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Analysis of MHD Flow and Heat Transfer over a Vertically Stretching Sheet with Thermal Slip

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Abstract – This paper investigates MHD flow and heat transfer over a vertically stretching sheet with thermal slip. The impact of viscous and Joules dissipation on MHD flow over a vertically stretching sheet with viscous and joules dissipation with hydrodynamic/thermal slip has been analysed. The system of ordinary differential non-linear Equation together with the conditions of the boundary Equations are similar and are solved numerically using the fourth order of the integration scheme of Runge kutta, accompanied by the shooting scheme. The effects of different governing parameters on velocity and temperature profiles were discussed in order to gain physical insight into the problem by assigning numerical values to the parameter, i.e. numerical calculations were performed for different values of magnetic parameter M, Grash of number Grx, Prandtl number Pr, Eckert number Ec, hydrodynamic slip parameter.

Keywords – Nanofluid, MHD, Runge-Kutta Method, Stretching Sheet, Thermal Slip.

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1. INTRODUCTION

Because of its various industrial applications, such as the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film, the condensation process of metal plate in a cooling bath and glass, and also in the polymer industries, the study of flow over a stretching sheet has generated much interest in recent years. Since the pioneering work of Sakiadis[1961], who researched the problem of moving plate movement, several authors such as Cortell[2008], Xu and Liao[2005], Hayat et al.[2008] etc. have investigated different aspects of the problem.

The study of two-dimensional boundary layer flow, heat, and mass transfer over a porous stretching surface is fascinating because it has numerous applications in various fields. Many metallurgical processes, in particular, necessitate the cooling of continuous strips or filaments by drawing them through a quiescent fluid and stretching them in the process. Viscous dissipation affects the temperature distribution by acting as an energy source that influences the rate of heat transfer. Whether the sheet is heated or cooled determines the effectiveness of the viscous dissipation effect.

The dissipation of Joules also serves as a volumetric heat source, apart from viscous dissipation. Due to its abundant applications, heat transfer analysis over porous surface is of high practical importance. Crystal growing is a few practical applications of flow over a stretching sheet to be more precise, heat-treated materials that move between a feed roll and wind-up roll or materials created by extrusion, glass-fiber and paper processing, cooling of metal sheets or electronic chips. The final result of the desired characteristics depends on the rate of cooling and also the rate of stretching in all these situations. In light of all of these factors, the current work explores the effect of viscous and Joules dissipation on MHD flow, heat, and mass transfer over a porous layer with partial slip. In these areas, many scientists have conducted research. Vajravelu and Hadjinicolaou [2006], for example, produced analytical results that included the effects of viscous dissipation and internal heat generation. Chiam[1977] investigated the thermal boundary layer in an electrically conducting fluid over a linearly stretching sheet in the presence of a continuous transverse magnetic field with suction or blowing at the ground.

Very recently, the energy equation has taken into account the dissipation of viscous and joules and the inner heat production. The non-like analytical solution for MHD flow and heat transfer in a third-grade fluid over a stretching layer was investigated by Sajid et al. [2007]. He found that as the magnetic parameter or the third grade parameter increases, the skin friction coefficient decreases. Abel et al. [2008] published a mathematical analysis of momentum and heat transfer properties in an incompressible, electrically conducting viscoelastic boundary layer fluid flow on a linear stretching board.

Pantokratoras [2008] examined MHD boundary layer flow over a heated stretching sheet with variable viscosity numerically. In the stagnation-point flow of an incompressible viscous fluid, Ishak et al. [2006] studied mixed convection boundary layers over a stretching vertical sheet. Hossain and Takhar[1996] studied the effect of radiation on the mixed convection boundary layer flow of an optically dense viscous incompressible fluid over a vertical plate with a uniform surface temperature.

