

A Study of Numerical Variable Method towards Pricing American Style of Asian Options

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Abstract – The early practice top for the Asian option in American style Note to rehearse decisions of an American style before development T whenever. The holder of a St. at the time t and its past has the option to practice it or hold it in consistence with the spot price $\{S_u, 0 \leq u \leq t\}$ before the time t . For American-style decisions, the qualification among "continuation" and "halting" territories assumes a significant job. In various flavors Asian options come. The normal can, for instance, be number-crunching or geometric. One talks about a plain vanilla alternative in Asia when the normal is determined over the full exchanging period, and a retrograde choice whenever determined over a right exchanging period. Ordinarily, this interim is fixed in time. The Asian alternative can either be solidified (when the cost of the strike is fixed), or skimming (when the strike is a normal in itself). At the point when the result is a weighted normal, it is viewed as flexible and is similarly weighted when each weight is equivalent. Prices are carefully examined when the result is the normal of an unambiguous arrangement of estimations of the basic resource (noted at discrete occasions) and ceaselessly tested if, over the time interim, the result is the fundamental of the benefit esteem separated when interim. The options right now the most widely recognized: fixed impact Asian options dependent on an arithmetical normal which is equivalent weighted and discretely inspected. Different kinds of discretely inspected asian options could likewise be changed in accordance with prices. Asian options ordinarily rely upon the normal of the spot cost of the base (number juggling or geometric). It can likewise be utilized as a valuable method to fence incredibly unpredictable items or resources. As a result of the basic value changes all through the lifetime of the alternative, the Asian choice holder can make sure about an unexpected value spring to an unfortunate locale (unreasonably high for the holder of the call choice or unreasonably low for the holder of the situation choice). Asian options are significant as they are very regular on business sectors, for example, cash and items, for example, oil. They are reliant on pathways. All the more explicitly, we are keen on the cost of an Asian American call and put options with $VT(S, A) = (S - A) + (A - S)$ and $(VT(S, A) = (A, -S)+$, individually. The strike value A_n is the normal of that which has been accounted for over time $[0, T]$.

Key Words: Numerical Variable, Pricing, American Style, Asian Options, Arithmetical, Options.

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INTRODUCTION

The price of Asian options based on average rates, according to Black and Scholes, includes the assessment of an indispensable one (numerical desire) for which there is no investigation arrangement. Pricing their partners in American style who offer early exercise openings displays the extra trouble of taking care of a unique improvement issue to decide the ideal practice technique. In blend mit limited material piece-by-piece-polynomial guess of the value function we are building up a numerical strategy to pricing Asian-style American options. Union, exactness, and unwavering quality demonstrate numerical analyses. There are likewise other hypothetical properties of the intrigue function

and the ideal exercise system. We've seen market shift in recent years in order to slow down stocks and other financial markets such as commodities and forex, not only during short periods of time and uncertainty. Options are used to hedge portfolios and to reduce the financial risk would become more expensive with all this uncertainty. Any less expensive options would be a good solution to reduce hedging costs. Alternatives, rather than their plain vanilla equivalent, such as a barrier or Asian alternative would be feasible. Asian options have been used since 1987 but we have no closed analytical pricing approach to this day. There have been various methods implemented for this role but in their study A Dynamic Programming Method to Pricing American Style Asian Options, the

methodology we have originally used was suggested by Hatem Ben-Ameur, Michèle Breton, and Pierre L'Écuyer.

