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An Analysis upon the Level-3 Reformulation-Linearization Technique (RLT) for the Quadratic Assignment Problem

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Abstract – We apply the level-3 Reformulation Linearization Technique (RLT3) to the Quadratic Assignment Problem (QAP). We then present our experience in calculating lower bounds using an essentially new algorithm, based on this RLT3 formulation. This algorithm is not guaranteed to calculate the RLT3 lower bound exactly, but approximates it very closely and reaches it in some instances. Calculating lower bounds for problems sizes larger than size 25 still presents a challenge due to the large memory needed to implement the RLT3 formulation. Our presentation emphasizes the steps taken to significantly conserve memory by using the numerous problem symmetries in the RLT3 formulation of the QAP.

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INTRODUCTION

The QAP is the various maximum hard combinatorial optimization issues. This is unlucky, considering the fact that a substantial array of programs arises in facility format and design. Solving widespread problems of length extra than N = 30 , i.E. With extra than 900 binary variables, is still computationally impractical.

Although the QAP is NP-tough, this complexity isn't always enough to explain its difficulty, as different classes of NP-difficult problems can be solved a long way extra efficaciously than the QAP. The majority of QAP take a look at problems have a homogeneous goal characteristic, and this contributes to their difficulty. Such homogeneity tends to provide bounds which are less powerful in pruning partial answers inside binary seek bushes. Among precise algorithms, department-and-bound methods are the maximum a success, however loss of tight lower bounds has been one of the most important hindrances.

The earlier computational revel in the use of in the beginning degree-1 and then degree-2 RLT QAP formulations has indicated promising studies directions. The resulting linear representations, Problems RLT1 and RLT2, are an increasing number of massive in size and especially degenerate. In order to solve those problems, Hahn and Grant (1998) and Adams et al. (2006) have offered a dual-ascent approach that exploits the block-diagonal structure of constraints within the degree-1 and level-2 bureaucracy, respectively. This strategy is a effective extension of that found in Adams and Johnson (1994).

Problem RLT2 , specially, offers sharp lower bounds. (2006), and therefore ends in very aggressive genuine answer methods. A hanging outcome is the notably few variety of nodes taken into consideration within the binary search tree to affirm optimality. This leads to marked fulfillment in solving difficult QAP times of size N ≥ 24 in record computational time.

In this chapter, I introduce the extent-3 RLT formulation of the QAP, and display that the hierarchy of quadratic, cubic and biquadratic challenge troubles is directly related to the RLT hierarchy. Also supplied are the superior decrease bounds supplied by means of the level-three RLT QAP version.

THE BIQUADRATIC ASSIGNMENT PROBLEM (BQAP)

The QAP minimizes a quadratic function over an task matrix. Burkard et al. (1994) gave the definition of the biquadrate challenge hassle (BQAP) which arose within the subject of VLSI synthesis. The BQAP is to minimize a weighted sum of products of four binary variables subject to a couple of choice constraints on those variables.

Given two 4-dimensional matrices of N^4 elements $\bar{A} = \left[\alpha_{ikpg}\right]$ and $\bar{B} = \left[\beta_{jnqh}\right]$, the BQAP can be written as

$$\min \left\{ \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{n=1}^{N}\sum_{p=1}^{N}\sum_{q=1}^{N}\sum_{g=1}^{N}\sum_{h=1}^{N} \alpha_{ikpg}\beta_{jnqh}x_{ij}x_{kn}x_{pq}x_{gh} \;:\; \boldsymbol{x} \in \boldsymbol{X}, \boldsymbol{x} \text{ binary} \right\}, \qquad (6\text{-}1a)$$

$$\text{where } \boldsymbol{x} \in \boldsymbol{X} \equiv \left\{ \boldsymbol{x} \geq 0 : \sum_{i=1}^{N} x_{ij} = 1, \forall (j = 1,\dots,N); \sum_{j=1}^{N} x_{ij} = 1, \forall (i = 1,\dots,N) \right\}. \qquad (6\text{-}1b)$$

Notice that the constraints of the BQAP are the same old a couple of preference constraints over the challenge matrix. Thus, the solution matrix X = $\left[x_{ij}\right]$ To the BQAP is also a permutation matrix. By representing the variables xij via a permutation of the set 1,..., N , one receives the following formulation in permutation as

$$\min_{\varphi \in \Gamma_N} \sum_{i=1}^{N}\sum_{k=1}^{N}\sum_{p=1}^{N}\sum_{g=1}^{N} \alpha_{ikpg}\beta_{\varphi(i)\varphi(k)\varphi(p)\varphi(g)}, \qquad (6\text{-}2)$$

whereΓ_N denotes the set of all permutations of {1,..., *N*} and ϕ Γ_N . If the coefficients $E = \left[e_{ijknpqgh}\right] \in \mathbb{R}^{N^8}$ are the prices associated with the goods of 4 binary variables, the BQAP becomes

$$\min \left\{ \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{n=1}^{N}\sum_{p=1}^{N}\sum_{q=1}^{N}\sum_{g=1}^{N}\sum_{h=1}^{N} e_{ijknpqgh}x_{ij}x_{kn}x_{pq}x_{gh} \;:\; \boldsymbol{x} \in \boldsymbol{X}, \boldsymbol{x} \text{ binary} \right\}. \qquad (6\text{-}3)$$

An alternative way is to show linear costs $\boldsymbol{B} = \left[b_{ij}\right] \in \mathbb{R}^{N^2}$, quadratic costs $\boldsymbol{C} = \left[c_{ijkn}\right] \in \mathbb{R}^{N^4}$,cubic costs$\boldsymbol{D} = \left[d_{ijknpq}\right] \in \mathbb{R}^{N^6}$and biquadratic costs$\boldsymbol{E} = \left[e_{ijknpqgh}\right] \in \mathbb{R}^{N^8}$ individually. Now the BQAP is

$$\min \left\{ \begin{aligned} &\sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij}x_{ij} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N} c_{ijkn}x_{ij}x_{kn} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N}\sum_{\substack{p=1\\p\neq i,k}}^{N}\sum_{\substack{q=1\\q\neq j,n}}^{N} d_{ijknpq}x_{ij}x_{kn}x_{pq} \\ &+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N}\sum_{\substack{p=1\\p\neq i,k}}^{N}\sum_{\substack{q=1\\q\neq j,n}}^{N}\sum_{\substack{g=1\\g\neq i,k,p}}^{N}\sum_{\substack{h=1\\h\neq j,n,q}}^{N} e_{ijknpqgh}x_{ij}x_{kn}x_{pq}x_{gh} \\ &: \boldsymbol{x} \in \boldsymbol{X}, \boldsymbol{x} \text{ binary} \end{aligned} \right\}. \qquad (6\text{-}4)$$

