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# Steady Flow of a Power Law Fluids with Uniform Suction and Injection between Given Two Parallel Plates

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***Abstract – My study is about the flow of a non-Newtonian power law fluid between two immovable parallel plates. The flow of power law fluid is encountered in various Industrial & Technological areas. A uniform suction and injection through the surface of the parallel plates are applied where the two plates are kept at different but constant temperature and pressure. Numerous of study are focussed on the investigation of flow. However, these studies are bounded to the consideration of the non-Newtonian fluid behaviour. They leave the viscous dissipation out of description. The research facts on the power law fluid flow with allowance for the diffraction are presented. The aim of this study work is to analyse the power law fluid flow decoration or influence under uniform suction and injection between two parallel plates, And for this, here we assumed & carry it some certain equation, and we will find the velocity profiles at different cases.***

***Keywords – Power Law Fluid Flow, Strain Rate, Velocity Profile, Suction & Injection Parameter.***

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## 1. INTRODUCTION

Non-Newtonian power law fluid refers to a specific category of fluid which exhibits variable viscosity under the action of applied force. It undoubtedly differs from the Newtonian fluid which follows the Newton's law of viscosity and bears constant viscosity under stress. The physical viscosity in non-Newtonian fluids could be depend on the magnitude of the shear stress. For example, shear-thickness (dilatants) fluids, Shear-thinning (Pseudoplastic) fluids, Bingham plastic etc. or even time-dependent (Thixotropic Liquid, rheopectic liquid) fluids. In reality and practically there are many fluids may present these non-Newtonian behaviors. e.g., Blood, Silicone oil, Printer ink, Polymers, Honey, Shampoo etc.

Various types of study work are focused on the unsteady & steady investigation of the flow between parallel plates. However, these works are limited to the cogitation of the non-Newtonian fluid behavior [1, 2, 3]. The research analysis on the power law and viscoelastic fluid flow through allowance for the dissipation are presented [4, 5]. Analytical solutions of steady flow of non-Newtonian fluids are limited to very specific geometries and they offered long algebraic development. Thus, numerical approaches have been profusely used. The most used numerical investigation to solve the equation of fluid flow is finite and different method.

The flow in duct of rectangular cross-section with uniform suction & injection has been examined by Rathy [6], Sai & Rao [7] and Erdogan & Imrak [8,9] Crane [10] & Terril and Thomas [11] started his work from the study stretching sheet by considering the non-Newtonian fluids under the assumption that velocity will vary in the direction of flow and that is must be linear to the distance from the specific point. Winter [12] investigated the flow in a pipe with uniform injection and suction shows a boundary layer character near the suction. Pinho [13] considering the asymptotic suction and Injection flows, has shown that this type of fluid expansion may lead to erroneous results.

## 2. MATHEMATICAL MODEL

We have focussed on the study of the steady flow of a power law fluid between two parallel plates given by the equations $y = 0$ and $y = h$. We assume that there is a constant pressure gradient in the x-direction (the direction of the main flow).We also assume that the velocity components u, v in the x and y directions are independent of the current length x measured parallel to the plates. Putting $\frac{\partial u}{\partial x} = 0.$, we find from the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad (2.1)$$

that v(x, y) is a constant $v_0$ (say), so that

$$v(x, y) = v_0 \qquad (2.2)$$

were $v_0$ is positive for injection at the plate y = 0. The only non-zero strain rate component has the value $\frac{1}{2}\frac{du}{dy}$. Because of suction and injection the velocity profile would not be symmetrical about $y = \frac{h}{2}$, but the maximum of velocity would shift. Let $y = y_0$ be the point where u is maximum. Then u increases from $u = 0$ to $u = u_{max}$ when y increases from ($y = 0$ to $y = y_0$) and then it decreases from $u = u_{max}$ to $u = 0$ as y increases from $y = y_0$ so $y = h$. Hence $\frac{du}{dy} = 0$ as $y = y_0$, $\frac{du}{dy} > 0$ in $0 \leq y \leq y_0$ and $\frac{du}{dy} < 0$ in $y_0 \leq y \leq h$

All the stress components except sxy are zero. Then the equation of motion.

$$fv_o = \frac{du}{dy} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} sxy \qquad (2.3)$$

gives

$$v_o \frac{du}{dy} = A + \upsilon \frac{d}{dy}\left[\left(\frac{du}{dy}\right)^n\right], \quad 0 \leq y \leq y_0 \qquad (2.4)$$

and

$$-v_o \frac{du}{dy} = -A + \upsilon \frac{d}{dy}\left[\left(-\frac{du}{dy}\right)^n\right], \quad y_0 \leq y \leq h \qquad (2.5)$$

where, constant

$$\upsilon = \frac{h}{\rho_2 n} \text{ and } -\frac{1}{\rho}\frac{\partial p}{\partial x} = A \qquad (2.6)$$

