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Static Charged Fluid Spheres in General Relativity

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***Abstract** – The present paper provides solution of Einstein-Maxwell field equations for static spherically symmetric metric by using a seditious choice of metric potential α and β (ie. g_{11} and g_{44}). The central and boundary conditions have been also discussed.*

***Keywords** : Charged, Fluid Sphere, Metric Potential, Maxwell Field.*

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INTRODUCTION

A number of authors have already studied the charged fluid distribution in equilibrium Bonnor [1], Effinger [3] and kyle and Martin [7] have considered the interior solution for a static charged sphere. As the fluid equations do not completely determine the system different solutions were obtained the Effinger [3], kyle and Martin [7] by using different conditions.

Some exact static solutions of Einstein- Maxwell equations representing a charged fluid sphere were obtained by Singh and Yadav [11], Shi-Chang [10] found some conformal flat interior solutions of the Einstein- Maxwell equations for a charged stable static sphere. These solution an exact solution satisfy physical conditions inside the sphere. Xingxiang [13] obtained by satisfying matter distribution and charge distribution. The metric is regular and can be matched to the Reissner-Nordstrom metric and pressure is finite. In the limit of vanishing charge, the soluteion can reduce to the interior solution of an uncharged sphere. Buchdahal [2] has also considered some regular general relativistic charged fluid spheres. Some other cases of the interior solutions for charged fluid sphere have been presented by Whitman and Burch [12], Krori and Barua [6], Junevicous [5], Nduka [8], Sah and Chandra [9], Fulare and Sah [4].

In this paper spherically symmetric metric, we have solved Einstein-Maxwell field equations by using different assumptions on metric potential α and β (ie. g_{11} and g_{44}). These solutions satisfy physical conditions. The central and boundary conditions have been also discussed. The pressure and density have been found for the distribution.

2. THE FIELD EQUATIONS

We use here the line element in the form

(2.1) $ds^2 = e^{\beta}\, dt^2 - e^{\alpha}\, dr^2 - r^2\ (d\theta^2 + sin^2\theta . d\emptyset^2)$

where α and β are function of r only.

(2.2) $R_{ij} - \frac{1}{2} . R\, g_{ij} = -8\pi\, T_{ij}$

(2.3) $F^{ij}_{;j} = 4\pi J^i = 4\pi\sigma u^i$

(2.4) $F_{[ij:k]} = 0$

where T_{ij} is the energy momentum tensor, J^i is the charged current four vector, R_{ij} is the Ricci tensor and R the scalar of curvature tensor.

For the system under study the energy momentum tensor T^i_j splits up into two parts viz $\overline{T^i_j}$ and E^i_j for matter and charge respectively

(2.5) $T^i_j = \overline{T^i_j} + E^i_j$

where

(2.6) $\overline{T^i_j} = \left[(\rho + p)u^i u_j - p\delta^i_j\right]$

with

(2.7) $u^i u_i = 1$

The non- vanishing components of $\overline{T_j^i}$ are

$$(2.8)\ \overline{T_1^1} = \overline{T_2^2} = \overline{T_3^3} = -p;\ \overline{T_4^4} = \rho$$

Here p is internal pressure, ρ and σ are densities of matter and charges respectively and u^i is the velocity vector of matter.

The static condition is given by

$$(2.9)\ u^1 = u^2 = u^3 = 0 \text{ and } u^4 = (g_{44})^{-\frac{1}{2}}$$

$$\text{i.e. } u^4 = e^{-\frac{\beta}{2}}$$

The electromagnetic energy momentum tensor E_j^i is given by

$$(2.10)\ E_j^i = -- F_{jk}F^{ik} + \frac{1}{4}\ \delta_j^i\ F_{im}F^{im}$$

We assume the field to be purely electrostatic, i.e. F_{ik} = 0 and F_{jk} = $\emptyset$, where $\emptyset$ is the electrostatic potential.

Thus the Einstein- Maxwell field equations are cast into the form

$$(2.11)\ e^{-\alpha}\left(\frac{1}{r^2} - \frac{\alpha'}{r}\right) - \frac{1}{r^2} = -8\pi\rho - E,$$

$$(2.12)\ \frac{1}{r^2} - e^{-\alpha}\left(\frac{1}{r^2} + \frac{\beta'}{r}\right) = -8\pi p + E,$$

$$(2.13)\ e^{-\alpha}\left[\frac{1}{4}\beta'\alpha' - \frac{1}{4}\beta'^2 - \frac{1}{2}\beta'' - \frac{1}{2}\left(\frac{\beta'-\alpha'}{r}\right)\right] = -8\pi p - E,$$

where

$$(2.14)\ E = -F^{41}F_{41}$$

and

$$(2.15)\ 4\pi\sigma = \left(\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + \frac{\alpha'+\beta'}{2}F^{41}\right)e^{\beta/2}$$

By the use of equations (2.11) – (2.13), we get the expressions for p, ρ and E as

$$(2.16)\ 8\pi p = \frac{e^{-\alpha}}{2}\left(\frac{3\beta'}{2r} + \frac{\beta''}{2} - \frac{\alpha'\beta'}{4} + \frac{\beta'^2}{4} - \frac{\alpha'}{2r} + \frac{1}{r^2}\right) - \frac{1}{2r^2},$$

$$(2.17)\ 8\pi\rho = e^{-\alpha}\left(\frac{5\alpha'}{4r} - \frac{\beta''}{4} + \frac{\alpha'\beta'}{8} - \frac{\beta'^2}{8} + \frac{\beta'}{4r} - \frac{1}{2r^2}\right) + \frac{1}{2r^2},$$

$$(2.18)\ 2E = e^{-\alpha}\left(\frac{\beta''}{2} - \frac{\alpha'\beta'}{4} + \frac{\beta'^2}{4} - \frac{\beta'}{2r} - \frac{\alpha'}{2r} - \frac{1}{r^2}\right) + \frac{1}{r^2}.$$

