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Mathematical Modeling on Cardiography

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Abstract – Modern medical sciences have led to new prospects of improved medical health care in which mathematical modeling becomes increasingly important and will predict behavior of the human body as a response to internal or external changes. Due to the physical complexity and difficult experimental accessibility of biological systems, mathematical modeling has not enjoyed a comparable interest. Consequently, medical treatment at present is still mainly based on consensus about the behavior of the human body and rarely supported by predictive modeling. The present paper provides some aspects of mathematical modeling in cardiography.

Key Wards – Electrocardiography (ECG), Blood Circular System, Cardiovascular System.

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1.1 INTRODUCTION

Mathematical modeling of the cardiovascular system is a relevant topic that has paying attention on outstanding interest from the mathematical society because of the intrinsic mathematical effort and due to the increasing effect of cardiovascular diseases worldwide.

The field of medical sciences deals with the study of the heart is called Cardiology. The nature and effects of vibrations of the heart as it pumps blood through the circulatory system of the body are a great source of mathematical applications [3]. An important aspect involves the recording of such vibrations known as cardiography. The instrument that records such vibrations is called an electrocardiography (ECG) [1, 2]. It translates the vibrations into electrical impulses which are then recorded. It is interesting to translate the heart vibrations into mechanical vibrations instead of translating these vibrations into electrical impulses.

1.2 CARDIOVASCULAR BIOMECHANICS

Biomechanics is one of the branches related to medical science in which the role of mathematical modeling is apparent. One of the founders of modern biomechanics, defined biomechanics as the mechanics applied to biology [4]. The research discipline that studies the mechanical properties of organisms and helps in understanding of their normal and pathological function, helps to predict their adaptation to changing circumstances and helps in finding methods for artificial intervention. Cardiovascular biomechanics is that part of biomechanics that attentions on the cardiovascular system, the heart and blood vessels. The mechanism of blood flow and wall motion in the heart and blood vessels and the exchange of nutrients and oxygen with waste products and carbon dioxide in the tissues of the organs

1.3 ANATOMY AND PHYSIOLOGY OF CARDIOVASCULAR SYSTEM

The representation given in Figure 1, it is shown that from a mechanical fact, the cardiovascular system includes of a heart acting as a four-chambered pump that propels blood around the circulatory system [5]. Cardiac impulse transmission of the depolarization wave determines the contraction of the heart muscle while the valves between the atria and ventricles, respectively, the ventricles and the main arteries take care of unidirectional blood flow [6]. The complete circulation consists of an arterial system branching in many arteries of decreasing diameter, a capillary micro-circulatory system in the tissues and a venous system with venules and veins amalgamation into larger vessels transferring blood back to the heart. A close look at the biological functioning of the cardiovascular system deals that it is extremely optimized and self-regulatory under the condition that all parts function properly. In the western world, however, the rate of mortality originating from cardiovascular disease is more than 40%.

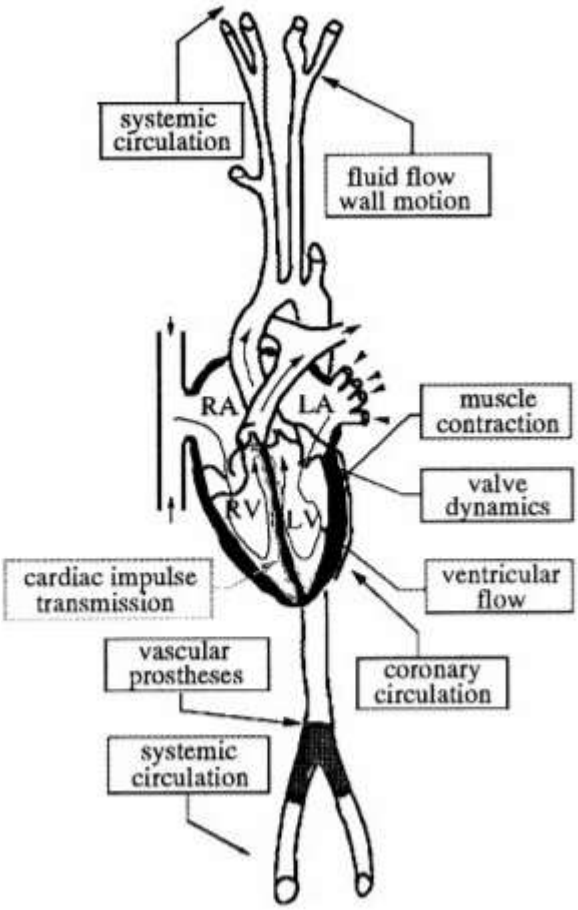

Figure 1. Schematic representation of the cardiovascular system

1.4 MATHEMATICAL EQUATION & SOLUTION

Suppose that a person rests on a horizontal table which has springs so that it can vibrate horizontally but not vertically. Then, due to the pumping of the heart, the table will undergo small vibrations, the frequency and magnitude of which will depend on the various limitations related with the heart. Thus, by investigating the motion of the table some important conclusions about the vibrations of heart can be drawn. Let x denote the horizontal displacement of some specified point of the table (as, for example, one end) from some fixed location (such as a wall). Let M denote the combined mass of the person and that portion of the table which is set into motion. If we assume that there is a damping force proportional to the instantaneous velocity and a restoring force proportional to the instantaneous displacement, then the differential equation describing the motion of the table is

(1.1) $$M\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} + \beta x = F$$

where α and β are constants of proportionality and F is the force on the system due to the pumping action of the heart. Suppose that m is the mass of blood pumped out of the heart during each vibration and y is the instantaneous centre of mass of this quantity of blood. Then by Newton's law, we have

(1.2) $$F = m\frac{d^2y}{dt^2}$$

As a first approximation it may be assumed that y can be expressed as a simple sinusoidal function of t given by

(1.3) $$y = k \sin \omega t$$

where k and ω are constants. Equation (1.3) suggests that there is only one frequency associated with the vibrations of the heart, whereas evidence shows that there are many frequencies. This leads us to replace equation (1.3) by

(1.4) $$y = k_1 sin\omega t + k_2 sin2\omega t + k_3 sin3\omega t + \dots$$

The series of terms on the right is called a Fourier series. The first term on the right of equation (1.4) represents a first approximation to the function, the sum of the first two terms a better approximation, and so on. Using only the first two terms of the series (1.4) in (1.2) and then putting the result into equation (1.1), we obtain

(1.5) $$M\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} + \beta x = -m\omega^2(k_1 sin\omega t + k_2 sin2\omega t)$$

Which can be solved subject to various possible conditions.

The general solution of Equation (1.5) consists of two parts :

(i) The general solution of the equation with the right side replaced by zero.

(ii) A particular solution.

The first part is the transient solution and will disappear rapidly provided α > 0. The second part will be the steady state solution in which we are interested. This steady state solution can be easily obtained as

$$x = \frac{m\omega^2 k_1[(M\omega^2 - \beta)sin\omega t + \alpha\omega cos\omega t]}{(M\omega^2 - \beta)^2 + \alpha^2\omega^2}$$

$$+\frac{4m\omega^2 k_2[(4m\omega^2 - \beta)sin(2\omega t) + 2\alpha\omega cos(2\omega t)}{(M\omega^2 - \beta)^2 + \alpha^2\omega^2}$$

A corresponding solution can be found assuming any number of terms in Equation (1.4). The solution is useful for concluding the vibrations in our heart.

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