}$ *such that* $\sigma(A_i) = B_i$.

Proof. Consider Π output by Algorithm 1 for $\{(A_i, B_i) | 1 \leq i \leq m\}$. For the sake of ease, we add (A_0, B_0) to Π, where $A_0 = B_0 =$ {1,….,n}. Clearly, from the algorithm, for any two (P_1,Q_1), $(P_2,Q_2) \in \Pi$, either Q_1 and Q_2 are disjoint, or one is contained in the other. In other words, they cannot be strictly intersecting. So to further refine Π, we consider the following tree, which can be called a containment tree. The nodes of this tree represent $(P,Q) \in \Pi$. Let (P_1,Q_1) and (P_2,Q_2) be theelements of Π associated with two nodes. There is an edge from the node corresponding to (P_1,Q_1) to the node corresponding to (P_2,Q_2) if and only if Q_1 is the largest interval that contains Q_2, among all the ordered pairs in Π. The root of the tree is the pair (A_0, B_0). Since the $Q_i s$ are intervals, this data structure is a tree which we denoteby T. We now refine Π as outlined in Algorithm 2 using the function call Post-Order-Traversal$(T, (A_0, B_0))$. Let the resulting set be Π_{end} which is a set of ordered pairs (P_i, Q_i), $\geq m$ is a finite number. In an ordered pair $(P_i, Q_i) \in \Pi_{end}$, Q_i is not necessarily an interval. However, for any two (P_1,Q_1), $(P_2,Q_2) \in \Pi_{end}$, $|P_1 \cap P_2| = |Q_1 \cap Q_2| = 0$, and $|P_i| = |Q_i|$. The other property is that $1 \leq j \leq m$ the image of A_j remains B_j. The reason is that each (A_j, B_j) is only *broken* into smaller sets in both Algorithm 1 and Algorithm 2. Therefore, any permutation that maps P_i to Q_i for each $(P_i, Q_i) \in \Pi_{end}$ satisfies all the constraints specified by the ICPIA $\{(A_i, B_i) | 1 \leq i \leq m\}$. Hence Π_{end} represents a family of permutations such that for each permutation $\sigma, \sigma(A_i) = B_i, 1 \leq i \leq m$.

Algorithm 2 Permutations from Π obtained from Algorithm 1

```
function Post-Order-Traversal(T,root-node)
if (root-node is a leaf) then
   return
end if
while (root-node has unexplored children) do
   next-root-node = an-unexplored-child-of-root-node
   Post-Order-Traversal(T,next-root-node)
end while
if (root-node has no unexplored children) then
   Let (P,Q) denote the element of Π associated with root-node
   Let (P1,Q1)...(Pk,Qk) be the pairs associated with the children of root-node
   Π ← Π \ {(P,Q)}
   Π ← Π ∪ {(P \ (P1 ∪ ... ∪ Pk), Q \ (Q1 ∪ ... ∪ Qk))}
   return
end if
```

Theorems 1 and 2 together prove that an interval assignment $\{(A_i, B_i) | 1 \leq i \leq m\}$ is feasible if and only if it is an ICPIA. We now use this result to characterize matrices whose rows can be rearranged to obtain desired interval-properties on the columns. The basic idea is to associatea set system with each columns based on the desired property, and then test if the resulting problem instance has an ICPIA.

CHARACTERIZING MATRICES WITH THE COP

Definition and Notation: An $m \times n$ matrix M with 0-1 entries is said to have the consecutive ones property (COP) if there is a permutation of the rows such that in the resulting matrix the ones occur consecutively in each column. Such a permutation is said to leave consecutive ones in the columns. Our characterization of matrices with the COP provides a new analysis of a recent Consecutive Ones Testing algorithm. For each $1 \leq i \leq m,$ let $A_i = \{p | M_{pi} = 1\}$ Let $r_i = |A_i|$ denote the number of ones in the *i-th* column. Let $B_i = \{l, l+1, \dots, l+r_i-1\}$ denote an interval assigned to the i-th column, where $1 \leq 1 \leq n$. From the results of the previous section, the following theorem holds as an application of the results obtained in the previous section in a more general setting.

