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A Study of Entropy Analysis for Fluid Flow and Heat Transfer with Boundary Layer

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Abstract - When thermal non-equilibrium causes a flow of thermal energy, it is known as heat transfer, and the heat flow per unit time measured at a control surface is what is generally used to describe it. Thermal convection, thermal conduction, thermal radiation, and energy transfer via phase exchanges are all types of thermal convection. There are many real-world instances of these mechanisms, such as heating the free end of a spoon dipped into hot fluid, boiling water in a pan, and cooling the hot food by blowing air into it. Heat transfer has a wide range of applications, particularly in mechanical engineering. IC engines, air compressors, brake cooling in cars, and bearing cooling in wind power plants are all examples of mechanical engineering applications where heat transmission is a major concern. Air conditioners and refrigerators are another important use. We are surrounded with awe-inspiring materials/substances. Solids, liquids, and gases are the three main kinds of matter. In everyday speech, liquids and gases are simply referred to as "fluids" to avoid confusion. A fluid is a substance that may flow freely and does not have a fixed shape. The study of fluids and their properties is the focus of the mechanical engineering specialty known as fluid mechanics. Fluid dynamics and fluid statics are subcategories of fluid mechanics. When thermal non-equilibrium causes a flow of thermal energy, it is known as heat transfer, and the heat flow per unit time measured at a control surface is what is generally used to describe it. Thermal convection, thermal conduction, thermal radiation, and energy transfer via phase exchanges are all types of thermal convection. There are many real-world instances of these mechanisms, such as heating the free end of a spoon dipped into hot fluid, boiling water in a pan, and cooling the hot food by blowing air into it.

Keywords - thermal, non-equilibrium Entropy, Boundary Layer, Fluid Flow, Heat Transfer

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INTRODUCTION

It is possible to mimic conical or diffusion flow problems using boundary layer flow in a cone. Wm. Choi and Ms. Choi (1977). For the heat transfer analysis of the axisymmetric flow inside of a cone, Rosenhead (1963) proposed the use of a similarity solution. Using the series solution, Ackerberg (1965) demonstrated the convergence of viscous fluid inside a cone. It was Takhar et al. (1986) that extended the electrically conducting fluid problem and studied the impact of heat and mass transfer on it. The transient instance was studied by Eswara et al. (2000). For the temperature-dependent fluid viscosity, Eswara and Bommaiah (2004) examined the same issue. By characterising the convergent axisymmetric potential flow as a point discharge enclosed by a cone with a vertex angle, Asatur and Koltan (1991) arrived at a numerical solution for the momentum boundary layer flow in an axisymmetric convergence duct. No attempt has been made to go into entropy analysis in any of these investigations, which are only focused on transport phenomena. We know that all genuine processes are irreversible because of physical factors. Relevant information is provided by the second law of thermodynamics. Entropy, in case you forgot, is a metric for how irreversible something is. Entropy production inside a cone for flow due to a point sink at the cone's apex has been studied in the preceding problem. Scientists anticipate that the research will help unlock new dimensions previously closed off by the first law of thermodynamics.

MATHEMATICAL MODEL

Consider the steady axisymmetric laminar flow of an incompressible fluid inside a circular cone at rest with a hole at the cone's apex. A porous fluid-saturated material is used to fill the cone. The hole, which is seen as a three-dimensional point sink, is what causes the boundary layer flow. So that it is independent of r, the cone's length has been taken to be semi-infinite. Schematic diagram Figure 1 depicts the flow model and the physical coordinate system.