THE CUBIC ASSIGNMENT PROBLEM (CAP) AND THE LEVEL-2 RLT FORMULATION OF THE QAP

Similarly, I outline for the primary time the cubic mission trouble (CAP), that's to limit the weighted sum of products of 3 binary variables over the identical undertaking matrix. If one uses the notations above, the CAP may be formulated as

$$\min \left\{ \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{n=1}^{N}\sum_{p=1}^{N}\sum_{q=1}^{N} d_{ijknpq}x_{ij}x_{kn}x_{pq} \;:\; \boldsymbol{x} \in \boldsymbol{X}, \boldsymbol{x} \text{ binary} \right\}, \qquad (6\text{-}5)$$

Or,

$$\min \left\{ \sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij}x_{ij} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N} c_{ijkn}x_{ij}x_{kn} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N}\sum_{\substack{p=1\\p\neq i,k}}^{N}\sum_{\substack{q=1\\q\neq j,n}}^{N} d_{ijknpq}x_{ij}x_{kn}x_{pq} \;:\; \boldsymbol{x} \in \boldsymbol{X}, \boldsymbol{x} \text{ binary} \right\}. \qquad (6\text{-}6)$$

If one introduces the variables yijkn and zijknpq to replacement the products of $y_{ijkn} = x_{ij}\, x_{kn}$ and $z_{ijknpq} = x_{ij}\, x_{kn}\, x_{pq}$ respectively, the system of the CAP is similar to the level-2 RLT version of the QAP, that is repeated under. (Construction details of Problem RLT2 given in Section 2.5.2).

[RLT2]

$$\min \sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij}x_{ij} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N} c_{ijkn}y_{ijkn} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N}\sum_{\substack{p=1\\p\neq i,k}}^{N}\sum_{\substack{q=1\\q\neq j,n}}^{N} d_{ijknpq}z_{ijknpq} \qquad (6\text{-}7a)$$

$$\text{s.t.} \quad \sum_{\substack{p=1\\p\neq i,k}}^{N} z_{ijknpq} = y_{ijkn} \qquad i,j,k,n,q = 1,\dots,N; q \neq n \neq j, k \neq i, \qquad (6\text{-}7b)$$

$$\sum_{\substack{q=1\\q\neq j,n}}^{N} z_{ijknpq} = y_{ijkn} \qquad i,j,k,n,p = 1,\dots,N; p \neq k \neq i, n \neq j, \qquad (6\text{-}7c)$$

$$z_{ijknpq} = z_{knijpq} = z_{ijpqkn} = z_{knpqij} = z_{pqijkn} = z_{pqknij} \qquad i,j,k,n,p,q = 1,\dots,N; p > k > i, q \neq n \neq j, \qquad (6\text{-}7d)$$

$$z_{ijknpq} \geq 0 \qquad i,j,k,n,p,q = 1,\dots,N; p \neq k \neq i, q \neq n \neq j, \qquad (6\text{-}7e)$$

$$\sum_{\substack{k=1\\k\neq i}}^{N} y_{ijkn} = x_{ij} \qquad i,j,n = 1,\dots,N; n \neq j, \qquad (6\text{-}7f)$$

$$\sum_{\substack{n=1\\n\neq j}}^{N} y_{ijkn} = x_{ij} \qquad i,j,k = 1,\dots,N; k \neq i, \qquad (6\text{-}7g)$$

$$y_{ijkn} = y_{knij} \qquad i,j,k,n = 1,\dots,N; k > i, n \neq j, \qquad (6\text{-}7h)$$

$$y_{ijkn} \geq 0 \qquad i,j,k,n = 1,\dots,N; k \neq i, n \neq j, \qquad (6\text{-}7i)$$

$$\boldsymbol{x} \in \boldsymbol{X}, \boldsymbol{x} \text{ binary}. \qquad (6\text{-}7j)$$

Since the additional constraints (6-7b)-(6-7i) in Problem RLT2 are derived absolutely from the substitutions of y_{ijkn} = xij xkn and z_{ijknpq} = xij xkn x_{pq} , you can use the extent-2 RLT QAP version to solve the CAP.

THE LEVEL-3 RLT FORMULATION OF THE QAP

One ought to assume the level-three RLT QAP to be used for solving the BQAP. The level-three RLT system is constructed as follows. In its reformulation step, multiply every of 2N equality constraints and every of N 2 non negativity regulations (which are rewritten in variables xkn) through every of N 2binary variables xij .Then, multiply every of 2N equality constraints and every of N 2nonnegativity restrictions (which can be rewritten in variables xpq) through each of N 2 (N −1)2 pair wise products of variables xij xkn (k ≠ i and n ≠ j) . Then, multiply each of 2N equality constraints and every of N 2 non negativity restrictions (which can be rewritten in variables xgh) via every of N2 (N −1).