3. SOLUTION OF THE FIELD EQUATIONS

We take α and β of the form

$$(3.1)\ \alpha = A\,r^{2m} + Br^2 + C$$

$$(3.2)\ \beta = \mu r^2 + \vartheta r + K$$

Where A,B,C, μ, ϑ and K are constants and m is a positive integer (m $\neq 0$). Then equations (2.15)-(2.18) yield

$$(3.3)\ 8\pi p = \frac{e^{-(Ar^{2m}+Br^2+C)}}{2}\left[\frac{(6\mu-2B)r+3\vartheta-2mAr^{2m-1}}{2r} + \frac{(4\mu+\vartheta^2)-2mAr^{2m-1}(2\mu r+\vartheta)-4\mu r^2(B-\mu)+2\vartheta r(2\mu-B)}{4} + \frac{1}{r^2}\right] - \frac{1}{2r^2}$$

$$(3.4)\quad 8\pi\rho = \frac{e^{-(Ar^{2m}+Br^2+C)}}{2}\left[\frac{10mAr^{2m-1}+10Br+2\mu r+\vartheta}{2r} + \frac{1}{4}\{4mA\mu r^{2m} + 2mA\vartheta r^{2m-1} + 2\vartheta r(B-2\mu) - 4\mu - \vartheta^2 + 4\mu r^2(B-\mu)\} - \frac{1}{r^2}\right] + \frac{1}{2r^2},$$

$$(3.5)\ E = \frac{e^{-(Ar^{2m}+Br^2+C)}}{2}\left[\frac{-2\mu r-2Br-\vartheta-2mAr^{2m-1}}{2r} + \frac{4\mu+\vartheta^2-2mAr^{2m-1}(2\mu r+\vartheta)-4\mu r^2(B-\mu)+2\vartheta r(2\mu-B)}{4} - \frac{1}{r^2}\right] + \frac{1}{2r^2},$$

$$(3.6)\ 4\pi\sigma = \left[\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F + {}^{41}\frac{1}{2}\{2mAr^{2m-1} + 2r(B+\mu) + \vartheta\}F^{41}\right]\left(e^{\frac{\mu r^2+\vartheta r+K}{2}}\right)$$

Now matching the solution with Reissner-Nordstrain metric at the boundary

R = r_b we have

$$(3.7)\ e^{-(Ar_b^{2m}+Br_b^2+C)} = \left(1 - \frac{2M}{r_b} + \frac{Q_b^2}{r_b^2}\right),$$

$$(3.8)\ e^{(\mu r_b^2+\vartheta r+K)} = \left(1 - \frac{2M}{r_b} + \frac{Q_b^2}{r_b^2}\right),$$

$$(3.9)\ (2\mu r_b + \vartheta)\ e^{(\mu r_b^2+\vartheta r+K)} = 2\left(\frac{M}{r_b^2} - \frac{Q_b^2}{r_b^3}\right),$$

In particular if we take A= $\vartheta = 0$

$$(3.10)\ 16\pi p = e^{-(Br^2+C)}\left[\frac{1}{r^2} - B - B\mu r^2 + \mu(\mu r^2 + 4)\right] - \frac{1}{r^2}$$

$$(3.11)\ 16\pi\rho = e^{-(Br^2+C)}\left[5B + B\mu r^2 - \mu^2 r^2 - \frac{1}{r^2}\right] + \frac{1}{r^2}$$

$$(3.12)\ 2E = e^{-(Br^2+C)}\left[\mu^2 r^2 - B(1+\mu r^2) - \frac{1}{r^2}\right] + \frac{1}{r^2}$$

$$(3.13)\ 4\pi\sigma = \left[\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + (B+\mu)F^{41}\right]e^{\frac{\mu r^2+K}{2}}$$

At r = 0, these results give (taking c= 0)

(3.14) $16\pi p_0 = 4\mu - B$,

(3.15) $16\pi \rho_0 = 5B$,

(3.16) $E_0 = -\frac{B}{2}$.

(3.17) $e^{-(Br_b^2+C)} = \left(1 - \frac{2M}{r_b} + \frac{Q_b^2}{r_b^2}\right)$,

(3.18) $e^{\mu r_b^2} = 1 - \frac{2M}{r_b} + \frac{Q_b}{r_b^2}$,

(3.19) $\mu e^{\mu r_b^2} = \frac{M}{r_b^3} - \frac{Q_b}{r_b^4}$,

DISCUSSION AND CONCLUSION:

For physically realistic solution pressure and density should by greater than or equal to zero so, we have from equation (3.4) and (3.15).

$$(3.20) \quad 4\mu \geq B \geq 0$$

Further for $\rho_0 \geq 3p_0$

$$(3.21) \quad 2B \geq 3\mu$$

From conditions (3.20) and (3.21) we have

$$(3.22) \quad 4\mu \geq B \geq \frac{3}{2}\mu$$

Thus the constant B lies between 4μ and $\frac{3}{2}\mu$.

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