Theorem 3. *A 0-1 matrix M has the COP if and only if thereexists an ICPIA* $\{(A_i, B_i\} | 1 \leq i \leq m\}$.

The problem of finding a permutation of the rows of a matrix such that each column is sorted in ascending order can be solved by creating a natural set system on the same lines as outlined for testing the COP.

STRUCTURAL CHARACTERIZATION OF MATRICES WITH AN ICPIA

In this section we address the question of whether a given set system $\{A_i \subseteq \{1, \dots, n\} | 1 \leq i \leq m\}$ has an ICPIA. Quite naturally we view the given set system as a $n \times m$ binary matrix M. In M, the *j*-th column corresponds to the set A_j and M_{ij} if and only if $i \in A_j$, otherwise $M_{ij} = 0$. Note that the columns of M are distinct. In the rest of this section, we say that a matrix M has an ICPIA if there is an ICPIA $\{(A_i, B_i) | 1 \leq i \leq m\}$ where $A_i = \{p | M_{pi} = 1\}$ We also refer to the *j*-th column of M as theset A_j. This is the word set in a matrix is used to refer to the set associated with a column. We recall the notion of matrix decomposition introduced .

An undirected graph on the columns of M: With the given matrix M, associate an undirected graph $G(M)$ where the vertices correspond to $A_i, 1 \leq i \leq m$. We assume that vertex v_i corresponds to set A_i. $\{v_i, v_i\} \in E(G(M))$ if an only if the corresponding sets intersect and neither is contained in the other. A maximal set of columns of M is called a prime submatrix of M if the corresponding subgraph of G is connected. Let us denote the prime submatrices by $M_1, \dots, M_p$. Clearly, two distinct matrices have a distinct set of columns. Let $col(M_i)$ be the set of columns in thesubmatrix M_i. We also introduce the notation for the support of a prime sub-matrix

$$M_i; supp(M_i) = \bigcup_{j \in col(M_i)} A_j.$$

For a set of prime sub-matrices X we define

$$supp(X) = \bigcup_{M \in X} supp(M).$$

A Partial Order on the prime sub-matrices: Consider the relation << on the prime sub-matrices $M_1, \dots, M_p$ defined as follows:

$\{(M_i, M_j) |$ A set $S \in M_i$ is contained in a set $S' \in M_j\} \cup \{(M_i, M_i) | 1 \leq i \leq p\}$

(1)

Lemma 3. *Let* $(M_i, M_j) \in <<$. *Then there in set* $S' \in M_j$ *such that for each* $S \in M_i, S \subseteq S'$.

Proof. Since $(M_i, M_j) \in <<$, it follows, by definition of $<<$, that there is an $S' \in M_j$ and $S \in M_j$ such that $S \subseteq S'$. We want to prove that each set of M_i is contained in S'. We prove this by contradiction. Let $T \in M_i$ be the first vertex in a path in $G(M_i)$ from $S \in M_i$ such that $T \not\subseteq S'$. Let $T' \in M^i$ be the neighbor of T on the path. Clearly, $T' \subseteq S'$ C S'. Further $T \in M_j$, and neither is contained in the other. Therefore, $T \cap S' \neq \emptyset$. By our assumption, $T \not\subseteq S'$. Therefore, $T \in M_j$. This is a contradiction to the fact that two distinct prime sub-matrices have distinct sets of columns. Therefore, our assumption of theexistenceof T is wrong. Hence the lemma.

Lemma 4. *For each pair of prime sub-matrices, either* $(M_i, M_j) \in << \; or \; (M_j, M_i) \in <<$

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Proof. The proof is by contradiction. If we assume that for two distinct i and j, $(M_i, M_j) \in <<$ and $(M_j, M_i) \in <<$, then from lemma 3 that there is an $S \in M_i$ such that each $S' \in M_i$ is contained in S. Since the columns of M are distinct, this is a contradiction to the definition of M_i. Therefore, our assumption is wrong. Hence the lemma is proved.

Lemma 5. *If* $(M_i, M_j) \in <<$ *and* $(M_j, M_i) \in <<$, *then* $(M_i, M_k) \in <<$.