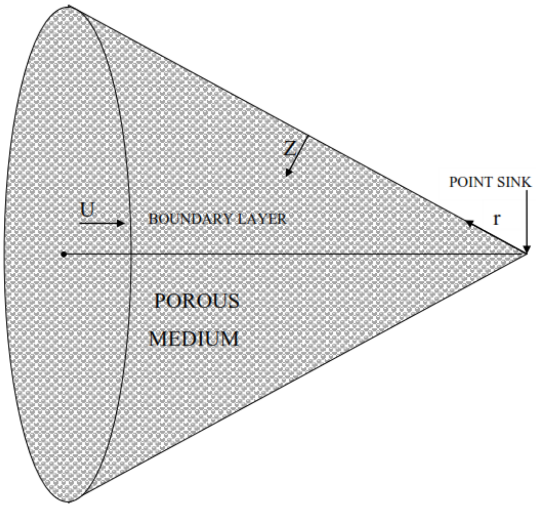

Figure 1 Schematic sketch of the problem

The boundary layer equations for the set up are

$$\frac{\partial(ru)}{\partial r}+\frac{\partial(rw)}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=U\frac{\partial U}{\partial r}+\frac{1}{\rho_\infty}\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)-\frac{\mu}{\rho_\infty K^*}(u-U) \tag{2}$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\frac{\kappa}{\rho_\infty C_p}\frac{\partial^2 T}{\partial z^2}+\frac{1}{\rho_\infty C_p}\left\{\mu\left(\frac{\partial u}{\partial z}\right)^2+\frac{\mu u^2}{K^*}\right\} \tag{3}$$

Together with the following boundary condition's

$$\begin{aligned} &z=0:\quad u=0,\ w=0,\ T=T_w \\ &z\to\infty:\quad u=U,\quad T=T_\infty \end{aligned} \tag{4}$$

where (u ,w) are the radial and axial velocities in directions of (r, z), ρ∞ is the density at free stream, T is the temperature, C_p is the specific heat at constant pressure, μ is the fluid viscosity, κ is the thermal conductivity, U isthe free stream velocity. Tw is the temperature at the wall, T∞ is the temperature of the free stream, K* is the permeability of the porous medium. The main stream flow is given by

$$U=-\frac{m}{r^2} \tag{5}$$

where 'r' is the distance measured along the cone from the vertex, m is the strength of point sink, m > 0. The fluid viscosity μ is assumed to be inverse linear function of temperature as follows [Lings and Dybbs, (1992)]

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma(T-T_\infty)\right] \tag{6}$$

$$\text{Or}\quad \frac{1}{\mu}=\alpha(T-T_r)\ \text{where}\quad \alpha=\frac{\gamma}{\mu_\infty},T_r=T_\infty-\frac{1}{\gamma}$$

where μ∞ is the viscosity of the fluid at free stream, both α and Tr are constant and their values depend on the reference state, γ is a viscosity variation constant based on thermal property of the fluid. We prescribe

$$\begin{aligned} &ru=\frac{\partial\psi}{\partial z},rw=-\frac{\partial\psi}{\partial r},\psi=-\left(2m\frac{\mu_\infty}{\rho_\infty}r\right)^{1/2}f(\eta), \\ &\eta=z\left(\frac{\rho_\infty m}{2\mu_\infty r^3}\right)^{1/2},u=Uf'(\eta),w=\left(\frac{m\mu_\infty}{2r^3\rho_\infty}\right)^{1/2}(f-3\eta f'), \end{aligned} \tag{7}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},U=-\frac{m}{r^2},m>0$$

We see that the equation of continuity (1) is identically satisfied and equations (2) and (3) take the following respective forms

$$f'''+\left(1-\frac{\theta}{\theta_r}\right)\left\{4(1-f'^2)-ff''\right\}+\frac{f''\theta'}{\theta_r-\theta}-K(f'-1)=0 \tag{8}$$

$$\theta''-\Pr f\theta'+\frac{Br\theta_r}{\theta_r-\theta}\left(f''^2+f'^2K\right)=0 \tag{9}$$

Where,

$$K=\frac{2\mu_\infty r^3}{\rho_\infty mK^*},\ \Pr=\frac{\mu_\infty C_p}{\kappa_\infty},\ Br=\frac{\mu_\infty U^2}{\kappa_\infty(T_w-T_\infty)}, \tag{10}$$

$$\theta_r=\frac{T_r-T_\infty}{T_w-T_\infty}=-\frac{1}{\gamma(T_w-T_\infty)}$$

are permeability parameter, Prandtl Number, Brinkman Number, viscosityparameter respectively,and the boundary conditions (4) becomes

$$\begin{aligned} &\eta=0:f'=0,\theta=1,f=0 \\ &\eta\to\infty:f'\to1,\theta\to0 \end{aligned} \tag{11}$$