A financial derivative is a contract that makes a future payout available to its holder, based on the price of at least one explicit resource, for example, stocks or monetary forms. In a free-showcase condition, as for a supposed hazard nonpartisan likelihood metric, the non-assertion idea permits one to communicate the factual desire for the subsidiary's limited future payout. Options are specific non-negative payout subordinates. Options of European style must be utilized at the expiry date, though options of the American kind give the holder early preparing chances. Explanatory estimations are accessible on the reasonable price of an option for simple cases, for example, European solicitations and options composed on stocks whose values are modeled in a geometric Brownian movement (GBM) as portrayed in Black and Scholes (1973), respectively. Nevertheless, analytical models are not available for more complicated derivatives that may include multiple properties, complex payoff functions, early exercise possibilities, stochastic model parameters that differ over time. The Asian options classes, for which payouts are based on the mathematical price average of the primary asset over certain duration, are a significant class of options that do not have an analysis structure even under the traditional Black Scholes GBM model. Such options are often used to defend against unwanted and harsh price changes. An Asian option can cope regularly over a certain period with the risks of a company selling or purchasing specific kinds of assets (crude materials, power, remote cash, and so on.). These agreements are less expensive than their standard forms and are generally less unstable than the hidden resource price itself. Asians are vigorously exchanged off - the-rack and their hypothetical prices frequently should be estimated on - the-ground for reasonable arrangements because of a possible lack of market depth.

Asian options are available in different flavours. The average may be arithmetical or geometric, for example. A simple vanilla asiatic option is discussed when the average is calculated during the entire trading period and a return-start option, if calculated over a correct subinterval. Typically, this interval has a set start. Asia can be either set (if the price is fixed) or floating (when the strike itself is an average). The Asian option is either fixed. When the payout is a weighted average and equal in weight all the weights equal, it is known as adjustable. The prices will be sampled discretely if the payoff is the average collection of subordinate value values (noted at the discrete epoch) which are sampled continuously if the payoff for a certain amount of time is the integral value of the commodity, divided up according to the period of that interval. These options are most common in this study: Fixed-strike asian options based on arithmetic averages that are equally weighted and discretely sampled. Certain kinds of

discretely sampled Asian choices may be tailored to our methodology. Asian (Eurasian) options can be implemented on the expiry date only, while an American (American) option provide earlier exercise incentives that can become intuitively appealing a call option and is above the current asset price average. We focus here on American call options, the values of which are harder to calculate than the Eurasian ones, as an optimization problem needs to be solved at the same time as mathematical expectations are calculated, giving the value of the choice.

The pricing of Eurasian options is comprehensive in the literature. Under the GBM model, the value of the discreet sampled Eurasian options is evaluated in closed form only if they are based on the geometric average. The idea is that the asset price has the lognormal distribution under the GBM model at any time, with a geometrical average of lognormal values being unusual. The Bessel procedures and derivative careful formulations for the Laplace are used by Geman and Yor (1993) to transform the value of the Eurasian continuing option. For the solution methods based on the arithmetic average, quasi-analytic methods of approximation based on Fourier's transform, expansion of Edgeworth, and Taylor and partial differential equation approaches.

EVOLUTION OF THE PRIMITIVE

Resource We expect a solitary crude resource whose value procedure $\{S(t), t \in [0, T]\}$ is a GBM, a system in which Black and Scholes (1973) follow traditional assumptions. Under these assumptions there are probability metrics (Q) known as risk-neutral, which comply with the stochastic differential equation with the primitive asset price $S(\cdot)$.

$$dS(t) = rS(t)dt + \sigma S(t)dW(t), \text{ for } 0 \leq t \leq T, \tag{1}$$

Where $S(0) > 0$, r is the hazard free rate, σ is the unpredictability parameter, T is the development date, and $\{W(t), t \in [0, T]\}$ is a standard Brownian movement. The arrangement of (1) is given by

$$S(t^p) = S(t^q)e^{(r-\frac{\sigma^2}{2})(t^p-t^q) + \sigma(W(t^p)-W(t^q))}, \text{ for } 0 \leq t^q \leq t^p \leq T. \tag{2}$$

where $\mu = r - \sigma^2 / 2$. An important feature is that the random variable $S(t_{00})/S(t_0)$ is lognormal with parameters $\mu(t_{00} - t_0)$ and $\sigma \sqrt{t_{00} - t_0}$, and independent of the σ -field $F(t_0) = \sigma \{S(t), t \in [0, t_0]\}$, i.e., of the trajectory of $S(t)$ up to time t_0 . This follows from the independent-increments property of the Brownian motion. In addition, from the noarbitrage property of the Black-Scholes

model, the discounted price of the primitive asset is a Q-martingale:

$$\rho(t')S(t') = E[\rho(t'')S(t'') | \mathcal{F}(t')], \text{ for } 0 \leq t' \leq t'' \leq T. \quad (3)$$

where $\{\rho(t) = e^{-rt}, t \in [0, T]\}$ is the rebate factor procedure and E is (up and down this examination) the desire as for Q. Insights regarding hazard impartial assessment can be found in Karatzas and Shreve (1998).