Proof. This follows from lemma 3 and the definition of containment. **Lemma 8.** *If* $(M_i, M_j) \in <<$ and $(M_i, M_k) \in <<$, then either $(M_j, M_k) \in << \; or \; (M_k, M_j) \in <<$

Proof. The proof is by contradiction. Let us assume that both (M_j, M_k) and (M_k, M_j) are not in $<<$. Along with the fact that M_j and M_k are prime sub-matrices, this implies that $supp(M_j)$ and $supp(M_k)$ are disjoint. Further, from lemma 3 we know that $supp(M_i)$ is strictly contained in $supp(M_j)$ and $supp(M_k)$. This is a contradiction to the conclusion that $supp(M_j) \cap supp(M_k) = \emptyset$ which follows from the assumption that (M_j, M_k) and (M_k, M_j) are not in $<<$. Hence the lemma.

Theorem 4. $<<$ *is a partial order on the set of prime sub-matrices of* M. *Further.it uniquely partitions the prime sub-matrices of* M *such that on each set in the partition << induces a total order.*

Proof. This follows from the previous four lemmas and the fact that $<<$ is reflexive by definition.

Lemma 7. *A* $0-1$ *matrix M has an ICPIA if and only if each prime sub-matrix has an ICPIA.*

Proof. If M has an ICPIA, then by definition each prime sub-matrix has an ICPIA. We now prove the reverse direction by construction. Let us assume that each $M_i, 1 \le i \le p$ has an ICPIA. Let $X_i, \ldots, X_1$ be the partition mentioned in theorem 4. From the definition of a prime sub-matrix and the definition of $<<$ it follows that $supp(X_r) \cap supp(X_s) = \emptyset$ for each $1 \le r \ne s \le l$. Therefore, to complete our construction, weidentify an interval $I(X_k), 1 \le k \le l$, and then prove our claim for a generic set in the partition. The interval $I(X_k)$ is written as $[l(X_k), r(X_k)]$. Here $I(X_1) = 1, r(X_k) = l(X_k) + |supp(X_k)| - 1$, for $1 \le k \le l$, and $+ \, l(X_k) = r(X_{k-1}) + 1$ for $2 \le k \le l$. Clearly, $I(X_k)$ is the interval which will contain the intervals assigned to the columns in the matrix formed by the prime sub-matrices in X_k. We next prove the claim for a generic set, Say X_k, in the partition. Let $M_{1k}, \ldots, M_{jk^k}$ be the sink to source order of the prime sub-matrices in theset X_k. Here j_k denotes the number of prime sub-matrices in X_k. From the definition of $<<$, for each $r, 2 \le r \le jk$, $supp(M_{rk})$ is contained in at least one set in $M_{(r-1)k}$. Therefore, it follows that $supp(X_k) = supp(M_{1k})$. For the construction, we associate an interval with each prime sub-matrix in X_k. For $jk \ge r \ge 2$, Let C_{rk} denotethe set of intervals assigned to those sets of $M_{(r-1)k}$ which contain $supp(M_{rk})$. We define

$$I(M_{rk}) = \bigcap_{I \in Crk} I.$$

The interval associated with M_{1k} is $I(M_{1k}) = [l(X_k), l(X_k) + |supp(M_{1k})| - 1]$.For $1 \le r \le jk$, let us consider the interval I' obtained by taking the union of intervals in an ICPIA associated with M_{rk}; we have this by the hypothesis. We know that $|I'| = |supp(M_{rk})|$ since I' is the set of intervals obtained from anICPIA assigned to the sets in M_{rk}. Further, for each $r, 1 \le r \le jk, |(supp(M_{rk})| \le |I(M_{rk})|$. Therefore, $|I'| \le |I(M_{rk})|$. To complete the construction, we order theelements of from the smallest point to the largest point, and map the i-th rank element of I' to the i-th rank element of $I(M_{rk})$. Clearly, this bijection takes each interval in the ICPIA given by the hypothesis and yields an

ICPIA that is completely contained in $I(M_{rk})$. This construction yields an ICPIA for the prime sub-matrices of X_k such that each interval in this assignment is contained in $I(X_k)$. Consequently, this yields an ICPIA for M. Hence the reverse direction is proved, and consequently the lemma is proved.