restrictions. Express the various resulting products in the order $x_{ij}x_{kn}$, $x_{ij}x_{kn}x_{pq}$ and $x_{ij}x_{kn}x_{pq}x_{gh}$. Substitute $x_{ij} = x_{ij}^2$, reduce $x_{ij}x_{ij}x_{kn}$ and $x_{kn}x_{ij}x_{kn}$ to $x_{ij}x_{kn}$, and reduce $x_{ij}x_{ij}x_{kn}x_{pq}$, $x_{kn}x_{ij}x_{kn}x_{pq}$ and $x_{pq}x_{ij}x_{kn}x_{pq}$ to $x_{ij}x_{kn}x_{pq}$. Set $x_{ij}x_{kn} = 0$ if $(k = i$ and $n \neq j)$ or $(k \neq i$ and $n = j)$ in all quadratic expressions. And set $x_{ij}x_{kn}x_{pq} = 0$ if $(p = i$ and $q \neq j)$, $(p = k$ and $q \neq n)$, $(p \neq i$ and $q = j)$ or $(p \neq k$ and $q = n)$ in all cubic expressions. And set $x_{ij}x_{kn}x_{pq}x_{gh} = 0$ if $(g = i$ and $h \neq j)$, $(g = k$ and $h \neq n)$, $(g = p$ and $h \neq q)$, $(g \neq i$ and $h = j)$, $(g \neq k$ and $h = n)$ or $(g \neq p$ and $h = q)$ in all biquadratic expressions. Then, in the linearization step, substitute every occurrence of each product $x_{ij}x_{kn}$ $(k \neq i$ and $n \neq j)$ with a single nonnegative continuous variable y_{ijkn}. And, substitute every occurrence of each product $x_{ij}x_{kn}x_{pq}$ $(p \neq k \neq i$ and $q \neq n \neq j)$ with a single nonnegative continuous variable z_{ijknpq}. Also, substitute every occurrence of each product $x_{ij}x_{kn}x_{pq}x_{gh}$ $(g \neq p \neq k \neq i$ and $h \neq q \neq n \neq j)$ with a single nonnegative continuous variable $v_{ijknpqgh}$. Enforce the symmetric restrictions that $y_{ijkn} = y_{knij}\ \forall (i,j,k,n = 1,...,N), k > i, n \neq j$. Also, enforce the symmetric restrictions that

$$z_{ijknpq} = z_{knijpq} = z_{ijpqkn} = z_{knpqij} = z_{pqijkn} = z_{pqknij}$$

$\forall (i,j,k,n,p,q = 1,...,N), p > k > i, q \neq n \neq j$, and enforce the symmetric restrictions that

$$\begin{aligned} v_{ijknpqgh} &= v_{knijpqgh} = v_{ijpqkngh} = v_{knpqijgh} = v_{pqijkngh} = v_{pqknijgh} = v_{ijknghpq} = v_{knijghpq} = v_{ijpqghkn} = v_{knpqghij} \\ &= v_{pqijghkn} = v_{pqknghij} = v_{ijghknpq} = v_{knghijpq} = v_{ijghpqkn} = v_{knghpqij} = v_{pqghijkn} = v_{pqghknij} = v_{ghijknpq} = v_{ghknijpq} \\ &= v_{ghijpqkn} = v_{ghknpqij} = v_{ghpqijkn} = v_{ghpqknij} \end{aligned}$$

$\forall (i,j,k,n,p,q,g,h = 1,...,N), g > p > k > i, h \neq q \neq n \neq j$, too. Then, the level-3 RLT

System of QAP is given under, where the coefficients d_{ijknpq} and $e_{ijknpqgh}$ discovered in the objective feature are all 0. Notice that Problem RLT3 allows nonzero d_{ijknpq} and $e_{ijknpqgh}$ values, so that it generally handles biquadratic mission problems.

[RLT3]

$$\begin{aligned} \min\ & \sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij}x_{ij} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N} c_{ijkn}y_{ijkn} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N}\sum_{\substack{p=1\\p\neq i,k}}^{N}\sum_{\substack{q=1\\q\neq j,n}}^{N} d_{ijknpq}z_{ijknpq} \\ & + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N}\sum_{\substack{p=1\\p\neq i,k}}^{N}\sum_{\substack{q=1\\q\neq j,n}}^{N}\sum_{\substack{g=1\\g\neq i,k,p}}^{N}\sum_{\substack{h=1\\h\neq j,n,q}}^{n} e_{ijknpqgh}v_{ijknpqgh} \end{aligned} \qquad (6\text{-}8a)$$

$$\text{s.t.} \quad \sum_{\substack{g=1\\g\neq i,k,p}}^{N} v_{ijknpqgh} = z_{ijknpq} \qquad i,j,k,n,p,q,h = 1,...,N; h \neq q \neq n \neq j, p \neq k \neq i, \qquad (6\text{-}8b)$$

$$\sum_{\substack{h=1\\h\neq j,n,q}}^{N} v_{ijknpqgh} = z_{ijknpq} \qquad i,j,k,n,p,q,g = 1,...,N; g \neq p \neq k \neq i, q \neq n \neq j, \qquad (6\text{-}8c)$$

$$\begin{aligned} v_{ijknpqgh} &= v_{knijpqgh} = v_{ijpqkngh} = v_{knpqijgh} = v_{pqijkngh} = v_{pqknijgh} \\ &= v_{ijknghpq} = v_{knijghpq} = v_{ijpqghkn} = v_{knpqghij} = v_{pqijghkn} = v_{pqknghij} \\ &= v_{ijghknpq} = v_{knghijpq} = v_{ijghpqkn} = v_{knghpqij} = v_{pqghijkn} = v_{pqghknij} \\ &= v_{ghijknpq} = v_{ghknijpq} = v_{ghijpqkn} = v_{ghknpqij} = v_{ghpqijkn} = v_{ghpqknij} \end{aligned}$$

$$i,j,k,n,p,q,g,h = 1,...,N; g > p > k > i, h \neq q \neq n \neq j, \qquad (6\text{-}8d)$$

$$v_{ijknpqgh} \geq 0$$

$$i,j,k,n,p,q,g,h = 1,...,N; g \neq p \neq k \neq i, h \neq q \neq n \neq j, \qquad (6\text{-}8e)$$

$$\sum_{\substack{p=1\\p\neq i,k}}^{N} z_{ijknpq} = y_{ijkn} \qquad i,j,k,n,q = 1,...,N; q \neq n \neq j, k \neq i, \qquad (6\text{-}8f)$$

