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Instantaneous Frequency Estimation Based on Robust Adaptive Spectrogram with Welch Periodogram

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Abstract – Robust m-Periodogram is used for the analysis of signals with heavy-tailed distribution noise. It is also used for the non-stationary signals in the form of Robust Spectrogram. Robust spectrogram is also used to estimate the instantaneous frequency (IF) of the signal. In this paper, we are introducing a Robust Spectrogram estimator based on Welch Periodogram for estimating the instantaneous frequency of the signal with a time varying window length.

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1. INTRODUCTION

Time Frequency Analysis (TFA) techniques are widely used in a variety of fields of Instrumentation and measurement such as power quality analysis, fault detection, biometric authentication etc. There are two types of spectral estimates which are used to find the various parameters required in these techniques: non-parametric and parametric. For non-parametric spectral estimation, the signal is assumed to be consists of sinusoidal components, and their magnitudes and phases are estimated by the use of Periodogram. In parametric methods, the signal is assumed to be generated by a certain model and spectrum is estimated from the model parameters. A complex-valued harmonic with the time varying phase is a key model of instantaneous frequency IF concept, as well as an important model in the general theory of time frequency distributions. It has been utilized to study a wide range of signals, including speech, music, acoustic, biological, radar, sonar and geophysical ones. An overview of the methods for the IF estimation, as well as interpretation of the IF concept, is presented in [2]-[3]. The frequency estimation technique, like Cohen class of quadratic distributions[4] and the Wigner distribution[23], short time Fourier Transforms[12],[14]-[15], represents a very efficient approach for estimating IF estimation.

In non-parametric spectral analysis, the various forms of Periodogram are used by various research workers in the various fields of science e.g. in astronomical space, in so many branches of engineering, metrology, biomedical science etc. the most commonly techniques of spectral estimation in the form of Periodogram are Fourier Periodogram[24], Bartlett Periodogram[1], Welch Periodogram[27], Lomb Periodogram[19], Scargle Periodogram[20] and many more. In 1998, V. Katkovnik[18] introduces the Robust M-Periodogram as a spectral estimate for stationary data.

In V.Katkovnik[7],[17] combine and develop two different ideas: the Robust M-Periodogram and the non-parametric approach for selection of the time varying window length in the corresponding Periodogram. In this paper, we are to develop a Robust Adaptive Spectrogram with Welch Periodogram (RAWP) which is an Instantaneous estimator with a time varying and data driven window length.

2. METHODOLOGY

2.1 Robust Spectrogram (RSPEC)

The Robust Spectrogram, of a signal $X(t)$, is based on the standard short-time Fourier transform (STFT) as

$$C_h(t,\omega) = \frac{1}{\sum_n W_h(nT)} \sum_n W_h(nT)\, X(t+nT)\, e^{p\omega nT} \qquad (2.1.1)$$

Where $p = \sqrt{-1}$, and window function is defined as $W_h(nT) = TW\left(\frac{nT}{h}\right), h \geq 0$; h is window length and also

$\sum_n W_h(nT) \to 1 \ as \ \frac{h}{T} \to \infty$ where sampling interval is denoted by T.

The $C_h(t,\omega)$ may be derived as a solution of the following optimization problem:

$$C_h(t,\omega) = arg\min_C J(\omega, C) \qquad (2.1.2)$$

Where

$$J(\omega, C) = \sum_n W_h(nT)|X(t+nT) - C_h(t,\omega)e^{p\omega nT}|^2 \qquad (2.1.3)$$

Here, the weighted square absolute error

$$F(e) = |e(nT)|^2 = |X(t+nT) - C_h(t,\omega)|^2 \qquad (2.1.4)$$

is used as a loss function and can be minimized. The Periodogram obtained using this loss function is called the Robust M-Periodogram and its corresponding RSPEC is given in the form:

$$I_A(t,\omega) = |C_{ih}(t,\omega)|^2 \qquad (2.1.5)$$

IF Estimation

Let $X(nT) = m(nT) + \in(nT), m(t) = A\,e^{p\omega t}$, where n is an integer, T is a sampling interval and $\in(nT)$ is a complex valued white noise $E(\in(nT)) = 0, E(|\in(nT)|^2 = \sigma^2$

Now, by definition, the IF is the first derivative of the phase $\Omega(t) = \phi'(t)$. Its estimate can be found as:-

$$\omega_h(t) = \arg max_{\omega \in Q_\omega} I_A(t,\omega)$$

Where for a window $W_h(nT)$ there are N samples within the interval $Q_\omega \in (-\pi,\pi)$. Thus the window $W_h(nT)$ implements the idea of nonparametric estimation of the time varying $\Omega(t)$, fitted by a constant ω, within the narrow window around the time-instant t.

2.1 Welch Periodogram

Welch(1967) introduced a method to the estimation of power spectra which involves sectioning the data, taking modified Periodograms of these sections and then averaging these modified periodograms. This averaged Periodogram is known as Welch Periodogram.

