Journal of Advances and Scholarly Researches in Allied Education
Vol. 16, Issue No. 6, May-2019, ISSN 2230-7540

A Study on the Applications of Stochastic Convergence

Prof. (Dr.) Rama Nand Singh*

HOD & Professor, Department of Mathematics, Jamunilal College Hajipur Vaishali, Bihar

Abstract – Concepts and hypothesis helpful in understanding the restricting conduct of stochastic processes. We start with an overall conversation of stochastic processes in measurement spaces. The focal point of this conversation is on quantifiable stochastic processes since most constraints of experimental processes in statistical applications are quantifiable. We next examine feeble convergence both by and large and in the particular instance of limited stochastic processes. One of the intriguing parts of the methodology we take to powerless convergence is that the processes considered need not be quantifiable besides in the breaking point. This is valuable in applications since numerous observational processes in insights are not quantifiable as for the uniform measurement. The last part of this section thinks about different modes of convergence, for example, in likelihood and external definitely, and their relationships to powerless convergence

Keywords – Applications Stochastic Convergence

- x -

INTRODUCTION

A wide scope of scientific and designing issues include multi scale wonders. Generally talking, each matter is portrayed by its own mathematical measurements which are all the time a few significant degree bigger. The investigation and the under remaining of these issues request the development of new numerical apparatuses and strategies. Homogenization hypothesis is such a device which presently possesses a focal spot in contemporary numerical exploration.

Deterministic issues in the periodic setting prominently highlighted in the first decade of the development of the hypothesis till the spearheading works of Kozlov, Papanicolaou and Varadhan in stochastic homogenization in the last part of the 1970s. From that point forward extreme examination exercises have been embraced with an incredible abundance of results as demonstrated by the huge existing writing to date, see e.g.,. It is important the intriguing work on stochastic homogenization with regards to the system of thickness arrangements by a few prominent mathematicians.

To manage deterministic homogenization hypothesis past the periodic setting, Nguetseng, following Zhikov and Krivenko, presented the idea of homogenization algebras. This hypothesis depends vigorously on ergodic hypothesis (yet not the ergodicity!) in light of the fact that in applications, the presumption of ergodicity of the homogenization polynomial math considered is principal. It is critical to take note of that there was a gap between the periodic homogenization hypothesis and the stochastic homogenization hypothesis, gap which was filled by Nguetseng's deterministic homogenization hypothesis.

Anyway as we will find in the current work, this new deterministic hypothesis can be seen as a unique instance of a summed up rendition of the stochastic homogenization hypothesis of Bourgeat et al. which we develop. In reality, Theorem 3 (see Section2) permits to expand on the range of a polynomial math with mean worth, a dynamical framework whose invariant measure is unequivocally the measure identified with the mean worth defined on the variable based math. Because of the previously mentioned theorem, we get a speculation of the relative multitude of results introduced in, those in being the unique case comparing to ergodic algebras; see Section 4. The two hypotheses referenced above have the specificity to be utilized to take care of either stochastic homogenization issues just (for the first one) or deterministic homogenization issues just (for the subsequent one). Sadly, as we probably are aware, in nature, not many marvels act, either arbitrarily or deterministically; the majority of these wonders carry on haphazardly in certain scales, and deterministically in different scales.

OBJECTIVES OF THE STUDY

1) Study On Applications Of Stochastic Convergence

2) Study on Preliminaries on dynamical systems and generalized Besicovitch spaces

Applications of Stochastic Convergence

Persuaded by this vision of the actual nature, we depend on these two hypotheses and henceforth on their related convergence techniques (the stochastic two scale convergence in the mean and the Σ-convergence to propose an overall strategy for addressing coupled - deterministic and stochastic - homogenization issues. Our strategy, the stochastic Σ-convergence, joins the macroscopic and microscopic [random and deterministic] scales, and has along these lines the benefit of taking both the effortlessness and the efficiency of the macroscopic models, just as the exactness of the coupled arbitrary deterministic microscopic models.

Additionally our multi scale approach is spurred by the way that the standard mono scale approach has demonstrated to be deficient as a result of restrictively enormous number of factors associated with each actual issue.

One can likewise give in any event two reasons very characteristic. Right off the bat, a scale can't be simultaneously deterministic and arbitrary. Also, the use of our outcomes to normal wonders; see Sections 5 and 6. To be more exact, our technique allows hereafter to treat deterministic homogenization issues without depending on the ergodicity supposition on one hand, and then again permit seeing the stochastic two-scale convergence in the mean in a more broad point as summing up the Σ-convergence.