$$\sum_{\substack{q=1\\q\neq j,n}}^{N} z_{ijknpq} = y_{ijkn} \qquad i,j,k,n,p = 1,...,N; p \neq k \neq i, n \neq j, \qquad (6\text{-}8g)$$

$$i,j,k,n,p,q = 1,...,N; p > k > i, q \neq n \neq j, \qquad (6\text{-}8h)$$

$$z_{ijknpq} \geq 0 \qquad i,j,k,n,p,q = 1,...,N; p \neq k \neq i, q \neq n \neq j, \qquad (6\text{-}8i)$$

$$\sum_{\substack{k=1\\k\neq i}}^{N} y_{ijkn} = x_{ij} \qquad i,j,n = 1,...,N; n \neq j, \qquad (6\text{-}8j)$$

$$\sum_{\substack{n=1\\n\neq j}}^{N} y_{ijkn} = x_{ij} \qquad i,j,k = 1,...,N; k \neq i, \qquad (6\text{-}8k)$$

$$y_{ijkn} = y_{knij} \qquad i,j,k,n = 1,...,N; k > i, n \neq j, \qquad (6\text{-}8l)$$

$$y_{ijkn} \geq 0 \qquad i,j,k,n = 1,...,N; k \neq i, n \neq j, \qquad (6\text{-}8m)$$

$$\boldsymbol{x} \in \boldsymbol{X}, \boldsymbol{x} \text{ binary}. \qquad (6\text{-}8n)$$

Just as one is capable of practice the level-2 RLT QAP version to remedy the CAP, one can resolve the BQAP the use of the level-three RLT QAP model, for the reason that extra constraints (6-8b)-(6-8m) inside the above formulation are derived completely from the substitutions of y_{ijkn} = xij x_{kn}, z_{ijknpq} = xij xkn x_{pq} and $v_{ijknpqgh}$ = xij xkn x pq x_{gh} .Therefore, the enlargement related to the multiplication of binary variables in the goal capabilities offers an n-hierarchy of assignment problems that has an immediate relation with the n-hierarchy of the RLT fashions of the QAP.

THE LEVEL-3 RLT DUAL-ASCENT PROCEDURE OF THE QAP

Just like Problems RLT1 and RLT2, Problem RLT3 is equal to the QAP whilst the binary constraints on x are enforced. While RLT1 implies y_{ijkn} = xij xkn and RLT2 additionally implies z_{ijknpq} = xij xkn xpq , RLT3 also implies $v_{ijknpqgh}$ = xij xkn x pq xgh . When relaxing the x binary constraints, RLT3 offers the tightest decrease bounds of all three RLT fashions, since RLT2 (RLT1) may be derived from RLT3 and not using a constraints imposed on v (and z). Although Problem RLT3 gives very sharp bounds, the formula is drastically larger in size than QAP, RLT1 and RLT2 . It is likewise exceedingly degenerate, due to the fact from all the severa equality constraints found in Problem RLT3 , simplest 2N constraints in x ∈ X have nonzero right-hand-facet (RHS) values. The undertaking is to extract tight bounds from this formula without paying a heavy computational fee. Fortunately, one want not solve Problem RLT3 to optimality, considering the reality that each dual viable solution gives a decrease bound. The approach is to speedy compute near-most efficient twin answers.

One can acquire a smaller formulation of RLT3 via the substitution suggested by using constraints (6-8d), (6-8h) and (6-8l) without affecting the bounding strength. Those last variables turnout to be $v_{ijknpqgh}$ (g > p > okay > i , h ≠ q ≠ n ≠ j) , z_{ijknpq} (p > k > i , q ≠ n ≠ j) and y_{ijkn} (okay > i , n ≠ j) . This makes constraints (6-8d), (6-8h) and (6-8l) needless. Here I do no longer carry out such operations, however alternatively exploit a block-diagonal structure gift inside Lagrangian sub problems. Suppose constraints (6-8d), (6-8h) and (6-8l) are dualized into the objective feature the use of a few Lagrangian multipliers. Let bij , cijkn , d_{ijknpq} and $e_{ijknpqgh}$ denote the objective coefficients associated with xij , yijkn , z_{ijknpq} and $v_{ijknpqgh}$ respectively, modified from bij , c_{ijkn} , dijknpq and $e_{ijknpqgh}$ within the original objective characteristic of RLT3 by means of dualizing (6-8d), (6-8h) and (6-8l). The changed model RLT3M is offered beneath.

$$\min\left\{\begin{array}{l}\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{b}_{ij}x_{ij}+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N}\bar{c}_{ijkn}y_{ijkn}+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N}\sum_{\substack{p=1\\p\neq i,k}}^{N}\sum_{\substack{q=1\\q\neq j,n}}^{N}\bar{d}_{ijknpq}z_{ijknpq}\\ +\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N}\sum_{\substack{p=1\\p\neq i,k}}^{N}\sum_{\substack{q=1\\q\neq j,n}}^{N}\sum_{\substack{g=1\\g\neq i,k,p}}^{N}\sum_{\substack{h=1\\h\neq j,n,q}}^{n}\bar{e}_{ijknpqgh}v_{ijknpqgh}\\ \text{s.t.}\\ (6\text{-}8b),(6\text{-}8c),(6\text{-}8e),(6\text{-}8f),(6\text{-}8g),(6\text{-}8i),(6\text{-}8j),(6\text{-}8k),(6\text{-}8m),\ x\in X\end{array}\right\} \quad (6\text{-}9)$$

The binary constraints on x in (6-9) are secure quickly, if you want to be defined later.

Now Problem RLT3M is prepared for decomposition. Adams et al. (2006) proved the following LEMMA.

Lemma 6-1. Consider any feasible and bounded linear program of the form [LP]

$$\hat{Z}=\min\left\{c^{T}x+g^{T}w: Bw\geq dx_{i}\ \text{for some chosen } i,\ Ax\geq b\right\}, \quad (6\text{-}10)$$

Where Bw ≥ d and Ax ≥ b denote possible and bounded polyhedral units, with Ax ≥ b implementing xi ≥ zero . Then an most suitable answer (xˆ , wˆ) to LP can be received through solving

$$\tilde{Z}=\min\left\{(c+\Delta e_{i})^{T}x: Ax\geq b\right\}, \quad (6\text{-}11)$$

Where

$$\Delta=\min\left\{g^{T}w: Bw\geq d\right\}, \quad (6\text{-}12)$$

And in which ei is the unit column vector having a 1 in role i and zeroes elsewhere. Here,xˆ = x and wˆ = w xi with x fixing (6-11) and w solving (6-12), in order that Z = Z .