Solution Equation

a section (Next) In order to solve the governing equations (8) and (9) numerically, the Runge-Kutta fourth order scheme and the shooting method were used. As a boundary value problem (BVP) is converted into an initial value system, systematic approximations are made for unknown values in order to meet the end requirements

$$f_3' = \left(\frac{f_4}{\theta_r} - 1\right)\left\{4\left(1 - f_2^2\right) - f_1 f_3\right\} - \frac{f_3 f_5}{\theta_r - f_4} + K\left(f_2 - 1\right) \tag{12}$$

$$f_5' = \Pr f_1 f_5 - \frac{Br\theta_r}{\theta_r - f_4}\left(f_3^2 + f_2^2 K\right) \tag{13}$$

with initial conditions

$$f_1(0) = 0, f_2(0) = 0, f_3(0) = ?, f_4(0) = 1, f_5(0) = ?, \tag{14}$$

Where,

$$f = f_1, f' = f_2, f'' = f_3, \theta = f_4, \theta' = f_5 \tag{15}$$

The numerical computation had two challenges. Firstly, f3(0) and f5(0) were not available; secondly η∞ i.e. maximum value of η for which end conditions f 1 ′ → and θ → 0 as η→∞ are satisfied , was not available beforehand. To complete the assignment, Runge-Kutta method of order four was used to integrate as η∞ (3.12)-(3.13) by making systematic estimations for f3(0) and f5(0) to take a finite value of as. As long as the required error tolerance of size 10-6 was met, the method was complete; if the end criteria were not met, we had to restart the process for another set of estimates.

In fact, these guesses were initially found on hit and trial basis and their refinement was achieved by Newton iteration method. Furthermore,' $\eta_{\max}$ i.e. η_∞ was also optimized together with grid independence. It was found that step size $\Delta\eta = 0.0005$ is satisfactory for desired convergence of magnitude 10-6.

Second Law Analysis

Equation Section (Next) The local volumetric rate of entropy generation SG for a viscous fluid flow is given as follows

$$S_G = \frac{\kappa}{T_\infty^2}\left(\frac{\partial T}{\partial z}\right)^2 + \frac{\mu}{T_\infty}\left\{\left(\frac{\partial u}{\partial z}\right)^2 + \frac{u^2}{K^*}\right\} \tag{16}$$

The equation (3.16) shows that entropy is generated by three different sources. Temperature transfer contributes to the first component, which is entropy generated by viscous dissipation of fluid friction. The second term illustrates the local entropy generated by the porous media, which offers resistance to fluid flow.

Here, we propose two new concepts: the typical temperature ratio, and the generation rate of entropy (SG0).

$$S_{G0} = \frac{\kappa(T_w - T_\infty)^2}{T_\infty^2}\left(\frac{\rho_\infty m}{2\mu_\infty r^3}\right), \omega = \frac{T_\infty}{T_w - T_\infty} \tag{17}$$

Thus, the non-dimensional entropy generation number Ns is given by

$$\begin{aligned} N_s &= \frac{S_G}{S_{G0}} = \theta'^2 + Br\omega\frac{\theta_r}{(\theta_r - \theta)}\left(f''^2 + f'^2 K\right) \\ &= HTI + FFI \end{aligned} \tag{18}$$

where,

heat transfer irreversibility (HTI)= θ'^2

and fluid friction irreversibility (FFI)= $Br\omega\frac{\theta_r}{(\theta_r - \theta)}\left(f''^2 + f'^2 K\right)$

The Bejan Number Be which is pertinent irreversibility parameter is defined as follows

$$Be = \frac{HTI}{HTI + FFI} \tag{19}$$

When frictional factors dominate irreversibility, a value of Be of 0 indicates no irreversibility; a value of Be of 1 indicates significant irreversibility due to heat transfer. The following equation computes the global entropy GNs by integrating the local entropy:

$$G_{N_s} = \int_0^{\eta_\infty} N_s d\eta \tag{20}$$

Table 1: Missing f''(0), θ'(0) whenBr=4,K=0.1,ω=0.6

θ_r	$f''(0)$	$\theta'(0)$
-4	2.550163	2.852006
-6	2.475837	2.974227
-8	2.438761	3.041776
-10	2.416602	3.084712
-13	2.396238	3.126119

