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Mathematical Modelling for Mechanism of Blood Flow as Newtonian Flow

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Abstract – The present paper provides model for mechanism of blood flow as Newtonian flow together with interpretation and limitation of the model. Here the term blood flow denotes that blood is carried from the heart to various parts of the body through a system of elastic tubes, e.g., the arteries, capillaries and vein and eventually returned to the heart without actually leaving the system. In order to form a mathematical model, we now formulate a simple mathematical model for blood flow in arteries by making assumption: blood is a homogeneous fluid, its flow is steady or laminar and arteries are rigid, long & straight. Here Newtonian flow is defined as that flow where shear stress is directly proportional to shear rate. Finally in interpretation of the model we get a formula is commonly used in laboratories for finding the viscosity of a given fluid and its velocity component.

Key words: Modelling, Blood Flow, Shear, Artery, Fluid

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1. INTRODUCTION:

The flow of blood in large arteries considered as Newtonian flow [21]. The Newtonian approximation for blood is acceptable in modelling flow in large arteries, normally, when shear rate of flow is greater than 100^{-1} [22] and [23]. Newtonian flow is defined as that flow where shear stress is directly proportional to shear rate. This mean that when shear stress is plotted against shear rate at a given temperature the plot shows a straight line with a constant slop, this slop is called the viscosity of fluid, also Newtonian flow can be defined as the flow in which the coefficient of viscosity is constant [24]. Aneurysm is an enlarged the size of arteries caused by a weakening of the arteries wall. This weakening can lead to rupture of blood artery at aneurysm location, which can cause internal bleeding and stooped the blood supply. For instance, rupture an aneurysm in the artery which is supplying the blood to brain, can bring strokes, likewise rupture in abdominal artery leads to death from internal bleeding [5].The relation between the aneurysm and hemodynamic of blood flow is considered the interest subject of researchers. [6] have suggested that, irregular shear stress of wall is one of the factors which contribute to the weakening of the wall tissue. [7] have reported that low wall shear stress is related to the growth of aneurysm, and rupture.[8] have observed that the aneurysm growth occur in regions of low wall shear stress. The blood flow in the aneurysm artery was considered as Newtonian flow [9], [10], [11], [12], [13], [14], [15], [16], [17],[19] and [20]. There are many physical reasons, why the increase radius of aneurysm can lead to rupture. Some other researchers are [1], [2], [3] and [4] in this field. All are related to the shear stress [18]. We all know that blood is carried from the heart to various parts of the body through a system of elastic tube, e.g., the arteries, capillaries and veins and eventually returned to the heart without actually leaving the system. This process is known as circulation of blood or flow of blood.

We also know that proper flow of blood is essential to transmit oxygen and other nutrients to various parts of the body in human beings as well as in all other animals. Any charge in the blood vessel or any change in characteristic of blood vessels can change the flow and cause damages ranging from minor discomfort to death.

In order to form a mathematical model, we first identify the essential characteristics of the blood flow:

(i) Blood is a non-homogeneous fluid.

(ii) Blood vessels are elastic and having branches repeatedly.

(iii) Blood flow is unsteady or pulsatile.

(iv) Blood flow is generally lanunar except for flow near the heart.

In this paper we used the laminar, incompressible, homogenous, fully developed, Newtonian flow having axially symmetric but radially symmetric lanunar in the presence of porous effects. Here the blood is represented as Poiseuille law of fluid model and flow model is shown by the Newtonian flow and the continuity equations.

2. THE FORMULATION OF THE PROBLEM AND ANALYSIS

We have derived the poiseuille law which gives a relation between the rate of flow and pressure difference existing while a fluid flows in a rigid tube of circular cross section.

Since the above essential characteristics of the blood flow will make the model very complicated therefore, we now formulate a simple mathematical model for blood flow in arteries by making certain assumption:

(i) The blood flow is modelled to be steady, one dimensional laminar and the nature of the flowing blood is incompressible, homogenous.

(ii) There is no external force acting on the flowing blood. The tubes (arteries) are rigid, long and straight.

(iii) The flow of Blood and Stenosis developed in the artery in an axially symmetric manner and depends upon the axial distance z and the height of its growth.

(iv) Radial velocity in the arteries region is very small in comparison to the axial velocity [11].

Thus, under these assumptions, Poiseuille law is applicable. Therefore, the velocity of the blood in such a configuration is

$$q_z(t) = \frac{(p_1 - p_2)}{4\mu l}(a^2 - t^2), \qquad 0 \le t \le \alpha \qquad (1)$$

and the rate of flow is

$$Q = \frac{\pi a^4 (p_1 - p_2)}{8\mu\mu}(\alpha^2 - t^2), \qquad 0 \le t \le \alpha \qquad (2)$$

Where p_1 & p_2 ($p_1 > p_2$) are fluid pressure at the end points of the tube , is the viscosity of the fluid, is velocity component of the fluid along z- axis, is length and a is radius of the tube.