Based upon LEMMA 5-1 and the decomposition of RLT2, it is easy to decompose RLT3M into a sequence of task troubles. Namely, the following THEOREM applies.

Theorem 6-2. Problem RLT3M (6-9) can be solved by the assignment problem

$$\min\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}(\bar{b}_{ij}+\gamma_{ij})x_{ij}: x\in X\right\}, \quad (6\text{-}13)$$

where for each (i j ,) , ij γ is computed as

$$\gamma_{ij}=\min\left\{\begin{array}{l}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N}(\bar{c}_{ijkn}+\eta_{ijkn})y_{ijkn}\\ \text{s.t.}\\ \sum_{\substack{k=1\\k\neq i}}^{N}y_{ijkn}=1,\forall(n\neq j)\\ \sum_{\substack{n=1\\n\neq j}}^{N}y_{ijkn}=1,\forall(k\neq i)\\ y_{ijkn}\geq 0,\forall(k\neq i,n\neq j)\\ k,n=1,\ldots,N\end{array}\right\}, \quad (6\text{-}14)$$

And where for each (*i* , *j* , *k* , *n*) with *k≠i* and *n≠j* , η_{ijkn} is computed as

$$\eta_{ijkn}=\min\left\{\begin{array}{l}\sum_{\substack{p=1\\p\neq i,k}}^{N}\sum_{\substack{q=1\\q\neq j,n}}^{N}(\bar{d}_{ijknpq}+\varphi_{ijknpq})z_{ijknpq}\\ \text{s.t.}\\ \sum_{\substack{p=1\\p\neq i,k}}^{N}z_{ijknpq}=1,\forall(q\neq j,n)\\ \sum_{\substack{q=1\\q\neq j,n}}^{N}z_{ijknpq}=1,\forall(p\neq i,k)\\ z_{ijknpq}\geq 0,\forall(p\neq i,k;q\neq j,n)\\ p,q=1,\ldots,N\end{array}\right\}, \quad (6\text{-}15)$$

And where for each (*i* , *j* , *k* , *n*, *p* , *q*) with *p ≠ k ≠ i* and *q ≠ n ≠ j* ,ϕ_{ijknpq} is computed as

$$\varphi_{ijknpq}=\min\left\{\begin{array}{l}\sum_{\substack{g=1\\g\neq i,k,p}}^{N}\sum_{\substack{h=1\\h\neq j,n,q}}^{N}\bar{e}_{ijknpqgh}v_{ijknpqgh}\\ \text{s.t.}\\ \sum_{\substack{g=1\\g\neq i,k,p}}^{N}v_{ijknpqgh}=1,\forall(h\neq j,n,q)\\ \sum_{\substack{h=1\\h\neq j,n,q}}^{N}v_{ijknpqgh}=1,\forall(g\neq i,k,p)\\ v_{ijknpq}\geq 0,\forall(g\neq i,k,p;h\neq j,n,q)\\ g,h=1,\ldots,N\end{array}\right\}. \quad (6\text{-}16)$$

Proof.

For any (i , j , k , n, p , q) with p ≠ ok ≠ i and q ≠ n ≠ j , treat the equality constraints (6-8b)-(6-8c) and the non-negativity constraints (6-8e) of (6-9) as Bw ≥ dxi of (6-10), with xi of (6-10) represented by variable z_{ijknpq} and w of (6-10) represented with the aid of variables $v_{ijknpqgh}$ with g ≠ p ≠ okay ≠ i and h ≠ q ≠ n ≠ j , and deal with the last variables and constraints of (6-nine) as x and Ax ≥ b respectively. Then follow LEMMA 6-1, in order that the ensuing problem of the form (6-11) includes no $v_{ijknpqgh}$ term for the selected (i, j, okay, n, p, q). Denote of (6-12) as ϕijknpq , in order that the goal coefficient of z_{ijknpq} changes from i_{jknpq} to d_{ijknpq} + ϕ_{ijknpq}. Now, (6-9) becomes.

$$\min\left\{\begin{array}{l}\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{b}_{ij}x_{ij}+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N}\bar{c}_{ijkn}y_{ijkn}\\ +\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{n=1\\n\neq j}}^{N}\sum_{\substack{p=1\\p\neq i,k}}^{N}\sum_{\substack{q=1\\q\neq j,n}}^{N}(\bar{d}_{ijknpq}+\varphi_{ijknpq})z_{ijknpq}\\ \text{s.t.}\\ (6\text{-}8f),(6\text{-}8g),(6\text{-}8i),(6\text{-}8j),(6\text{-}8k),(6\text{-}8m),\ x\in X\end{array}\right\}. \quad (6\text{-}17)$$

The relaxation of proof is to use LEMMA 6-1 twice, which was blanketed within the THEOREM proof from the RLT2 decomposition with the aid of Adams et al. (2006).