Different researchers have investigated this issue of non-linear stretching sheets for various fluid flow scenarios. Vajravelu [2006] explored fluid flow over a nonlinear stretching sheet. Cortell[2007] focused on viscous flow and heat transfer over a non-linear stretching sheet. Raptis et al [2006] examined viscous flow over a non-linear stretching sheet in the presence of a chemical reaction and a magnetic field. Abbas and Hayat[2008] addressed the influence of radiation on MHD flow induced by a stretching layer in porous space. Cortell [2008] studied the influence of the similarity solution on the flow and heat transfer of a quiescent fluid over a non-linear stretching surface. Awang and Kechil [2008] used a chemical reaction and a magnetic field to produce a series solution for flow over a nonlinear stretching layer. Cortell [2008] studied the influence of the similarity solution on the flow and heat transfer of a quiescent fluid over a non-linear stretching surface.

The research of fluid conducting magneto hydrodynamics finds applications in a number of astrophysical and geophysical issues. Romig[1964], Elbashbeshy[1998], and considered heat transfer over a stretching surface with a variable surface heat flux, presented the influence of the magnetic field on the normal convection heat transfer. The convective heat transfer on a stretching surface in an electrically conducting fluid was studied by Vajravelu and Hadjinicolaou [1997]. Other research dealing with hydro magnetic flows can be found in Takhar and Ram[1994], Duwairi and Damseh[2004] and Grandet et al[1992]. An exact solution of the two-dimensional boundary layer equations was obtained by Crane [1970]. The flow field over a stretching surface has attracted considerable attention after his pioneering work and a good amount of literature on this problem has been produced.

The present study therefore investigates the impact of viscous and joules dissipation on MHD flow over a viscous and joules dissipation vertically stretching sheet with hydrodynamic/thermal slip.

2. MATHEMATICAL FORMULATION

For investigation, two-dimensional, steady, MHD laminar boundary layer flow with heat transfer of a viscous, incompressible and electrically conducting fluid over a hydrodynamic/thermal slip vertical stretching board, embedded in the presence of transverse magnetic field including viscous and dissipation of Joules, is considered. A uniform transverse magnetic field of strength $\frac{\sigma B^2}{\rho}$ that, as in a polymer extrusion process, emerges out of a slit at x = 0, y = 0 and then is stretched. let us assume that the speed on the plate at a given point is proportional to the power of the distance from the break, and that the boundary layer approximations are true. The induced magnetic field, the external electric field, and the electric field are all believed to be zero in the following calculations due to charge polarization.

Consider a steady, two-dimensional free flow of convection adjacent to a vertical sheet stretching immersed in an electrically conducting viscous temperature fluid incompressible. The stretching velocity and surface temperature are constant. Under these conditions, momentum, energy with buoyancy, viscous and Joules dissipation equations with hydrodynamic slip are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{.......................(3.1)}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma B^2}{\rho}u \qquad \text{...........(3.2)}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\sigma B^2}{\rho c_p}\right)u^2, \qquad \text{...........(3.3)}$$

And the boundary conditionsa are,

$$u(x,y)=L\frac{\partial u}{\partial y}+\alpha x,\ v=0,\ T=T_\infty+a_1\left\{\frac{x}{x_1}\right\}^2+h\left(\frac{\partial T}{\partial y}\right)\ \text{at } y=0$$

$$u\to 0,\ T\to T_\infty\ \ as\ \ y\to\infty, \qquad \text{..................(3.4)}$$

Where u and v are the elements of velocity along the axes of x and y, respectively. Further μ, ρ, α, β, T and g are the dynamic viscosity, fluid density, thermal diffusivity, thermal expansion coefficient, fluid temperature in the boundary layer, and acceleration due to gravity, respectively. Introducing the following similarity transformations,

$$\eta=\left(\frac{a}{\nu}\right)^{1/2}y, \qquad u(x,y)=axf'(\eta), \quad v(x,y)=-\sqrt{\nu a}\,f(\eta) \qquad \text{.........(3.5)}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \text{..........(3.6)}$$

Eqns. (3.1) to (3.3) takes the following form of nonlinear ordinary differential equatuions.

$$f''' = ff'' - f'^2 - Gr\theta + Mf', \quad \text{......(3.7)}$$

$$\theta'' = P_r f\theta' - 2P_r f'\theta - Ec\Pr(f''^2 + Mf'^2) \quad \text{......(3.8)}$$