FORMULATION OF PARTIAL DIFFERENTIAL EQUATION

Let t indicate the existing time and T indicate the expiry date of an Asian Floating Call Option contract with a continuous geometric average asset price. This Asian's terminal payout function call option is given by

$$C(S_T, G_T, T) = \max(S_T - G_T, 0), \quad (1)$$

Where S_T at T and G_T is the asset price, the average asset price starts at zero and a continuous geometrical average. Accordingly, G_t is defined by

$$G_t = \exp\left(\frac{1}{t} \int_0^t \ln S_u du\right), \quad 0 < t \leq T. \quad (2)$$

The present resource price S_t is expected to follow the hazard impartial lognormal progression:

$$dS_t = (r - q)S_t dt + \sigma S_t dZ(t). \quad (3)$$

The normal Viennese method is r, q and τ , which refers, respectively, to the continuous riskless rate, constant dividend yield and continuous volatility.

$$\ln S_T = \ln S_t + \left(r - q - \frac{\sigma^2}{2}\right)(T - t) + \sigma[Z(T) - Z(t)], \quad (4)$$

And,

$$\ln G_T = \frac{t}{T} \ln G_t + \frac{1}{T} \left[(T - t) \ln S_t + \left(r - q - \frac{\sigma^2}{2}\right) \frac{(T - t)^2}{2} + \frac{\sigma}{T} \int_t^T [Z(u) - Z(t)] du \right], \quad (5)$$

where it is known that [2]

$$Z(T) - Z(t) = \phi(0, \sqrt{T - t}), \quad (6a)$$

$$\int_t^T [Z(u) - Z(t)] du = \phi\left(0, \frac{1}{\sqrt{3}}(T - t)^{\frac{3}{2}}\right). \quad (6b)$$

Here, $\mu(\mu, \hat{S})$ indicates the mean μ and standard eccentricity of the distribution normal. The above relationships demonstrate that GT is also distributed lognormally.

The leading calculation of the interest for the Americans from the Asian language is the riskless hedging and the non-arbitrage statement call is given by

$$\frac{\partial c}{\partial t} + \left(\frac{G}{t} \ln \frac{S}{G}\right) \frac{\partial c}{\partial G} + \frac{\sigma^2}{2} S^2 \frac{\partial^2 c}{\partial S^2} + (r - q)S \frac{\partial c}{\partial S} - rc = 0, \quad 0 < t < T. \quad (7a)$$

With terminal condition:

$$c(S, G, T) = \max(S - G, 0). \quad (7b)$$

Let $S^*(G, t)$ indicate the best exercise asset price above which the American Asian option is optimally exercised. The governing equation of the above choice for the American Asian is obtained with the use of the delay exercise compensation statement advocated by Jamshidian[7] by modifying the Eq. (7a) as follow:

$$\frac{\partial C}{\partial t} + \left(\frac{G}{t} \ln \frac{S}{G}\right) \frac{\partial C}{\partial G} + \frac{\sigma^2}{2} S^2 \frac{\partial^2 C}{\partial S^2} + (r - q)S \frac{\partial C}{\partial S} - rC = \begin{cases} 0 & \text{if } S \leq S^*(G, t) \\ -qS - \frac{dG}{dt} + rG & \text{if } S > S^*(G, t) \end{cases} \quad (8)$$

The above incomplete differential condition detailing includes two spatial factors: S and G, and furthermore the ideal exercise resource cost relies upon G and t. The non-homogeneous term in Eq. (8) contains the additional term, This compares to the difference in the strike rate because of the normal resource worth's brief pace of progress.

