Journal of Advances and Scholarly Researches in Allied Education
Vol. 16, Issue No. 5, April-2019, ISSN 2230-7540

Electric and Scalar Charged Fluid Sphere in General Relativity

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Abstract – The paper provides interior solution of an electric and scalar charged fluid sphere in general relativity under certain assumptions.

Key Word: Scalar, Geometric mass, Density, Pressure, Flatness

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1. INTRODUCTION

An interior solution of electroscalarly charged static 'dust' sphere has been found by Teixeira et al. [4] and it has been shown that the geometric mass of an electroscalarly charged sphere is

$$m = \left(q^2 - \gamma_b^2\right)^{1/2}$$

where q is the electric charge, b the scalar charge and $\gamma = \pm 1$.

Florides [1] has shown that the geometric mss of a charged sphere is

$$m = \mu(a) + \varepsilon(a),$$

where, (a) is the contribution from the mass density and (a) is that from electric charge. So et al. [3] have found Reissner Nordstrom solution from the Schwarzschild solution by a co-ordinate transformation and have shown that the geometric mass of a charged sphere is

$$m = \left(m_s^2 + q^2\right)^{1/2}$$

where, m_s is the Schwarzschild mass and q is the charge of the sphere. Paul [2] has obtained an interior solution of electroscalarly charged fluid sphere and has shown that the geometric mass of the sphere has contribution from its mass density and scalar electric charges. In the present paper, we have investigated the interior solution of an electric and scalar charged fluid sphere in general relativity under certain assumptions.

2. THE FIELD EQUATIONS AND THEIR SOLUTIONS

We consider the spherically symmetric metric given by

$$(2.1)\quad ds^2 = e^{2\eta}dt^2 - e^{2\alpha}dr^2 - r^2e^{\alpha-\eta}d\theta^2 - r^2e^{\alpha-\eta}\sin^2\theta d\phi^2$$

where r, and t are numbered 1, 2, 3 and 4 respectively. Here □ and □ are functions of alone.

Einstein – Maxwell scalar field equations are

$$(2.2)\quad R_\mu^\nu = -8\pi\left(T_\mu^\nu - \frac{1}{2}\delta_\mu^\nu T\right)$$

$$(2.3)\quad T_\mu^\nu = (\rho + p)u_\mu u^\nu - \delta_\mu^\nu P + \frac{1}{4\pi}\left[-F^{\nu\alpha}F_{\mu\alpha} + \frac{1}{4}\delta_\mu^\nu F^{\alpha\beta}F_{\alpha\beta}\right] + k_\mu^\nu$$

$$(2.4)\quad 4\pi\gamma K_\mu^\nu = s^{;\nu}s_{;\mu} - \frac{1}{2}\delta_\mu^\nu S^{;\alpha}S_{;\alpha},$$

$$(2.5)\quad F^{\mu\nu};v = 4\pi\,\sigma u^\mu,$$

$$(2.6)\quad F_{[\mu\nu;\alpha]} = 0$$

$$(2.7)\quad S^{;\mu}_{;\mu} = -4\pi\gamma\beta$$

where P, and q are the mass density, pressure, charge density and scalar charge density respectively, s is the scalar field. The matter is at rest in the co-ordinate system of (1) so that

$$u^\mu = \delta_4^\mu\left(\delta_{44}\right)^{-1/2}$$

The electric field is such that only $F_{41} = +\psi_1$ exists along radial direction where ψ is the electric potential. The suffix 1 indicates differentiation with respect to r. equation (2.2) with the help of equation (2.1) gives

$$(2.8)\quad \eta_{11} + \frac{2\eta_1}{r} = 4\pi(\rho + 3P)e^{2\alpha} + e^{-2\eta}\psi_1^2$$

$$\alpha_{11} - \frac{\alpha_1^2}{2} - \alpha_1\eta_1 + \frac{3}{2}\eta_1^2 - \frac{2\eta_1}{r}$$

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$$(2.9)\quad = -4\pi(\rho - P)e^{2\alpha} + e^{-2\eta}\psi_1^2 - 2\gamma s_1^2$$

$$(2.10)\quad \frac{1}{2}\eta_{11} - \frac{1}{2}\alpha_{11} + \frac{\eta_1}{r} - \frac{\alpha_1}{\theta} - \frac{1}{r^2} + \frac{1}{r^2}e^{\eta+\alpha}$$

$$= 4\pi(\rho - P)e^{2\alpha} + e^{-2\eta}\psi_1^2$$

$$(2.11)\quad \frac{d}{dr}\left(r^2 e^{-2\eta}\psi_1\right) = -4\pi r^2 e^{2\alpha-\eta}$$

$$(2.12)\quad \frac{d}{dr}\left(r^2 S_1\right) = -4\pi\gamma\beta r^2 e^{2\alpha},$$