Let $X(j), j = 0,1,2,3,\ldots\ldots N-1$ be a sample from stationary sequence and let $X(j)$ has power spectral density $P(\omega), |\omega| \le \frac{1}{2}$. The segments, possibly overlapping, of length L with starting points of their segments D units apart. Let $X_1(j) = X(j), j = 0,1,2,\ldots\ldots, L-1$ be the first such segment then similarly $X_2(j) = X(j+D), j = 0,1,2,\ldots\ldots, L-1$ be the second one and finally *X$_k$(j)=X(j+(K-1)D),j=0,1,2,……….L-* thus these are K segments $\{X_1(j), X_2(j), \ldots\ldots\ldots, X_K(j)\}$ which covers the entire record and $(K-1)D + L = N$

Welch suggested choosing either window from the following two types of windows:-

$$W_1(j) = 1 - \left[\frac{j - \frac{L-1}{2}}{\frac{L+1}{2}}\right]^2 ; \quad W_1(j) = 1 - \left|\frac{j - \frac{L-1}{2}}{\frac{L+1}{2}}\right| ; \; j = 0,1,2,\ldots\ldots\ldots, L-1$$

And finite Fourier transforms of K segments are $FT_1(n), FT_2(n), \ldots\ldots\ldots, FT_K(n)$ defined as:-

$FT_K(n) = \frac{1}{L}\sum_{j=0}^{L-1} X_K(j)W(j)e^{-2Kijn/L}$ and $i = \sqrt{-1}$; Finally K modified periodograms are defined as :-

$$I_p(\omega_n) = \frac{L}{U}\left|FT_p(n)\right|^2 ; p = 1,2,\ldots\ldots, K$$

$$\text{Where } \omega_n = \frac{n}{L}; n = 0,1,2,\ldots\ldots,\frac{L}{2} \text{ and } U = \frac{1}{L}\sum_{j=0}^{L-1} W^2(j)$$

The spectral estimate is the average of these periodograms i.e.

$$P(\omega) = \frac{1}{K}\sum_{p=1}^{K} I_p(\omega_n) \qquad (2.2.1)$$

This function is known as Welch Periodogram.

2.2 Robust Spectrum with Welch Periodogram

Let $X(t_j), j = 0,1,2,3,\ldots\ldots N-1$ be N discrete time samples of a signal (t). Let $X_1(t_j) = X(t_j), j = 0,1,2,\ldots\ldots, L-1$ be the first such segment then similarly $X_2(t_j) = X(t_{j+D}), j = 0,1,2,\ldots\ldots, L-1$ be the second one and finally $X_K(t_j) = X(t_{j+(k-1)D}), j = 0,1,2,\ldots\ldots, L-1,$ thus, there are K segments $\{X_1(t_j), X_2(t_j), \ldots\ldots\ldots, X_K(t_j)\}$ of length L with the starting points of these segments D units apart and $(K-1)D + L = N$

Now, we calculate RSPEC foe each segment as the following:-

$$C_{ih}(t,\omega) = \frac{1}{\sum_j W_h(t_j)}\sum_j W_h(t_j)\,X_i(t_j)\,e^{p\omega t_j} \qquad (2.3.1)$$

Where $p = \sqrt{-1}$, and window function is defined as $W_h(NT) = TW\left(\frac{NT}{h}\right), h \ge 0$; h is window length and also

$$\sum_n W_h(NT) \to 1 \ as \ h/T \to \infty$$

Sampling interval is denoted by T.

The $C_{ih}(t,\omega)$ may be derived as a solution of the following optimization problem:

$$C_{ih}(t,\omega) = arg\ min_C\ J_i(\omega, C) \qquad (2.3.2)$$

Where

$$J_i(\omega, C) = \sum_j W_h(t_j)\left|X_i(t_j) - C_{ih}(t,\omega)e^{p\omega t_j}\right|^2$$

Now we have Robust Adaptive Welch Periodogram(RAWP) as spectral estimate can be defined as

$$I_A(t,\omega) = \frac{1}{K}\sum_i |C_{ih}(t,\omega)|^2 \qquad (2.3.3)$$

IF Estimation

Let $X(NT) = m(NT) + \in(NT), m(t) = A\ e^{p\omega t}$, where n is an integer, T is a sampling interval and $\in(NT)$ is a complex valued white noise $E(\in(NT)) = 0, E(|\in(NT)|^2 = \sigma^2$

Now, by definition, the IF is the first derivative of the phase $\Omega(t) = \phi'(t)$. Its estimate can be found as:-

$$\omega_h(t) = \arg max_{\omega\in Q_\omega} I_A(t,\omega)$$

Where for a window $W_h(NT)$ there are N samples within the interval $Q_\omega \in (-\pi, \pi)$

3. DATA DRIVEN WINDOW LENGTH CHOICE

The estimation error provides the basic idea as getting at least for asymptotic case, it can be represented as a sum of deterministic component (bias) and random component as following:-

$$|\Omega(t) - \omega_h(t)| \le |\text{bias}(t,h)| + \aleph\sigma(h) \ (3.1)$$

With $\sigma^2(h) = var(\Delta\omega_{h_s}(t))$ where $\aleph$ is corresponding quantile of standard Gaussian distribution N(0,1).

Let $h = h_s$ be so small that $|\text{bias}(t,h)| \le \aleph\sigma(h_s)$ then

$$|\Omega(t) - \omega_h(t)| \le 2\aleph\sigma(h) \qquad (3.2)$$

It is obvious that, for small h_s

$$D_s = [\omega_h(t) - 2\aleph\sigma(h), \omega_h(t) + 2\aleph\sigma(h)]$$

Have a point in common, namely $\Omega(t)$. Consider an increasing sequence of $h_s, h_1 < h_2 < h_3 < \ldots\ldots\ldots$.

Let h_{s+} be the largest of those h_s for which the segments D_{s-1} and D_s have a point in common. Let us call this window length 'optimal' and determines IF estimate with data driven optimal window length. The basic idea of window length choice is as explained: if D_{s-1} and D_s do not have a common point then it means that one of the inequality (3.2) does not hold, i.e. bias is too large as compared with σ. Thus the statistical hypotheses to be tested for the bias is given in the form of the sequence of inequalities (3.2) and the largest length h_s for which these inequalities have a point in common is considered as bias-variance compromise.[22]

4. CONCLUSION

The Robust Adaptive Welch Periodogram (RAWP) as a time-varying form of the robust M-periodogram, with the varying adaptive window length, is developed. The intersection of confidence intervals rule is applied for varying window length selection.

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