A few endeavors were made by looking for the fitting factors scaling[4, 9] to lessen the element of the administering condition.

$$y = t \ln \frac{G}{S}, \quad V(y, t) = \frac{C(S, G, t)}{S}, \quad (9)$$

Where S is used as numeras for asset price. the current similarity variables

$$\frac{\partial V}{\partial t} + \frac{\sigma^2 t^2}{2} \frac{\partial^2 V}{\partial y^2} - \left(r - q + \frac{\sigma^2}{2}\right) t \frac{\partial V}{\partial y} - qV = \begin{cases} 0 & \text{if } y \geq y^*(t) \\ -q + re^{y/t} + \frac{y}{t^2} e^{y/t} & \text{if } y < y^*(t) \end{cases} \quad (10a)$$

And,

$$V(y, T) = \max(1 - e^{y/T}, 0). \quad (10b)$$

$$V(y, t) = 1 - e^{y/t}, \quad y \leq y^*(t). \quad (10c)$$

The above new course of action liberates the course for the persuading deduction from the esteeming formula of the American Asian decision.

Early exercise integral premium representation

Formal representation of the US appeal solution obtained from the price model above can be given as an indispensable factor for the green capacity of the management equation. Let $G(y, t; Y, T)$ be the green ability to meet the corresponding reduced equation:

$$\frac{\partial V}{\partial t} + \frac{\sigma^2 t^2}{2} \frac{\partial^2 V}{\partial y^2} - \left(r - q + \frac{\sigma^2}{2} \right) t \frac{\partial V}{\partial y} = 0. \tag{11}$$

The Green function is found to be

$$G(y, t; Y, T) = n \left(\frac{Y - y + \mu \int_t^T u du}{\sigma \sqrt{\int_t^T u^2 du}} \right), \tag{12}$$

where $\mu = r - q + \frac{\sigma^2}{2}$ and $n(x)$ is the standard typical thickness work. The answer for the overseeing Eqs. (10a) and (10b) can be officially spoken to by

$$V(y, t) = e^{-r(T-t)} \int_{-\infty}^{\infty} \max(1 - e^{-y^2/T}, 0) G(y, t; Y, T) dY + \int_t^T e^{-r(u-t)} \int_{-\infty}^{\infty} \left(y - \mu y^{1/2} - \frac{Y}{\sigma^2} y^{3/2} \right) G(y, t; Y, u) dY du. \tag{13}$$

Where $y^*(u)$ is the precarious value of y at time u , such that when $y \leq y^*(u)$. The intrinsic value of the American option assumes. Eq's first term. (13), if S is multiplied, the American counterpart option value of the existing American Asian call option is given. The value of the American equivalent is shown by instinctive strength incorporation of the first essential.

$$c(S, G, t) = S e^{-r(T-t)} N(d_1) - G e^{q(T-t)} S e^{-q(T-t)} e^{-Q} N(d_2), \tag{14}$$

Where,

$$d_1 = \frac{t \ln \frac{S}{G} + \frac{\sigma}{2} (T^2 - t^2)}{\sigma \sqrt{\int_t^T u^2 du}}, \quad d_2 = d_1 - \frac{\sigma}{T} \sqrt{\frac{T^2 - t^2}{3}}, \quad Q = \frac{\mu T^2 - t^2}{2} - \frac{\sigma^2 T^2 - t^2}{6} \tag{15}$$

$$c(S, G, t) = C(S, G, t) - c(S, G, t).$$

In this case, in Eq, $c(S, G, t)$ is given. The solution to Eq is(14) and $C(S, G, t)$. 8]. Allow the second component in Eq. The following (13) is $Ve(y, t)$, so $e(S, G, t) = S Ve(y, t)$. Again, we achieve the essential illustration of the initial exercise exceptional as follows by carrying out the integration accordingly:

$$e(S, G, t) = S \int_t^T \left\{ \rho e^{-\rho(u-t)} N(d_1) - \left(\frac{G}{S} \right)^{1/\rho} e^{-\rho(u-t)} \left[(r + d_1) N(d_2) - \frac{\sigma}{u^2} n(d_2) \right] \right\} du. \tag{16}$$