If τ_{tz} is the shear stress, then by Newton formula, we have

$$\tau_{tz} = \mu\left(\frac{\delta q_1}{\delta r}\right) = \mu\left[\frac{(p_1 - p_2)}{4\mu l}(-2t)\right], \qquad \text{(Using(1))}$$

$$\tau_{rz} = \left[\frac{(p_1 - p_2)}{2l}(-t)\right] \qquad (3)$$

The shear stress of the wall, is

$$\tau_{tz} = \left[\frac{(p_1 - p_2)}{2l}(-a)\right] \qquad (4)$$

[putting t = a in (3)]

Example: Consider arterial blood viscosity = 3/110 poise. If the length of the artery is 4 cm, and radius 8 x 10^{-3}cm and P_1-P_2 = 4 x 10^3 dynes/cm^2, then,

(i) Find q_z (t) and the maximum peak velocity of blood.

(ii) Find the shear stress at the wall.

Solution: (i) since the velocity of the blood is

$$q_z(t) = \left[\frac{(p_1 - p_2)}{4\mu l}(a^2 - t^2)\right] \qquad (5)$$

Here P_1-P_2 4 x 10^3 dynes/cm^2, L= 4 cm, a = 8 x 10^{-3} cm = 3/110 poise putting these values in (1), we get

$$q_z(t) = \frac{(4 \times 10^3)}{4 \times 0.07 \times 4}(64 - 10^{-6} \times t^2) \qquad (6)$$

The velocity will maximum at t = 0. This velocity $(q_z)_{max}$ is

$$= \frac{(16 \times 10^{-3})}{0.027} = \frac{16}{27}$$

=0.5925 cm/sec.

(ii) The shear stress on the wall is

$$\tau_{tz} = \left[\frac{(p_1 - p_2)}{2L}(-a)\right] \qquad (7)$$

$$= \frac{-8 \times 10^{-3} \times 4 \times 10^3}{2 \times 4}$$

$$= -4 dynes / cm^2$$

3. INTERPRETATION AND LIMITATION OF THE MODEL

Interpretation

Since the rate of the blood flow is

$$Q = \frac{\pi a^4 (p_1 - p_2)}{4\mu l}, \qquad (8)$$

Here we get When increasing the radius of aneurysm lead to increase the pressure difference and decrease the velocity. While wall shear stress, wall shear rate and flow rate of blood are irregular.

Obviously the rate of the flow depend on l, p_1-p_2, a and. In formula the only unexpected term is the geometric term a^4 (fourth power of the radius of the artery) which is important in physiological flows. This gives an effective control of blood flow by arteries.

The blood vessels and the radius of the lumen go on decreasing towards the peripheral regions. For a given pressure and for given constant viscosity, if the radius of the arteries is decreased to half the mark, then the flow of the blood in the arteries reduces to sixteenth of the original value which effects the regulation of normal metabolic activities of the body.

If = is average velocity over the cross section, then

$$p_1 - p_2 = \frac{8\mu l}{a^2} (q_z)_{av}$$

This formula is commonly used in Laboratories for finding the viscosity of the given fluid.

In this paper we used the laminar, incompressible, homogenous fully developed, Newtonian flow having axially symmetric but radially symmetric, lanunar in the presence of porous effects. Here the blood is represented as Herschel-Bulkley fluid model and flow model is shown by the Newtonian flow and the continuity equations.

Limitation:-

During the formulation of the model we have made certain assumptions, these assumptions are satisfied only partly.

Therefore, we have the following limitations over the assumptions:

(i) We have assumed that blood has been taken homogeneous which obeys the Newton's law of friction. But this law holds good if the flow occurs in tubes in which the internal diameter is large compared with the size of the red cells. If we consider the small blood vessels, the cell structure should be included resulting non-linear terms of higher degree in the model.

(ii) In the other assumption, flow is steady which is not satisfied in general, because the flow in all large arteries and the into thoracic veins is shown to be pulsative which is infact a time dependent flow and not steady in this case, thus the above model is not applicable in such cases.

(iii) Flow is assumed to be laminar which is not taken always because at rate of flow above critical value, the flow becomes turbulent and turbulent flow may occur is very large blood vessels i.e., near the heart and aorta; Also the flow near the heart and aorta is pulsative that is flow is unsteady there.

(iv) In the last assumption, the arteries are assumed to be long, which is true, because in the circulating system there are so many branches of the arteries, therefore, there is no artery is sufficiently long enough for the model to be valid.

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