Similarly the boundary conditions (3.4), becomes

$$f(0) = f_w, \quad f'(0) = 1 + \gamma f''(0), \quad \theta(0) = 1 + K\theta'(0), \quad f'(\infty) = 0, \quad \theta(\infty) = 0, \quad (3.2.9)$$

Where.

$$K = K_1\sqrt{\frac{a}{\nu}} \text{ and } Gr_x = \frac{g\beta(T_w - T_\infty)}{\nu^2} \quad \gamma = l\sqrt{\frac{a}{\nu}}$$

3. NUMERICAL SOLUTION

The problem with the nonlinear limit value represented by Eqs. Using the Fourth-order Runge Kutta shooting technique, (3.2.7) to (3.2.9) are numerically solved.

The system of ordinary differential non-linear Eqs . (3.2.7) and (3.2.8) together with the conditions of the boundary Eq. (3.2.9) are similar and are solved numerically using the fourth order of the integration scheme of Runge kutta, accompanied by the shooting scheme. To initiate the shooting process, making an initial guess for the values of $f''(0)$, $\theta'(0)$, is very crucial in this process. The success of the method relies heavily on how good this guess is. For several physical parameter values, i.e. magnetic parameter M, Prandtl number Pr, hydrodynamic slip parameter γ,thermal slip parameter K, Grashof number G_{rx},, and Eckert number, numerical solutions are obtained (Ec).

In order to satisfy the convergence criterion in all cases, we have selected a step size h of order 0.01. Each iteration loop was found to have a maximum value of 0.01. If the value of the unknown boundary conditions at, y=0, is not changed to a successful loop with an error less than 10^{-6}, the maximum value h for each group of parameters is determined.

4. RESULTS AND DISCUSSION

The effects of different governing parameters on velocity and temperature profiles were discussed in order to gain physical insight into the problem by assigning numerical values to the parameter, i.e. numerical calculations were performed for different values of magnetic parameter M, Grashof number G_{rx}, Prandtl number Pr, Eckert number Ec, hydrodynamic slip parameter.

The influences on the longitudinal velocity profile of the magnetic parameter are shown in Fig1. Increasing magnetic parameters can be seen to reduce the distribution of velocity in the boundary layer, resulting in the thinning of the thickness of the boundary layer, and thus inducing an increase in the absolute value of the surface velocity gradient.

The influences on the temperature profile of the thermal slip parameter K are shown in Fig 2. It can be seen that the increasing thermal slip parameter improves temperature, resulting in thickening of thermal boundary layer thickness in the thermal boundary layer region.

An increase in the number of Prandtl *Pr* is associated with a decrease in the distribution of temperature shown in Fig. 3, which is consistent with the fact that the thickness of the thermal boundary layer decreases by increasing the number of Prandtl values. With the increasing values of the Prandtl number, the rate of heat transfer increases. As *Pr* improves, the boundary layer edge is reached faster.

For some distinct hydrodynamic slip parameter values, the dimensionless velocity profile is presented in Fig.4. It is easily seen that it has a significant influence on the alternatives. In fact, with the no-slip solution for and towards full slip, the amount of slip increases monotonically as tends to infinity. The latter limiting case means that the frictional resistance between the viscous fluid and the surface is eliminated and there is no longer any fluid movement imposed by the stretching of the sheet.

The effects of longitudinal velocity on dimensionless velocity are shown graphically in Fig 5 and the effects of buoyancy force (Grashof number G_{rx}) are discovered with a small *Pr*. Thus, fluid with a smaller *Pr* is more prone to the effects of buoyancy force.

The velocity and temperature profiles shown in Figures 1-5, show that the conditions of the far field boundary are met asymptotically, supporting the validity of the numerical results presented.