First, efficiently do away with allz_{ijknpq} with $p \neq k \neq i$ and $q \neq n \neq j$, where (6-17) becomes

$$\min \left\{ \begin{array}{l} \sum_{i=1}^{N}\sum_{j=1}^{N}\bar{b}_{ij}x_{ij} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\substack{k=1 \\ k\neq i}}^{N}\sum_{\substack{n=1 \\ n\neq j}}^{N}\left(\bar{c}_{ijkn} + \eta_{ijkn}\right) y_{ijkn} \\ \text{s.t.} \\ \text{(6-8j),(6-8k),(6-8m), } \boldsymbol{x} \in X \end{array} \right\}. \qquad \text{(6-18)}$$

And then remove all y_{ijkn} with $k \neq i$ and $n \neq j$, till the problem is finally reduced to (6-13) only in variables x_{ij} . This completes the proof.↑

The premier (binary) way to (6-9) can be obtained as follows.Let x , y , z and v denote the computed most desirable (intense factor) answers to (6-13), (6-14), (6-15) and (6-16), respectively. By LEMMA 6-1, (solves (6-18) in which

— — —

$$\bar{\boldsymbol{x}} = \tilde{\boldsymbol{x}}, \qquad \text{(6-19a)}$$

$$\bar{y}_{ijkn} = \tilde{y}_{ijkn}\tilde{x}_{ij}, \ \forall (i,j,k,n) \text{ with } k \neq i \text{ and } n \neq j; \qquad \text{(6-19b)}$$

Also by LEMMA 6-1, given that $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$ is optimal to (6-18), $(\bar{\bar{\boldsymbol{x}}}, \bar{\bar{\boldsymbol{y}}}, \bar{\bar{\boldsymbol{z}}})$ solves (6-17) where

$$\bar{\bar{\boldsymbol{x}}} = \bar{\boldsymbol{x}}, \qquad \text{(6-20a)}$$

$$\bar{\bar{\boldsymbol{y}}} = \bar{\boldsymbol{y}}, \qquad \text{(6-20b)}$$

and $$\bar{\bar{z}}_{ijknpq} = \tilde{z}_{ijknpq}\bar{y}_{ijkn}, \ \forall (i,j,k,n,p,q) \text{ with } p \neq k \neq i \text{ and } q \neq n \neq j; \qquad \text{(6-20c)}$$

Again by LEMMA 6-1, given that $(\bar{\bar{\boldsymbol{x}}}, \bar{\bar{\boldsymbol{y}}}, \bar{\bar{\boldsymbol{z}}})$ is optimal to (6-17), $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}, \hat{\boldsymbol{v}})$ solves (6-9),

$$\hat{\boldsymbol{x}} = \bar{\bar{\boldsymbol{x}}}, \qquad \text{(6-21a)}$$

$$\hat{\boldsymbol{y}} = \bar{\bar{\boldsymbol{y}}}, \qquad \text{(6-21b)}$$

$$\hat{\boldsymbol{z}} = \bar{\bar{\boldsymbol{z}}}, \qquad \text{(6-21c)}$$

and $$\hat{v}_{ijknpqgh} = \tilde{v}_{ijknpqgh}\bar{\bar{z}}_{ijknpq},$$

$$\forall (i,j,k,n,p,q,g,h) \text{ with } g \neq p \neq k \neq i \text{ and } h \neq q \neq n \neq j. \qquad \text{(6-21d)}$$

Altogether, one obtains $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}, \hat{\boldsymbol{v}})$, which is optimal to (6-9), in terms of $(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{y}}, \tilde{\boldsymbol{z}}, \tilde{\boldsymbol{v}})$ as

$$\hat{\boldsymbol{x}}: \ \hat{\boldsymbol{x}} = \tilde{\boldsymbol{x}}, \qquad \text{(6-22a)}$$

$$\hat{\boldsymbol{y}}: \ \hat{y}_{ijkn} = \tilde{y}_{ijkn}\tilde{x}_{ij}, \ \forall (i,j,k,n) \text{ with } k \neq i \text{ and } n \neq j, \qquad \text{(6-22b)}$$

$$\hat{\boldsymbol{z}}: \ \hat{z}_{ijknpq} = \tilde{z}_{ijknpq}\tilde{y}_{ijkn}\tilde{x}_{ij}, \ \forall (i,j,k,n,p,q) \text{ with } p \neq k \neq i \text{ and } q \neq n \neq j, \qquad \text{(6-22c)}$$

and $$\hat{\boldsymbol{v}}: \ \hat{v}_{ijknpqgh} = \tilde{v}_{ijknpqgh}\tilde{z}_{ijknpq}\tilde{y}_{ijkn}\tilde{x}_{ij},$$

$$\forall (i,j,k,n,p,q,g,h) \text{ with } g \neq p \neq k \neq i \text{ and } h \neq q \neq n \neq j. \qquad \text{(6-22d)}$$

Now, on account that (6-13)-(6-16) are undertaking problems, the extreme factors are binary so that x , y , z and v are binary, which makes (x, y, z , v) an most suitable binary solution to (6-9).

THEOREM 5-2 and its proof show a way to decompose into one RLT3M task hassle (6-13) of size N, N2 task issues (6-14) of length N −1, N 2 (N −1)2 task troubles (6-15) of length N − 2 , and N 2 (N − 1) 2 (N − 2)2 project problems (6-sixteen) of length N − three. This motivates a Lagrangian method for determining the most useful set of twin multiplier values for constraints (6-8d), (6-8h) and (6-8l), and for this reason for acquiring the surest goal value of the continuous rest Problem RLT3 . I gift underneath a twin-ascent method, a good deal similar to that hired in Adams et al. (2006) for Problem RLT2 , which offer a monotonic non decreasing series of decrease bounds for the QAP thru Problem RLT3 . Notice that with the symmetric constraints (6-8d) connecting the twenty-4 v variables, it doubtlessly results in extracting as much as viable from the associated price matrix E , thereby increasing the lower certain Z . This observation also applies to the cost matrix D by means of constraints (6-8h) and to the cost matrix C via constraints (6-8l). Here are the stairs.

1. Initialize (6-9) by assigning $\bar{e}_{ijknpqgh} = e_{ijknpqgh} = 0$ for $\forall (i,j,k,n,p,q,g,h)$ with $g \neq p \neq k \neq i$ and $h \neq q \neq n \neq j$, $\bar{d}_{ijknpq} = d_{ijknpq} = 0$ for $\forall (i,j,k,n,p,q)$ with $p \neq k \neq i$ and $q \neq n \neq j$, $\bar{c}_{ijkn} = c_{ijkn}$ for $\forall (i,j,k,n)$ with $k \neq i$ and $n \neq j$, and $\bar{b}_{ij} = b_{ij}$ for $\forall (i,j)$, where $e_{ijknpqgh}$, d_{ijknpq}, c_{ijkn} and b_{ij} are objective coefficients taken from $\overline{\text{RLT3}}$. Set the initial lower bound $Z = 0$. Set the iteration counter to be 0.