We trust that the hypothesis created in the current paper will find applications in the arising field of homogenization of stochastic fractional respectful conditions attempted in the papers in the periodic case and in on account of non-periodically punctured areas

The paper is coordinated as follows. In Section 2 we give some fundamental outcomes identified with the hypothesis of dynamical frameworks on conceptual likelihood spaces. We additionally define and give some essential properties of summed up Besicovitch spaces. Area 3 is given to the investigation of the idea of stochastic Σ-convergence. We demonstrate in that some compactness results. In Sections 4, 5 and 6, we give a few applications of the prior outcomes. We start in Section 4 by showing how the after effects of Section 3 apply and how they sum up the current outcomes; this is shown by the investigation of a fairly basic linear operator in dissimilarity structure. We at that point contrast our outcomes and the generally existing ones. In Section 5 we study the homogenization issue for the notable nonlinear Reynolds condition. One significant accomplishment of our outcomes is gotten in Section 6 where we tackle the coupled stochastic-deterministic homogenization issue identified with the accompanying

Stokes equation:

$$-\sum_{i,j=1}^{N}\frac{\partial}{\partial x_i}\left(a_{ij}(x,T(x/\varepsilon_1)\omega,x/\varepsilon_2)\frac{\partial \mathbf{u}_\varepsilon}{\partial x_j}\right)+\mathbf{h}^\varepsilon\times\mathbf{u}_\varepsilon+\operatorname{grad}p_\varepsilon=\mathbf{f}\text{ in }Q$$

$$\operatorname{div}\mathbf{u}_\varepsilon=0\text{ in }Q$$

$$\mathbf{u}_\varepsilon=0\text{ on }\partial Q.$$

We get the following homogenization result which is, to our knowledge, new

Theorem 1.

Assume

(1) $a_{ij}(x, \omega, \cdot) \in A$ for all $(x, \omega) \in Q \times \Omega$, $1 \leq i, j \leq N$, and $h \in L^\infty(\Omega; A)^N$.

For each $0 < \varepsilon < 1$ and for a.e. $\omega \in \Omega$ let $u_\varepsilon(\cdot, \omega) = (u^k_\varepsilon(\cdot, \omega)) \in \mathbf{H}^1_0(Q)$

be the (unique) solution of the above Stokes equation.

Then as ε → 0,

$u_\varepsilon \to u_0$ stoch. in $L^2(Q \times \Omega)^N$ -weak

$$\frac{\partial u_\varepsilon^k}{\partial x_j}\to\frac{\partial u_0^k}{\partial x_j}+\overline{D}_{j,\omega}u_1^k+\frac{\overline{\partial}u_2^k}{\partial y_j}\ \textit{stoch. in } L^2(Q\times\Omega)\textit{-weak }\Sigma\ (1\leq j,k\leq N)$$

where u = (u_0, u_1, u_2) is the unique solution to the following variation problem:

$$a(\mathbf{u},\mathbf{v})+\iint_{Q\times\Omega}(\tilde{\mathbf{h}}\times\mathbf{u}_0)\cdot\mathbf{v}_0\,dx\,d\mu=\langle\mathbf{f},\mathbf{v}_0\rangle\ \textit{for all }\mathbf{v}=(\mathbf{v}_0,\mathbf{v}_1,\mathbf{v}_2)\in\mathbb{F}_0^1$$

With

$$a(\mathbf{u},\mathbf{v})=\sum_{i,j,k=1}^{N}\iint_{Q\times\Omega\times\Delta(A)}\widehat{a}_{ij}(x,\omega,s)\left(\frac{\partial u_0^k}{\partial x_j}+\overline{D}_{j,\omega}u_1^k+\widehat{\partial_j u_2^k}\right)\times\left(\frac{\partial v_0^k}{\partial x_i}+\overline{D}_{i,\omega}v_1^k+\widehat{\partial_i v_2^k}\right)dx\,d\mu\,d\beta;$$

$$\tilde{\mathbf{h}}(\omega)=\int_{\Delta(A)}\widehat{\mathbf{h}}(\omega,s)\,d\beta;$$

$$\langle\mathbf{f},\mathbf{v}_0\rangle=\int_{\Omega}(\mathbf{f}(\cdot,\omega),\mathbf{v}_0(\cdot,\omega))_{H^{-1}(Q)^N,H_0^1(Q)^N}\,d\mu$$

and

$$\overline{\widehat{\partial_j u_2^k}}=\mathcal{G}_1(\overline{\partial}u^k_2/\partial y_j)\ \textit{(and }\overline{a}\textit{ same definition for }\widehat{\partial_i v_2^k}\textit{)}.$$

Unless otherwise species, vector spaces throughout are assumed to be complex vector spaces, and scalar functions are assumed to be complex valued. We shall always assume that the numerical spaces Rmand their open sets are each equipped with the Lebesgue measure

We first prove the weak law of large numbers. There exist different versions of this theorem since more assumptions on X_n can allow stronger statements.