GRAPHS

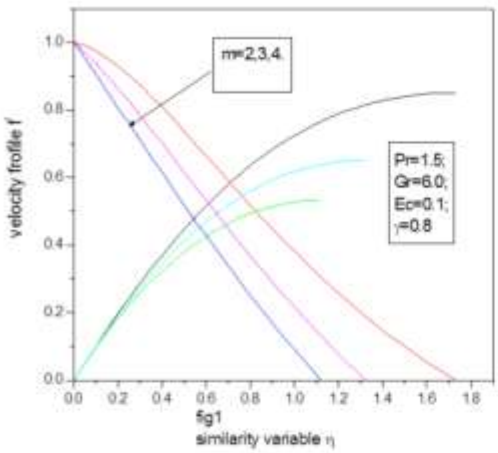

Fig. 1 The influences of the magnetic parameter Vs velocity profile

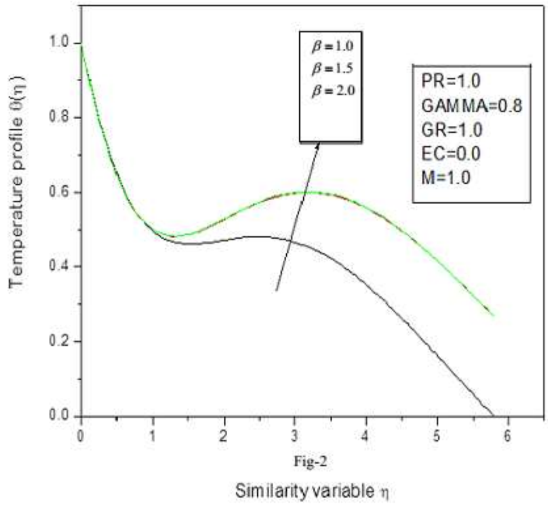

Fig 2 The influence of thermal slip parameter on temperature profile

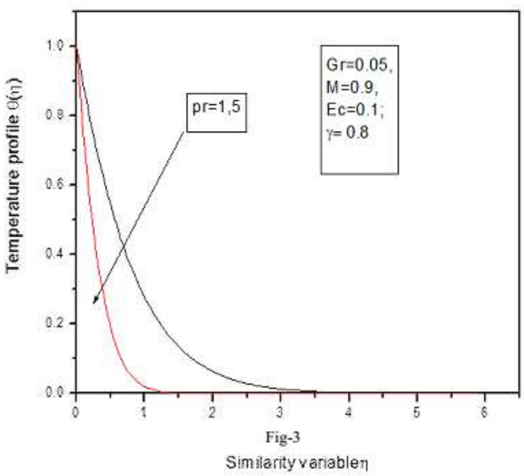

Fig 3 The effect of Prandtl number Pr Vs temperature distribution

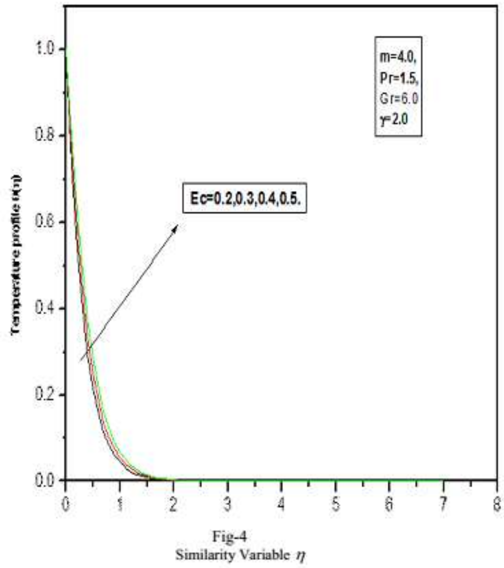

Fig 4 The effects of temperature profile for various values of Eckert number Ec

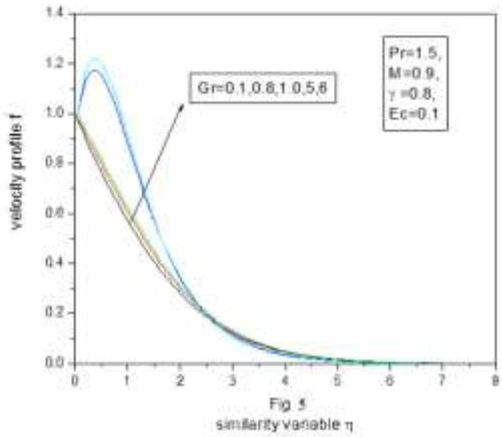

Fig.5. The effects of dimensionless velocity and the buoyancy force (Grash of number G_{rx})

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