2a. For each (i,j), distribute the coefficient $\bar{b}_{ij}$ among the $(N-1)^2$ coefficients $\bar{c}_{ijkn}$ for all $k \neq i$ and $n \neq j$ by increasing each such $\bar{c}_{ijkn}$ by $\bar{b}_{ij}/(N-1)$ and decreasing $\bar{b}_{ij}$ to 0. This is equivalent, for each (i,j), to adding $\bar{b}_{ij}/(N-1)$ times each of the $N-1$ equations $\sum_{n\neq j} y_{ijkn} - x_{ij} = 0$ for all $k \neq i$ found in (6-8k) to the objective of (6-9).

2b. For each (i,j,k,n) with $i \neq j$ and $k \neq n$, distribute the updated coefficient $\bar{c}_{ijkn}$ among the $(N-2)^2$ coefficients $\bar{d}_{ijknpq}$ for all $p \neq i,k$ and $q \neq j,n$ by increasing each such $\bar{d}_{ijknpq}$ by $\bar{c}_{ijkn}/(N-2)$ and decreasing $\bar{c}_{ijkn}$ to 0. This is equivalent, for each (i,j,k,n) with $i \neq j$ and $k \neq n$, to adding $\bar{c}_{ijkn}/(N-2)$ times each of the $N-2$ equations $\sum_{q\neq j,n} z_{ijknpq} - y_{ijkn} = 0$ for all $p \neq i,k$ found in (6-8g) to the objective of (6-9).

2c. For each (i,j,k,n,p,q) with $i \neq j$, $k \neq n$ and $p \neq q$, distribute the updated coefficient $\bar{d}_{ijknpq}$ among the $(N-3)^2$ coefficients $\bar{e}_{ijknpqgh}$ for all $g \neq i,k,p$ and $h \neq j,n,q$ by increasing each such $\bar{e}_{ijknpqgh}$ by $\bar{d}_{ijknpq}/(N-3)$ and decreasing $\bar{d}_{ijknpq}$ to 0. This is equivalent, for each (i,j,k,n,p,q) with $i \neq j$, $k \neq n$ and $p \neq q$, to adding $\bar{d}_{ijknpq}/(N-3)$ times each of the $N-3$ equations $\sum_{h\neq j,n,q} v_{ijknpqgh} - z_{ijknpq} = 0$ for all $g \neq i,k,p$ found in (6-8c) to the objective of (6-9).

3. Use THEOREM 6-2 to sequentially solve (6-9) as $N^2(N-1)^2(N-2)^2 + N^2(N-1)^2 + N^2 + 1$ assignment problems.

3a. Solve $N^2(N-1)^2(N-2)^2$ assignment problem (6-16) of size $N-3$ to obtain $\tilde{\boldsymbol{v}}$ and the value φ_{ijknpq} as follows. Sequentially consider all (i,j,k,n,p,q) with $p \neq k \neq i$ and $q \neq n \neq j$, beginning with those (i,j,k,n,p,q) for which $\bar{d}_{ijknpq}$ prior to step 2c was 0. For a selected (i,j,k,n,p,q), change the coefficient $\bar{e}_{ijknpqgh}$ for each $g \neq i,k,p$ and $h \neq j,n,q$ to a percentage of the sum of $\bar{e}_{ijknpqgh}$, $\bar{e}_{knijpqgh}$, $\bar{e}_{ijpqkngh}$, $\bar{e}_{knpqijgh}$, $\bar{e}_{pqijkngh}$, $\bar{e}_{pqknijgh}$, $\bar{e}_{ijknghpq}$, $\bar{e}_{knijghpq}$, $\bar{e}_{ijpqghkn}$, $\bar{e}_{knpqghij}$, $\bar{e}_{pqijghkn}$, $\bar{e}_{pqknghij}$, $\bar{e}_{ijghknpq}$, $\bar{e}_{knghijpq}$, $\bar{e}_{ijghpqkn}$, $\bar{e}_{knghpqij}$, $\bar{e}_{pqghijkn}$, $\bar{e}_{pqghknij}$, $\bar{e}_{ghijknpq}$, $\bar{e}_{ghknijpq}$, $\bar{e}_{ghijpqkn}$, $\bar{e}_{ghknpqij}$, $\bar{e}_{ghpqijkn}$, and $\bar{e}_{ghpqknij}$, and equally adjust the latter twenty-three values so that the sum stays constant. Upon solving this assignment problem, place the corresponding equality constraints (6-8b) and (6-8c) into the objective function with the optimal dual multipliers, effectively readjusting the $\overline{e}_{ijknpqgh}$ values for $g \neq i,k,p$ and $h \neq j,n,q$ and increasing $\overline{d}_{ijknpq}$ by φ_{ijknpq}. Proceed through all such (i,j,k,n,p,q) indices where $p \neq k \neq i$ and $q \neq n \neq j$.

3b. Solve $N^2(N-1)^2$ assignment problem (6-15) of size $N-2$ to obtain $\tilde{z}$ and the value η_{ijkn} as follows. Sequentially consider all (i,j,k,n) with $k \neq i$ and $n \neq j$, beginning with those (i,j,k,n) for which $\overline{c}_{ijkn}$ prior to step 2b was 0. For a selected (i,j,k,n), change the coefficient $\overline{d}_{ijknpq}$ for each $p \neq i,k$ and $q \neq j,n$ to a percentage of the sum of $\overline{d}_{ijknpq}$, $\overline{d}_{knijpq}$, $\overline{d}_{ijpqkn}$, $\overline{d}_{knpqij}$, $\overline{d}_{pqijkn}$, and $\overline{d}_{pqknij}$, and equally adjust the latter five values so that the sum stays constant. Upon solving this assignment problem, place the corresponding equality constraints (6-8f) and (6-8g) into the objective function with the optimal dual multipliers, effectively readjusting the $\overline{d}_{ijknpq}$ values for $p \neq i,k$ and $q \neq j,n$ and increasing $\overline{c}_{ijkn}$ by η_{ijkn}. Proceed through all such (i,j,k,n) indices where $k \neq i$ and $n \neq j$.