Definition.

A sequence of random variables Y_n converges in probability to a random variable

Y, if for all $\epsilon > 0$,

$$\lim_{n\to\infty} \mathrm{P}[|Y_n - Y| \geq \epsilon] = 0 .$$

One calls convergence in probability also stochastic convergence.

Remark

If for some $p \in [1,\infty)$, $\|X_n - X\|_p \to 0$, then $X_n \to X$ in probability since by the Chebychev-Markov inequality $\mathrm{P}[|X_n - X| \geq \epsilon] \leq \|X - X_n\|^p / \epsilon^p$

Exercise

Show that if two random variables $X, Y \in \mathcal{L}^2$ have non-zero variance and satisfy $|\mathrm{Corr}(X,Y)| = 1$, then $Y = aX + b$ or some real numbers a, b.

Preliminaries on dynamical systems and generalized Besicovitch spaces

Stochastic vector calculus. We begin by recalling the definition of the notion of a dynamical system Let $(\Omega, \mathcal{M}, \mu)$ denote a probability space An N-dimensional dynamical system on Ω is a family of invertible mappings $T(x) : \Omega \to \Omega, x \in \mathbb{R}^N$, such that the following conditions hold

i) (Group property T (0) = id_{Ω} and T (x + y) = T (x)◦T (y) for all x, y ∈ $\mathbf{R}^N$;

ii) (Invariance) the mappings T(x):Ω→Ω are measurable and μ- measure preserving i.e., μ (T (x) F) = μ (F) for each x ∈ $\mathbf{R}^N$ and every F ∈ **M**;

iii) (Measurability) for each F ∈ **M**, the set (x, ω) ∈ **RN** × Ω: T (x) ω is measurable with respect to the product σ-algebra **L** $\otimes$ **M**

iv) We recall that in (i) above, the symbol ◦ denotes the usual composition of map-pings, and in

v) **L** $\otimes$ **M** is the σ-algebra generated by the family **{**L **×** M: L **∈ L** and M **∈ M}**, L **×** M being the Cartesian product of the sets L and M.

vi) If Ω is a compact topological space, by a continuous N -dimensional dynamical system on Ω is meant any family of mappings T (x) : Ω → Ω, x ∈ $\mathbf{R}^N$, satisfying the above group property (i) and the following condition: The mapping (x, ω) T (x)ω is continuous from $\mathbf{R}^N$ **×** Ω to Ω.

Let 1 ≤ p ≤ ∞. An N -dimensional dynamical system T (x): Ω → Ω induces a N -parameter

group of isometries U (x) : $L^p(\Omega) \to L^p(\Omega)$ defined by

(U (x) f) (ω) = f (T (x)ω) , f ∈ $\mathbf{L^p(\Omega)}$

(i) which is strongly continuous, i.e., U (x) f → f in $L^p(\Omega)$ as x → 0; see [22, p. 223] or. We denote by $D_{i,p}$ (1 ≤ i ≤ N) the generator of U (x) along the ith coordinate direction, and by $\mathbf{D_{i,p}}$ its domain. Thus, for f ∈ $L^p(\Omega)$, f is in $\mathbf{D_{i,p}}$ if and only if the limit $D_{i,p}f$ defined by

$$D_{i,p} f(\omega) = \lim_{\tau\to 0} \frac{f(T(\tau e_i)\omega) - f(\omega)}{\tau}$$

exists strongly in $L^p(\Omega)$, where e_i denotes the vector $(\delta_{ij})_{1\leq j\leq N}$, δ_{ij} being the Kronecker δ. One can naturally define higher order derivatives by setting $D_p^{\alpha} = D_{1,p}^{\alpha_1} \cdot\cdot\cdot D_{N,p}^{\alpha_N}$ for α = (α_1, ..., α_N) ∈ $\mathbf{N}^N$, where $D_{i,p}^{\alpha_i} = D_{i,p} \circ \cdot\cdot\cdot \circ D_{i,p}$, α_i-times.