Table 2: Missing f''(0), θ'(0) whenK=0.1,ω=0.6,θr= -13

Br	f″(0)	θ′(0)
2	2.372107	1.671839
3	2.362943	2.728798
4	2.354001	3.775738
5	2.345272	4.813045
6	2.336745	5.841080

Table 3: Missing f''(0), θ'(0) when Br=4, θr = -13,ω=0.6

K	f″(0)	θ′(0)
0	2.335877	3.618484
0.1	2.354001	3.775738
0.2	2.371901	3.932822
0.3	2.389584	4.089711
0.4	2.407058	4.246385
0.5	2.424330	4.402824
0.6	2.441406	4.559010

RESULTS AND DISCUSSION

In Tables 1 to 3, the values of missing quantities for various values of parameters are shown. Skin friction increases when K rises, whereas it reduces as θ´r and Br rise numerically in the range of 0.01 to 0.01. We can also see that θ´(0) rises with higher amounts of Br, K, and θr.

There is an increase in K, but reduction in u as a function of increasing rθr.

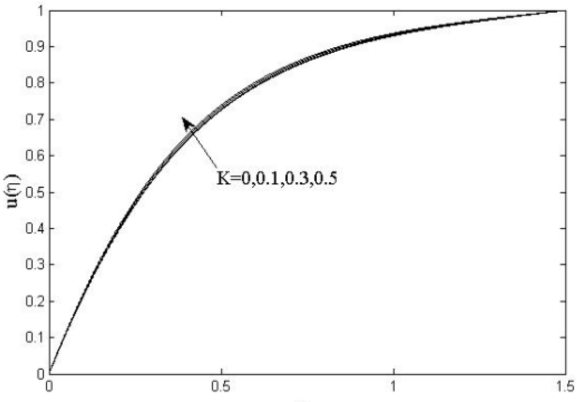

Figure 3.2: Velocity profiles for varying K, for θ_r= -13, Br=4,ω=0.6

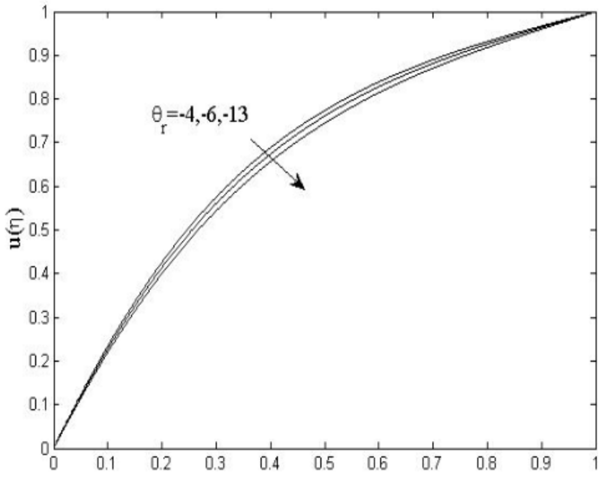

Figure 3.3: Velocity profiles for varying θ_r for Br=4, K=0.1,ω=0.6

CONCLUSION

While a linear relationship produces a straight line, a nonlinear relationship does not, but instead forms a curve. Non-linearity examines the links between causes and effects. Plasma physics, solid state physics, optical fibres, biology, fluid dynamics, and chemical kinetics are all examples of non-linear phenomena. Non-linear systems exhibit a wide range of properties, including thermal convection, Taylor vortex flow, turbulence, the creation of coherent light by laser, and chemical oscillations. An understanding of connected physical events must be expressed mathematically in order to accelerate fluid flow, heat transfer, and other processes. Almost all interesting physical phenomena may be explained using non-linear partial or ordinary differential equations that state conservation principles. Analytical fluid flow and heat transfer solutions are still very important in science and engineering even in the digital era, despite the complexity of the non-linear differential equations that can be solved numerically in many instances. As a result, analytical solutions have the distinct advantage of revealing the factors that have an impact on the final result. Contact with the solid wall raises the temperature of the fluid particles to that of the wall. Fluid particles in touch with the wall exchange heat with those in adjoining layers if the wall temperature is higher than the surrounding fluid. This results in the formation of thermal gradients in the fluid. Hydrodynamic and thermal boundary layers are significant in fluid mechanics because velocity is an important component in mass, momentum and energy equations while temperature gradient in the thermal boundary layer affects heat transfer.

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