3c. Solve N^2 assignment problem (6-14) of size $N-1$ to obtain $\tilde{y}$ and the value γ_{ij} as follows. Sequentially consider all (i,j), beginning with those (i,j) for which $\overline{b}_{ij}$ prior to step 2a was 0. For a selected (i,j), change the coefficient $\overline{c}_{ijkn}$ for each $k \neq i$ and $n \neq j$ to a percentage of the sum of $\overline{c}_{ijkn}$ and $\overline{c}_{knij}$, and then adjust $\overline{c}_{knij}$ so that the sum stays constant. Upon solving this assignment problem, place the corresponding equality constraints (6-8j) and (6-8k) into the objective function with the optimal dual multipliers, effectively readjusting the $\overline{c}_{ijkn}$ values for $k \neq i$ and $n \neq j$ and increasing $\overline{b}_{ij}$ by γ_{ij}. Proceed through all such (i,j) indices.

3d. Solve the assignment problem (6-13) of size N to obtain $\tilde{x}$. Upon doing so, place the equality constraints of X into the objective function with the optimal dual multipliers, adjusting the value of $\overline{b}_{ij}$ and the lower bound Z. Here, Z is increased by the nonnegative objective value to the minimization problem of (6-13). Proceed to step 4.

4. If the binary optimal solution $(\hat{x},\hat{y},\hat{z},\hat{v})$, computed as (6-22), to (6-9) is feasible to RLT3, i.e., if it satisfies (6-8d), (6-8h) and (6-8l), stop with $(\hat{x},\hat{y},\hat{z},\hat{v})$ optimal to problem QAP. If it is not feasible to RLT3, stop if some predetermined number of iterations has been performed. Otherwise, increase the iteration counter by 1 and return to step 2a.

The dual-ascent procedure will produce a nondecreasing sequence of lower bounds since Step 1 is input with all variables having nonnegative reduced costs.

COMPUTATIONAL RESULTS OF THE LEVEL-3 RLT LOWER BOUND CALCULATION

Several years in the past Professor Hahn coded in FORTRAN a initial twin-ascent algorithm that calculates QAP level-three RLT root lower bounds. To demonstrate the capability of level-three RLT, I tested this code and done a sequence of experiments the usage of benchmark QAP times. Table 1 compares the overall performance of the QAP degree-3 128 RLT algorithm with the quality lower bound values achieved for these instances by using some other methods. For every test trouble, Table 1 affords the extent-3 RLT lower bound finished after seven hundred iterations of the set of rules. The Optimum column gives the most efficient value. The Lower Bound column lists the extent-three RLT decrease sure for each QAP instance. The Run time column shows the CPU seconds normalized to a Dell 7150 gadget on a single 733MHz CPU. The Best Previous and Method columns provide the preceding best lower bounds and its attaining set of rules from Table 1 of Loiola et al. (2006).

Table 1. The QAP level-3 RLT lower bounds

Instance	Optimum	Lower Bound	Runtime	Best Previous	Method
Nug12	578	577.15*	1,468	578	RLT2
Nug15	1,150	1,149.74*	16,671	1,150	RLT2
Nug18	1,930	1,930**	86,951	1,905	RLT2
Nug20	2,570	2,569.19*	304,274	2,508+	RLT2
Had16	3,720	3,718.11*	~15,000	3,672	RLT2
Had18	5,358	5,357.67*	44,680	5,299	RLT2
Had20	6,922	6,919.1	48,020	6,811	RLT2
Rou15	354,210	354,210**	951	350,207	RLT2
Rou20	725,520	725,314.4	252,282	695,123	RLT2
Tai20a	703,482	703,482**	254,432	671,685	RLT2

Optimum demonstrated by using the level-3 RLT lower sure code.

Problem solved precisely through the extent-3 RLT lower certain code. + Recently corrected end result via the level-2 RLT lower sure code.

As stated before, the wide variety of variables grows dramatically with RLT degree. RLT 2 code already run into memory barriers for off-the-contemporary technology of computer systems for trouble instances larger than N = 36 . Those limitations have made it hard, if now not impossible to calculate degree-three RLT decrease bounds for trouble instances large than N = 20 , even though RLT 3 code has proven even greater promise for decreasing the range of nodes that have to be taken into consideration for providing optimal. Figure 1 below demonstrates the boom in random get entry to reminiscence (RAM) with hassle example length, required for stage-3 root lower certain calculations. The linear extrapolation is based totally on statistics from the decrease certain experiments on 4 Nugent times reported in Table 1.

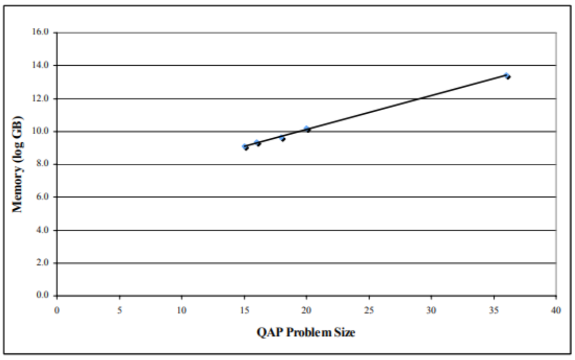

Figure 1. Memory required for level-3 RLT lower bound calculations.

CONCLUSION

This section reports the level-3 reformulation-linearization procedure (RLT) detailing of the quadratic task issue (QAP) and its primer lower bound estimations. By broadened meanings of the cubic and biquadrate task problems, I am ready to build up a chain of command of zero-one task problems parallel to the RLT progressive system of the QAP. Consequently, another hypothetical method for interfacing the QAP and its related problems is set up through their answer techniques. RLT systems, while demonstrating extraordinary guarantee, need to date got little examination as far as handy application. My motivation in this part is to demonstrate that viable methods can be conceived to make these procedures helpful, for settling the QAP, yet for explaining enormous classes of comparably troublesome combinatorial improvement problems.

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