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Static Fluid Sphere with Pressure Equal to Energy Density

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Abstract – This research paper provides solution of Einstein's field equation with special equation of state i.e. Stiff matter) To overcome the difficulty of infinite density at the centre, it is assumed that the distribution has a core of radius r0 and constant density □0 which is surrounded by the fluid with pressure equal to energy density.

Keywords:-Exact Solution, Perfect Fluid, Density, Core, Pressure.

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INTRODUCTION

Solution with a simple equation of state have been found in various cases, e.g. for □ = P (Letelier[6], Letelier and Tabenaky [7], □ + 3p = constant □ = 3p (Klein [5], Singh and Abdussattar [8], (Buchdahl and Land [3], and for $\rho = (1+a)\sqrt{p} - ap$ (Buchdahl [2]). But if one takes e.g. polytropic flud sphere $p = a\rho^{1+\frac{1}{n}}$ (Klein [4]. Topper [11]. Buchdahl [1]) or a mixture of ideal gas and radiation (Suhonen [10]), one soon has to use numerical method. Singh and Yadav [9] have also studied the static fluid sphere with the equation of state p = □ (i.e. Zeldowich fluid). Further study in this line has been done by Yadav and saini [12] which is more general than one due to Singh and Yadav [9].

In this paper, we have obtained an exact, static spherically symmetric solution of Einstein's field equation for the perfect fluid with p = □ (i.e. stiff matter). To overcome the difficulty of infinite density at the centre, it is assumed that the distribution has a copy of redius r_0 and constant density $□_0$ which is surrounded by the fluid with pressure equal to energy density.

2. THE FIELD EQUATION AND THEIR SOLUTIONS

We take the static spherically symmetric metric in the form

$$(2.1) \quad ds^2 - e^{\beta}dt^2 - e^{\alpha}dr^2 - r^2de^2 - r^2\sin^2\theta\, d\,\phi^2$$

where □ and □ are functions of r only. The field equations.

$$(2.2) \quad R^i_j - \frac{1}{2}R\,\delta^i_j = -8\pi T^i_j$$

For the element (2.1) are

$$(2.3) \quad -8\pi T^1_1 = e^{-\alpha}\left(\frac{\beta'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}$$

$$(2.4) \quad -8\pi T^2_2 = -8\pi T^3_3 = e^{-\alpha}\left(\frac{\beta''}{2} - \frac{\alpha'\beta'}{4} + \frac{\beta'^2}{4} + \frac{\beta' - \alpha'}{2r}\right)$$

$$(2.5) \quad 8\pi T^4_4 = e^{-\alpha}\left(\frac{\alpha'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$$

Where a prime denotes differentiation with respect to r. Throughout the investigation we set velocity of light C and gravitational constant K to be unity. A zeldovich fluid can be regarded as a perfect fluid having the energy momentum tensor.

$$(2.6) \quad T^i_j = (\rho + p)u^i u_j - \delta^i_j P$$

Characterized by the equation of state

$$(2.7) \quad \rho = p$$

We use commoving co-ordinates so that

$$u^1 = u^2 = u^3 = 0 \text{ and } u^4 = e^{-\beta/2}$$

The non – vanishing components of the energy momentum tensor are

$$T_1^1 = T_2^2 = T_3^3 = -P \text{ and } T_4^4 = \rho$$

$$(2.8) \quad 8\pi p = e^{-\alpha}\left(\frac{\beta'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}$$

$$(2.9) \quad 8\pi p = e^{-\alpha}\left(\frac{\beta''}{2} - \frac{\alpha'\beta'}{4} + \frac{\beta'^2}{4} + \frac{\beta' - \alpha'}{2r}\right)$$

$$(2.10) \quad 8\pi\rho = e^{-\alpha}\left(\frac{\alpha'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$$

Using equation (2.7), (2.8) and (2.10), we have

$$(2.11) \quad e^{-\alpha}\left(\frac{\beta'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} = e^{-\alpha}\left(\frac{\alpha'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$$

From (2.11) we see that if □ is known □ can be obtained.