## 3. INTEGRATION OF EQUATION

Integrating equations (2.4) and (2.5) once and applying the condition $\frac{du}{dy} = 0$ and $u = u_{max}$ when $y = y_0$, we get

$$\upsilon\left(\frac{du}{dy}\right)^n = A(y_0 - y) - v_0(u_{max} - u), \; 0 \leq y \leq y_0 \qquad (3.1)$$

and

$$\upsilon\left(-\frac{du}{dy}\right)^n = A(y - y_0) + v_0(u_{max} - u), \; y_0 \leq y \leq h \qquad (3.2)$$

We make the equations dimensionless by substituting

$$z = \frac{y}{h} \text{ and } w = \frac{u}{h\left(\frac{Ah}{\upsilon}\right)^{1/n}} \qquad (3.3)$$

and obtain

$$\left(\frac{dw}{dz}\right)^n = (z_0 - z) - c(w_{max} - w), \; 0 \leq z \leq z_0 \qquad (3.4)$$

and

$$\left(-\frac{dw}{dz}\right)^n = (z - z_0) + c(w_{max} - w), \; z_0 \leq z \leq 1 \qquad (3.5)$$

Where $z_0 = \frac{y_0}{h}$, $w_{max}$ the corresponding value of w and $C = \frac{v_0}{A}\left(\frac{Ah}{\upsilon}\right)^{-1/n}$ is a dimensionless number

For solving equation (3.4) we substitute

$$W^p = C^n\left[z_0 - z - C(W_{max} - W)\right] \qquad (3.6)$$

where $n = \frac{p}{q}$, p and q being prime so each other, and get

$$C^n(z - z_0) = p\int_0^w \frac{W^{p-1}}{W}\frac{dW}{-1}, \; 0 \leq z \leq z_0 \qquad (3.7)$$

Putting z = 0, we obtain

$$-z_0 C^n = p\int_0^{W_0} \frac{W^{p-1}dW}{W^q - 1} \qquad (3.8)$$

where

$$W_0 = C^{1/q}\left[z_0 - C W_{max}\right]^{1/p} \qquad (3.9)$$

Similarly substituting

$$W^p = C^n\left[z - z_0 + c(W_{max} - w)\right] \qquad (3.10)$$

is equation (3.5) and integrating, we get

$$C^n(z - z_0) = p\int_0^w \frac{W^{p-1}dW}{W^q + 1}, \; z_0 \leq z \leq 1 \qquad (3.11)$$

Putting z = 1, we obtain

$$C^n(1 - z_0) = p\int_0^{W_1} \frac{W^{p-1}dW}{W^q + 1} \qquad (3.12)$$

Where

$$W_1 = C^{1/q}\left[1 - z_0 + CW_{max}\right]^{1/p} \qquad (3.13)$$

Equation (3.8) and (3.12) may be regarded as two simultaneous equation for determining the values of $z_0$ and $W_{max}$. Then equations (3.4) and (3.11) will determine the velocity profile. All the integral entering into the problem can be analytically integrated for general values of p and q, but the simultaneous equations for $z_0$ and $W_{max}$ would be too complicated to solve, hence in such situations numerical integration would be preferred.

## 4. CALCULATION OF VELOCITY PROFILES

We have found the velocity profiles for $n = \frac{1}{2}$, 1 and 2 as particular examples to show the following work.

### (4.1) Case, (When) $n = \frac{1}{2}$

On integration of equations (3.7) and (3.11) & we get

$$C^{1/2}(z_0 - CW_{max} - z + CW) = \tanh\left[C^{1/2}(z_0 - z)\right], 0 \le z \le z_0 \qquad (4.1.1)$$

and

$$C^{1/2}\left[z - z_0 + C(W_{max} - w)\right] = \tanh\left[C^{1/2}(z - z_0)\right],\ z_0 \le z \le 1 \qquad (4.1.2)$$

The equations for determining z, and $w_{max}$ are

$$C^{1/2}(z_0 - CW_{max}) = \tanh\left[C^{1/2}z_0\right] \qquad (4.1.3)$$

and

$$C^{1/2}(1 - z_0 + CW_{max}) = \tanh[C^{1/2}(1 - z_0)] \qquad (4.1.4)$$

Eliminating $W_{max}$ & we get

$$C^{1/2} = \tan[c^{1/2}(1 - z_0)] + \tanh[c^{1/2}z_0] \qquad (4.1.5)$$

A little consideration would show that in order to have only one value of $z_0$ (which is physically feasible) between 0 and 1, C has to be between 0 and $\frac{\pi^3}{4}$. If C is greater than $\frac{\pi^3}{4}$, there would be more than one value of $z_0$ and the flow of the type (laminar flow) is not possible.