Now since there are five equations and eight variables, let us assume

$$(2.13)\quad \left.\begin{aligned} 2\alpha &= Ar \\ 2\eta &= Br \\ s &= Cr \end{aligned}\right\}$$

where A, B and C are constant. The first two assumptions of equation (2.13) ensure flatness at the centre and the third makes the scalar field zero at r = 0

Now with the help of equation (2.13) equations (2.8) to (.12) give

$$(2.14)\quad 16\pi p = \frac{e^{-Ar}}{r^2}\left[(A+B)\frac{r}{2} - {}^{(A+B)r} + 1\right]$$

$$(2.15)\quad 8\pi\rho = \frac{e^{-Ar}}{8r^2}\Big[(14B - 2A)r - \left(3B^2 - A^2 - 2AB + 16\gamma c^2\right)r^2$$

$$4\left(e^{(A+B)r/2} - 1\right)\Big]$$

$$(2.16)\quad \psi_1^2 e^{-2\eta} = \frac{1}{2\gamma^2}\left(e^{(A+B)\frac{r}{2}} - 1\right)$$

$$+\frac{1}{16}\left(3B^2 - A^2 - 2AB - 16\gamma c^2\right) - \frac{1}{4r}(A+B)$$

$$(2.17)\quad 4\pi\sigma = \left(\frac{d}{dr}F^{41} + \left(\frac{2}{r} + A\right)F^{41}\right)e^{\frac{Br}{2}}$$

$$(2.18)\quad 4\pi\beta = 2F\frac{e^{-Ar}}{\gamma r}$$

Now we see that e, p, i.e. mass density, pressure, charge density and scalar charge density all become infinite at the centre r = 0. So there is singularity in these quantities at centre.

However if we choose

$$(2.19)\quad 2\alpha = Ar^2,\ 2\eta = Br^2,\ s = cr^{\frac{2}{8\pi\eta_0}} = \frac{1}{4}(17B - 7A)$$

Then at the centre r = 0, we get

$$(2.20)\quad 8\pi\rho_0 = \frac{1}{2}(17B - 7A)$$

$$(2.21)\quad 16\pi p_0 = \frac{3}{2}(A+B)$$

$$(2.22)\quad 4\pi\sigma_0 = \left(3B^2 - A^2 - 2AB + 16\gamma c^2\right)^{1/2}$$

$$(2.23)\quad 4\pi\beta_0 = 6\frac{C}{\gamma}$$

Here if we assume that constant A and B are positive then □0 and p0 are both positive if

$$B > \frac{7}{17}A$$

Also □0 is is real if

$$(2.24)\quad 3B^2 - A^2 - 2AB + 16\gamma c^2 > 0$$

The geometric mass of the sphere is

$$(2.25)\quad M = \int_0^a \int_{\theta=0}^{\pi} \int_{\theta=0}^{2\pi} \rho\, dv$$

where dv (the proper elementary volume)

$$(2.26)\quad dv = r^2 e^{2\alpha-\eta}\sin\theta\, d\theta\, d\phi\, dr.$$

Thus, the geometric mass of the sphere [for the choice equation (19)] is given by

$$(2.27)\quad M=\int_0^a\left[\frac{1}{4}(17B-7A)+\left(\frac{3}{2}A^2-\frac{9}{2}B^2+3AB\right)r^2\right]\times e^{-\eta}r^2dr$$

$$+\frac{1}{4}\int_0^a(2e^{-2\eta}\psi_1^2+6\gamma s_1^2)e^{-\eta}r^2dr$$

$$=\mu(a)+\varepsilon(a)+s(a)$$

Thus the mass density, electric charge and scalar charge densities contribute to the geometrical mass of the charged fluid sphere as is evident from equation (2.27).

3. REFERENCE:

1. Florides, P.S. (1977). The complete field of a general static spherically symmetric distribution of charge Il Nuovo Cim., 42A., pp. 343.

2. Paul, B.B. (1980). Electro-scalarly charged fluid spheres in general relativity. Indian Jour. Pure. Appl. Math. 11(8), pp. 1055.

3. Som, M.M.; Santos, N.O. and Teixeira, A.F. daF (1977). 0 Geometrical mass in the Reissner-Nordstrom solution. Phy. Rev., D16 , pp. 2417.

4. Teixeira, A.F. daF.; Wolk Idel and Som, M.M. (1976). Generalised charged static dust spheres in general relativity. J. Phys. A9, pp. 1267.

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