Now we need to define the stochastic analog of the smooth functions on $\mathbf{R}^N$. To this end, we set $\mathbf{D_p}(\Omega) = \cap^N_{i=1}\mathbf{D_{i,p}}$ and define

$$\mathcal{D}_p^{\infty}(\Omega) = \left\{ f \in L^p(\Omega) : D_p^{\alpha} f \in \mathcal{D}_p(\Omega) \text{ for all } \alpha \in \mathbb{N}^N \right\}.$$

It is a fact that each element of $\mathbf{D}_\infty^{\infty}(\Omega)$ possesses stochastic derivatives of any order that are bounded. So as in [1] we denote it by the suggestive symbol $\mathbf{C}^{\infty}(\Omega)$, and also as in[1] it can be shown that $\mathbf{C}^{\infty}(\Omega)$ is dense in $L^p(\Omega)$, 1 ≤ p < ∞.

At this level, one can naturally define the concept of stochastic distribution: by a stochastic distribution on Ω is meant any continuous linear mapping from $\mathbf{C}^{\infty}(\Omega)$ to the complex field **C**. We recall that $\mathbf{C}^{\infty}(\Omega)$ is endowed with its natural topology defined by the family of semi norms $N_n(f) = \sup_{|\alpha|\leq n} \sup_{\omega\in\Omega} |D_\infty^{\alpha} f(\omega)|$ (where **|**α**|** = α_1 + ... + α_N for α = (α_1, ..., α_N) ∈ $\mathbf{N}^N$). We denote the space of stochastic distributions by $(\mathbf{C}^{\infty}(\Omega))'$. One can also define the stochastic weak derivative of f ∈ $(\mathbf{C}^{\infty}(\Omega))'$ as follows: For any α ∈ $\mathbf{N}^N$, $D^{\alpha}f$ stands for the stochastic distribution defined by

$$(D^{\alpha} f)(\phi) = (-1)^{|\alpha|} f(D^{\alpha}_{\infty} \phi) \quad \forall \phi \in \mathcal{C}^{\infty}(\Omega).$$

As **C**$^\infty$(Ω) is dense in L ^{p}(Ω) (1 ≤ p < ∞), it is immediate that L^p(Ω) ⊂ (**C** $^\infty$(Ω))$'$ so that one may define the stochastic weak derivative of any f ∈ L^p(Ω), and it verifies the following functional equation:

$$(D^{\alpha} f)(\phi) = (-1)^{|\alpha|} \int_{\Omega} f D^{\alpha}_{\infty} \phi \, d\mu \text{ for all } \phi \in \mathcal{C}^{\infty}(\Omega).$$

In particular, for f ∈ **D**$_{i,p}$ we have **−-Ω-**f D-**i,∞-**φdμ =-**Ω-**φD-**i,p-**f dμ for all φ-∈ **C-**$^\infty$(Ω) **α**$_i$ so that we may identify D$_{i,p}$f with D--f , where α$_i$ = (δ$_{ij}$)$_{1≤j≤N}$. Conversely, if f ∈ L^p (Ω) is such that there exists f$_i$ ∈ L^p (Ω) with (D$^\alpha$i f) (φ) = **−** $_\Omega$ f$_i$φdμ for all φ ∈ **C**$^\infty$(Ω), then f ∈ **D**$_{i,p}$ and D$_{i,p}$f = f$_i$. Therefore, endowing **D**$_p$(Ω) with the natural graph norm

APPLICATION TO THE HOMOGENIZATION OF A LINEAR PARTIAL DIFFERENTIAL EQUATION

We need to show how the previous outcome emerges in the homogenization of halfway respectful conditions. To outline this we start by concentrating on the fairly straightforward instance of an elliptic linear respectful operator of request two, in uniqueness structure, in particular, we consider the accompanying boundary-esteem issue

$$-\sum_{i,j=1}^{N} \frac{\partial}{\partial x_i}\left(a_{ij}(x, T(x/\varepsilon_1)\omega, x/\varepsilon_2)\frac{\partial u_\varepsilon}{\partial x_j}\right) = f \text{ in } Q$$
$$u_\varepsilon = 0 \text{ on } \partial Q$$