AN ALGORITHM FOR FINDING AN ICPIA

Here we show that it is possible to find an ICPIA to the columns of a given binary matrix M in polynomial time, provided there is one. The algorithm 3 is based on the structural characterization described above in this section and the algorithm 4. In algorithm 3 the function ICPIA($M', I(M')$) assigns an ICPIA to a prime sub-matrix M' in theinterval $I(M') = [l(M'), r(M')]$. Basically, the function ICPIA$(M', I(M'))$ is a loop that calls Algorithm 4 for each column of M'.

Algorithm 3 Algorithm to find an ICPIA for a matrix M

Identify the prime sub-matrices. This is done by constructing the strict overlap graph and identify connected components. Each connected component yields a prime sub-matrix.
Construct the partial order << on the set of prime sub-matrices.
Construct the partition $X_1, \ldots, X_l$ of the prime sub-matrices induced by << and find $I(X_k)$.
Construct the total order on each set in the partition.
for $(k = 1; k \leq l; k++)$ **do**
  Let $I(M_{1k}) = [l(X_k), l(X_k) + supp(M_{1k}) - 1]$
  $ICPIA(M_{1k}, I(M_{1k}))$
  for $(r = 2; r \leq j_k; r++)$ **do**
    Construct C_{rk} from the ICPIA assigned to sets of $M_{(r-1)k}$.
    Let $I(M_{rk}) = \bigcap_{I \in C_{rk}} I$
    $ICPIA(M_{rk}, I(M_{rk}))$
  end for
end for

In algorithm 4, theelements of the set $\{S^1, \ldots, S^p\}$ are the sets corresponding to p columns, of M', that have been assigned an ICPIA among them. Let this ICPIA be $\{I^1, \ldots, I^p\}$. Further, let S' be the set such that the sets of M' that intersect with it have a pairwise non-empty intersection. The interval I' assigned to S' is $[l(M'), l(M') + |S^1| - 1]$. Now, let S denote the set corresponding to the *j*-th column such that S has a non-empty intersection with some $S^i, 1 \leq i \leq p$, and $S \not\subseteq S^i, S^i \not\subseteq S$. The algorithm 4 describes how S is assigned an interval I such that $\{I^1, \ldots, I^p, I\}$, is an ICPIA for $\{S^1, \ldots, S^p, I\}$.

Theorem 5. *Algorithm 4 outputs an ICPIA to a prime matrix M' iff there is an ICPIA for M'.*

Proof. The only-if part of the theorem is straightforward.We now show that if there is an ICPIA for M', then Algorithm 4 will indeed discover it. The key fact is that in M' for each set S, there is another set $T \in M'$ such that $S \cap T \neq \emptyset$, and S and T are not contained in each other. Due to this fact, there areexactly two ICPIAs for M'. The two distinct ICPIAs differ based on the interval assigned to S_1, see Algorithm 4.If It is assigned to S_1, then we get one, and the other ICPIA is obtained by assigning I_r to S_1. Fbr each subsequent set, say S^j, the interval to be assigned is forced. It is forced due to the fact that the interval assigned to S^j is based on the interval assigned to S^i, where S^i n $S^i \cap S^j \neq \emptyset$, and $S^i \not\subseteq S^j$, and $S^j \not\subseteq S^i$. Given the fact that the algorithm is anexact implementation of these observations, it follows that Algorithm 4 finds an ICPIA if there is one.

CONCLUSION

We have presented the thought of an ICPIA formally and have demonstrated that an interval assignment is practical on the off chance that and just on the off chance that it is an ICPIA. We at that point utilize this perception to describe matrices that have the consecutive ones property, in this manner giving a more up to date comprehension of Hsu's algorithm for COT. This combinatorial seeing additionally prompts a portrayal of matrices whose rows can be permuted with the goal that every column is arranged. At long last, we have additionally introduced an algorithm to test if a set framework has an ICPIA utilizing approaches created by.

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