So we choose

$$(2.12) \quad e^{\beta} = Ar$$

where A is constant.

Using (2.12) equation (2.11) takes the form

$$(2.13) \quad \frac{{}^{-\alpha}e\alpha^1}{r^2} - \frac{3e^{-\alpha}}{r^2} + \frac{2}{r^2} = 0$$

Putting $Z = e^{-\alpha}$ the equation (2.13) is reduced to

$$(2.14) \quad \frac{dz}{dr} + \frac{3z}{r} = \frac{2}{r^2}$$

which is a linear differential equation whose solution is

$$(2.15) \qquad z = \frac{2r^3}{3} + c$$

Therefore we get

$$(2.16) \qquad e^{-\alpha} = \frac{2}{3}r^3 + c$$

where c is an integration constant.

Consequently the metric (2.1) can be put into the form

$$(2.17) \qquad ds^2 = Ar\,dt^2 - \left(\frac{2r^3}{3} + c\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

If we set the integration constant c = 0 then absorbing the constants A and $\frac{2}{3}$ in co-ordinate differentials dt and dr respectively, we get.

$$(2.18) \qquad ds^2 = rdt^2 - r^{-3}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

Also for the metric (2.18) the fluid velocity u^i is given by

$$(2.19) \qquad u^1 = u^2 = u^3 = u_1 = u_2 = u_3 = 0$$

$$\text{and} \qquad u^4 \frac{1}{\sqrt{r}}, \quad u_4 = \sqrt{r}$$

The scalar of expansion $\theta - u^1;i$ is identically zero. The non-vanishing components of the tensor of rotation w_{ij} defined by

$$(2.20) \qquad w_{ij} = u_{i,j} - u_j,i$$

are

$$(2.21) \quad w_{14} = -w_{41} = -\frac{1}{2\sqrt{r}}$$

The components of the shear tensor σ_{ij} defined by

$$(2.22) \qquad \sigma_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \frac{1}{3}\theta h_{ij}$$

With the projection tensor

$$h_{ij} = g_{ij} - u_i\, u_j$$

are

$$(2.23) \qquad \sigma_{14} = \sigma_{41} = -\frac{1}{2\sqrt{r}}$$

The other components being zero.

3. SOLUTION FOR THE PERFECT FLUID CORE

Pressure and density for metric (2.18) are

$$(3.1) \qquad 8\pi\rho = 8\pi\rho = -4r + \frac{1}{r^2}$$

It follows from (3.1) that the density of the distribution tends to infinity as r tends to zero. In order to get rid of the singularity at r = 0 in the density we visualize that the distribution has a core of radius r_0 and constant density ρ_0. The field inside the core is given by the Schwarzschild interial solution.

$$(3.2) \qquad e^{-\alpha} = 1 - \frac{r^2}{R^2}$$

$$e^{\beta} = \left[A - B\left(1 - \frac{r^2}{R^2}\right)^{1/2} \right]^2$$

$$8\pi P = \frac{1}{R^2}\left[\frac{3B\left(1-\frac{r^2}{R^2}\right)^{1/2} - A}{A - B\left(1-\frac{r^2}{R^2}\right)^{1/2}} \right]$$

where A and B are constants and

$$R^2 = \frac{3}{8\pi\rho_0}$$

The constants appearing in the solution can be evaluated by the continuity conditions for the metric (2.18) and (3.2) at the boundary r = r_0

4. DISCUSSION

In this paper the equation of state for the fluid has been taken as p = ρ which describes several important cases, e.g. radiation, relativistic degenerated Fermal gas and probably very dense baryon matter

Further if the fluid satisfies the equation of stae p = ρ and if in addition its motion is irrotational, then such a source has the same stress energy tensor as that of a massless field.

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