where Q is a bounded open subset in **R**^{N} , f ∈ L$^\infty$(Ω; H^{-1}(Q)) = L$^\infty$(Ω; W $^{-1,2}$(Q)), a$_{ij}$ ∈ **C**(Q; L $^\infty$(Ω; **B**(**R** ^N_y))), a$_{ij}$ = a$_{ji}$ (the complex conjugate of a$_{ji}$), and (a$_{ij}$)$_{1≤i,j≤N}$ satisfies the following ellipticity condition: there exists a constant α > 0 such that a$_{ij}$ (x, ω, y)λ$_i$λ$_j$ ≥ α |λ|^2 for all (x, y) ∈ Q × **R**^{N} , for $d\mu$-almost all ω ∈ Ω and for all λ ∈ **C**^{N} . It is a well-known fact that for each ε > 0 (4.1) uniquely determines u$_\varepsilon$ = u$_\varepsilon$(x, ω) ∈ H$_0^1$(Q; L^2(Ω)) in such a way that we have in hands a generalized sequence (u$_\varepsilon$)$_{\varepsilon>0}$. The fundamental problem in homogenization theory is the study of the asymptotic behaviour of such a sequence under a suitable assumption made on the coefficients a$_{ij}$ of the operator in. Here, as we will see in the sequel, it will be sufficient to make this assumption with respect to the variable y ∈ **R**^{N}.

Prior to this, it is worth to recall the following facts: firstly, in the case when the functions a$_{ij}$ do not depend on the variable y, the homogenization of _has been conducted in ; secondly, in the case when the coefficients a$_{ij}$ depend only on the variables x, y (i.e. the functions a$_{ij}$ (x, ·, y) are constants), it is commonly known that under the periodicity assumption on the functions a$_{ij}$ (with respect to y), the homogenization problem for _has already been solved by many authors and the results are available in the literature.

In the same case, it is also known that in the general framework of deterministic homogenization theory the same results are available in the ergodic environment; see e.g. However, in contrast with the ergodic setting, no result is available in the non-ergodic framework so far. The following theorem provides us with a general homogenization result in all settings: the stochastic one, the coupled stochastic-deterministic one and the deterministic one as well.

CONCLUSION

Spurred by the way that in nature practically all wonders carry on haphazardly in certain scales and deterministically in some different scales, we develop a system reasonable to handle both deterministic and stochastic homogenization issues at the same time, and furthermore independently. Our methodology, the stochastic sigma-convergence, can be seen either as a multi scale stochastic methodology since deterministic homogenization hypothesis can be viewed as an uncommon instance of stochastic homogenization hypothesis (see Theorem), or as a combination of the stochastic and deterministic methodologies, both taken around the world, yet additionally each independently. One of the principle applications of our outcomes is the homogenization of a model of rotating liquids.

REFERENCES

[1] K.T. Andrews, S. Wright (1998). Stochastic homogenization of elliptic boundary-value problems with Lp-data, Asympt. Anal. 17, pp. 165-184.

[2] Bensoussan, Homogenization of a class of stochastic partial differential equations. Composite media and homogenization theory (Trieste, 1990), pp. 47–65, Progr. Nonlinear Differential Equations Appl., 5, Birkh¨auser Boston, Boston, MA, 1991.

[3] A.S. Besicovitch (1954). Almost periodic functions, Cambridge, Dover Publications,

[4] X. Blanc, C. Le Bris, P.L. Lions (2007). Stochastic homogenization and random lattices, J. Math. Pures Appl. 88, pp. 34-63.

[5] H. Bohr (1947). Almost periodic functions, Chelsea, New York.

[6] N. Bourbaki (1966). Int´egration, Chap. 1-4, Paris.

[7] N. Bourbaki (1971). Topologie generale, Chap. 1-4, Hermann, Paris.

[8] Bourgeat, A. Piatnitski (2004). Approximations of effective coefficients in stochastic omogenization. Ann. Inst. H. Poincare Probab. Statist. 40, pp. 153-165.

[9] Bourgeat, S.M. Kozlov, A. Mikelic (1995). Effective equations of two-phase flow in random media. Calc. Var. Part. Differ. Equ. 3, pp. 385-406.

[10] Bourgeat, A. Mikelic, S. Wright (1994). Stochastic two-scale convergence in the mean and applications, J. Reine Angew. Math. 456, pp. 19-51.

[11] L.A. Caffarelli, P.E. Souganidis, L. Wang (2005). Homogenization of fully nonlinear, uniformly elliptic and parabolic partial differential equations in stationary ergodic media. Commun. Pure Appl. Math. 58, pp. 319–361.

[12] L.A. Caffarelli, P.E. Souganidis (2010). Rates of convergence for the homogenization of fully nonlinear uniformly elliptic pde in random media. Invent. Math. 180, pp. 301–360.

Corresponding Author

Prof. (Dr.) Rama Nand Singh*

HOD & Professor, Department of Mathematics, Jamunilal College Hajipur Vaishali, Bihar