We have determined the value of $z_0$, $w_{max}$ for various values of C.

### 4.2 Case, (When) n = 2

Equations (3.7) and (3. 11) give

$$\frac{C^2}{2}(z_0 - z) = -\left[W + \log(-W)\right], 0 \le z \le z_0 \qquad (4.2.1)$$

and

$$\frac{C^2}{2}(z - z_0) = \left[W - \log(1 + W)\right], z_0 \le z \le 1 \qquad (4.2.2)$$

The equations for deserting $z_0$ and $W_{max}$ are

$$\frac{-C^2}{2}z_0 = Cs^{1/2} + \log(1 - Cs^{1/2}) \qquad (4.2.3)$$

and

$$\frac{C^2}{2}(1 - z_0) = c(1 - s)^{1/2} - \log[1 + C(1 - s)]^{1/2} \qquad (4.2.4)$$

where

$$s = z_0 - CW_{max} \qquad (4.2.5)$$

Equation (4.2.3) and (4.2.4) can be solved for determining the values of s and $z_0$, then $w_{max}$ would be obtained from equation.

Then from equation (4.2.1) & (4.2.2) we can determine the value of z and w for various values of W. When $0 \le z \le z_0$, the value of W for which z and w would be determined will lie between 0 and $W_0$ determined by the equation.

$$W_0 + \log(1 - W_0) + \frac{C^2}{2}z_0 = 0 \qquad (4.2.6)$$

Similarly when $z_0 \le z \le 1, W$ lies between 0 and $W_1$, where $W_1$ is the root of the equation.

$$W_1 - \log(1 + W_1) - \frac{C^2}{2}(1 - z_0) = 0 \qquad (4.2.7)$$

We have determined the value of $z_0$, $W_{max}$ for various value of C.

### 4.3. Case, (When) n = 1

In this case w, $z_0$ and $w_{max}$ are given by the following equations.

$$Cw_1 = z - \frac{e^{cz} - 1}{e^c - 1}, \qquad (4.3.1)$$

$$z_0 = \frac{1}{c}\log\left(\frac{e^c - 1}{c}\right) \qquad (4.3.2)$$

and

$$W_{\max} = \frac{1}{c}\left[\frac{1}{e^{c}-1} + z_0 - \frac{1}{c}\right] \qquad (4.3.3)$$

## 5. RESULTS & CONCLUSIONS

[a] The value of $z_0$ and $w_{\max}$ for different value of n and c have been given in table 1.

**Table – 1**

| | C = | 0 | 0.5 | 1.0 | 8.0 |
|---|---|---|---|---|---|
| n= ½ | $Z_0$ = | 0.5000 | 0.5090 | 0.5166 | 0.8387 |
| | $W_{\max}$ = | 0.04166 | 0.04150 | 0.04156 | 0.04003 |
| n = 1 | $Z_0$ = | 0.5000 | 0.6307 | 0.6417 | 0.5507 |
| | $W_{\max}$ = | 0.1360 | 0.1245 | 0.1233 | 0.1191 |
| n = 2 | $Z_0$ = | 0.5000 | 0.5297 | 0.6587 | 0.6143 |
| | $W_{\max}$ = | 0.3387 | 0.2342 | 0.2292 | 0.2107 |

[b] It is seen that for any value of c, $z_0$ increases with n, that is the point of maximum velocity shifts more and more towards the wall z = 1 with the increases of n. We also see that the variation of $z_0 – [z_0]_{c=0}$ with c, for any fixed n is almost linear. Hence for any fixed n, $z_0$ increases with h and $v_0$ decreases with the increase of $\upsilon$.

[c] It is found that for fixed value of [illegible] decreases with the increases of C. Hence $u_{\max}$ will decreases with the increase of $v_0\left(\frac{h}{A}\right)\left(\frac{Ah}{\upsilon}\right)^{1/n}$. Therefore $u_{\max}$ increases with the suction velocity $v_0$ and h. Again it decreases with the increase of the pressure gradient for n < 2 and increases with the pressure gradient for n > 2.

[d] It can be seen that for a fixed n, the velocity always decreases with the increases of C in the region 0 < z < ½ and in the neighbourhood of the wall z = 1, the velocity increases with the increases of C. Now C increases with $v_0$ (the velocity of injection) and hence it is seen that the velocity in the neighbourhood of the wall z = 1, where the liquid is withdrawn, increases with the increase of suction velocity $v_0$ where as it decreases in the region (0 < z < ½) with the increases of $v_0$.

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