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A Study on the Implications of Spline Methods as Numerical Solution for Partial Differential Equations

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Abstract – In the modern era, fractional order differential equations have gained a significant amount of research work due to their wide range of applications in various branches of science and engineering such as physics, electrical networks, fluid mechanics, control theory, theory of viscoelasticity, neurology, and theory of electromagnetic acoustics.

The spline approximation techniques have been applied extensively for numerical solution of ODEs and PDEs. The spline functions have a variety of significant gains over finite difference schemes. These functions provide a continuous differentiable estimation to solution over the whole spatial domain with great accuracy. The straightforward employment of spline functions provides a solid ground for applying them in the context of numerical approximations for initial/boundary problems.

Keywords: Spline, Numerical, Partial

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INTRODUCTION

Various techniques of spline methods and their application have been developed to obtain the numerical solution of the differential equations. They possess some of advantages and are worth on using in the numerical techniques. So, spline procedures exhibit the following the desirable features: (1) obtained governing system is always diagonal which permits easy algorithms; (2) it provides low computer cost and easy problem formulation. The requirement of the continuity up to the second degree are guaranteed at the mesh points over the domain and the first and second degree of the derivatives are directly evaluated.

The mathematical model describing the transport and diffusion processes is the one-dimensional advection-diffusion equation. Mathematical modeling of heat transport, pollutants and suspended matter in groundwater involves the solution of a convection–diffusion equation. When the velocity field is complex and changing in time, transport processes can't be analytically calculated, numerical approximations are necessary.

The method has widely become popular in recent years thanks to its simplicity of application. The fundamental idea behind the method is to find the weighting coefficients of the functional values at the nodal points by using base functions, derivatives of which are already known at the same nodal points over the entire region.

Partial differential equation solves by spline collocation and finite difference method. While numerical solution obtained. Therefore, two common questions are encountered, first is about its acceptance whether it is sufficiently close to true solution or not. If one has an analytic solution then this can be answered very clearly but in either case it is not so easy. One has to be careful while concluding that a particular numerical solution is acceptable when an analytic solution is not available. Normally a method is selected which does not produce an excessive error.

We have obtained a numerical solution of the problem by using spline collocation technique. In the investigated mathematical model we consider that the ground water recharge takes place over the large basin of such geological location that the sides are limited by rigid boundaries and the bottom by a thick layer of water table.

In this case the flow may be assumed vertically downwards through unsaturated porous media. Here the average diffusivity coefficient of the whole range of moisture content is regarded as constant and the permeability of the moisture content is assumed to have a parabolic distribution. The theoretical formulation of the problem yields a nonlinear partial differential equation for the moisture content.

METHODS USED FOR NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATION WITH A REFERENCE OF SPLINE METHODS

In this equation, a nonlinear dispersion term replaces the nonlinear dispersion term in the Korteweg-de Vries (KdV) equation, coming about with.

$$K(m,n): \qquad u_t + (u^m)_x + (u^n)_{xxx} = 0\ , \qquad m > 0,\ 1 < n \le 3\ .$$

For certain values of m and n, the K(m, n) equation has solitary waves which are compactly supported. In particula, the variant K(2, 2),

$$K(2,2): \qquad u_t + (u^2)_x + (u^2)_{xxx} = 0\ ,$$

has a fundamental "compacton" solution of the form

$$u(x,t) = \frac{4\lambda}{3}\left[\cos\left(\frac{x-\lambda t}{4}\right)\right]^2, \qquad |x-\lambda t| \le 2\pi\ .$$

After the first appearance of the compactons, it turned out that similar structures emerge as solutions for a much larger class of nonlinear PDEs, among which is, e.g.,

$$u_t + (u^m)_x + (u(u^n)_{xx})_x = 0\ , \quad m > 1\ , \qquad m = n+1\ ,$$

which we consider with m = 2, n = 1 as our non-linear model problem.

In this work we are mostly intrigued by creating tools for approximating numerically solutions of equations which produce non-smooth structures. Because of the irregularity in the subordinates on the fronts of these developing structures, standard numerical methods, for example, limited contrasts and pseudo-ghostly methods create spurious oscillations on the fronts. Controlling these oscillations requires a numerical separating of the higher modes, which may bring about the disposal of fine scales from the arrangement.

Theorem 1.

Let $\gamma = 2$ and $n \ge 4$.

Let $\varphi \in H^1$ be radially symmetric. If $\|\varphi\|_{H^1}$ is sufficiently small, then there exists a unique radial solution $u \in C_b H^1$ such that $|x|^{-1}u \in L^2 L^2$ to Moreover, there exist radial functions φ^+ and φ^- such that

$$\int_{S^{n-1}} |re_1 - \rho\sigma|^{-\theta}\, d\sigma \le M(r,\rho) < \infty,$$

$$\|u(t) - U(t)\varphi^{\pm}\|_{H^1} \to 0 \quad \text{as} \quad t \to \pm\infty$$

For GWP we use a fractional integral estimate on the unit sphere such that

$$\int_{S^{n-1}} |re_1 - \rho\sigma|^{-\theta}\, d\sigma \le M(r,\rho) < \infty,$$

where $e_1 = (1,0,\cdots,0)$. The result of Theorem E corresponds to the case $\theta = \gamma + \frac{1}{2}$

If n = 3, then the finiteness of integralenforces γ to be less than $\frac{3}{2}$ as in Theorem H since the integral is finite only when $n-2-\theta > -1$. For details see Lemma El and Lemma SI In Theorem El we treated the case $\theta = 2$ for which the integral is not finite if n = 3. However, the three dimensional GWP can be slightly improved up to $\frac{5}{3}$ by using another Strichartz estimate on a hybrid Sobolev space. It will be worthy of trying to fill the gap $\frac{5}{3} \le \gamma \le 2$ for n = 3.

The Klein-Gordon equation (2) was initially studied by (W. Strauss, 2001). In (T. Motai, 2008), the GWP is considered for $\lambda < 0$ and $0 < \gamma \le 4$. It was proved that the scattering operator for (2) is well-defined on some domain if $n \ge 2,\ 4/3 < \gamma \le 4n/(n+1)$ and $\gamma < n$. Furthermore, (K. Mochizuki, 2009) showed that if $n \ge 3,\ 2 \le \gamma \le 4$ and $\gamma < n$, then the scattering operator can be defined on some neighborhood near zero in the energy space.

In this study the small data scattering of radial solutions is successfully treated below energy space, provided $\frac{3}{2} < \gamma < 2$. To state precisely, let us define a weighted spaces $W_{s,\varepsilon}$ and a data space $D_{\alpha,\beta}$ by

$$W_{s,\varepsilon} = \{\psi \in L^2 : \|\psi\|^2_{W_{s,\varepsilon}} \equiv \||\cdot|^{-s-\varepsilon}\psi\|^2_{L^2(|x|\le 1)}$$

$+ \||\cdot|^{-s+\varepsilon}\psi\|^2_{L^2(|x|>1)} < \infty\}$ and $D_{\alpha,\beta} = H^{\alpha-\frac{1}{2}} \cap L^{\frac{2n}{n+2-2\beta}}$, respectively, where $\varepsilon > 0$ is sufficiently small.

CONCLUSION

Splines provide high-quality approximations, lead to a sparse structure of the system operator represented by shift-invariant separable kernels in the domain, and are mesh-free by construction. Further, high-order bases can easily be constructed from Spline.

In order to demonstrate the advantageous numerical performance of Spline methods, we studied the solution of a large-scale heat-equation problem (consisting of roughly 0.8 billion nodes!) on a heterogeneous workstation consisting of multi-core CPU and GPUs.

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