Where,

$$\hat{d}_1 = \frac{u \ln \frac{G}{S(G,u)} - t \ln \frac{G}{S} + \frac{\sigma}{2} (u^2 - t^2)}{\hat{\sigma}}, \quad \hat{d}_2 = \hat{d}_1 - \frac{\hat{\sigma}}{u},$$

$$\hat{d}_3 = \frac{t \ln \frac{G}{S} - \frac{\sigma}{2} (u^2 - t^2) + \frac{\sigma^2}{u}}{u^2},$$

$$\hat{Q} = \frac{\hat{\sigma}^2}{2u^2} - \frac{\mu(u^2 - t^2)}{2u}, \quad \hat{\sigma}^2 = \frac{\sigma^2}{3} (u^3 - t^3).$$

The above premium integral exercise is similar to an American vanilla option. In the subsequent examination on training policy, the availability of the premium term in analytical fashion has proven to be valuable.

AMERICAN FORM OF ASIAN OPTIONS PROBABILISTIC APPROACH

This section has as its primary purpose an integral equation to estimate an Asian option paying continuous dividend at an early exercise cap. Hansen and Jørgensen, 2000, follow the ideas of derivation. Their formula for a fluctuating incursion was based on martingale theory and the pre-conditioned values. We broaden their plan to include Asian options for a non-nil disbursement return underpayment and a general average floating attack. We discuss geometrical, arithmetic and weighted averaging operators in greater detail. The price model is based on the premise that the underlying asset is stochastic in time. During the whole analysis we must assume that S_t is driven by a stochastic cycle that satisfies following stochastic differential equation

$$dS_t = (r - q) S_t dt + \sigma S_t dW_t^P, \quad 0 \leq t \leq T. \tag{1}$$

The obvious price is $S_0 > 0$. It starts. The parameter $r > 0$ here is consistent with the hazard free loan charge, while $q > 0$ is an uninterrupted benefit rate. In addition, the consistent parameter σ depicts the instability of the hidden resources as the standard impartial probability measure P is a common Brownian movement. An equation answer (1) corresponds to the Brownian symmetrical movement

$$S_t = S_0 e^{(r-q-\frac{1}{2}\sigma^2)t + \sigma W_t^P}, \quad 0 \leq t \leq T.$$

We will draw an important equivalence which determines with a floating strike the value of an Asian American-style option. The payout of the option is set by if we determine the optimum stop time as T / T

$$V_{T^*} = \left(\rho(S_{T^*} - A_{T^*}) \right)^+,$$

If V_t is the value of the option at times of t , the value of the asset underlying at intervals $[0, t]$

and $c=1$ for the option call and $\mu=1$ for the option put is a continuous average. Many forms of averages described in table 1 may be considered.

Table 1: Average method classification

arithmetic average	geometric average	weighted arithmetic average
$A_t = \frac{1}{T} \int_0^t S_u du$	$\ln A_t = \frac{1}{T} \int_0^t \ln S_u du$	$A_t = \frac{1}{\int_0^t a(s) ds} \int_0^t a(t-u) S_u du$

On account of a weighted numerically arrived at the midpoint of coasting strike Asian option the part function $a(\cdot) \geq 0$ with the property $\int_0^\infty a(s) ds < \infty$ is frequently definite as $a(s) = e^{-\lambda s}$ where $\lambda > 0$ is constant.

The dependent statements of the US-style can be rated, according to (Hansen and Jørgensen 2000). The choice price can be calculated by taking into account all the probable interval stop times $[t, T]$

$$V(t, S, A) = \text{esssup}_{s \in \mathcal{T}_{[t, T]}} \mathbb{E}_t^P \left[e^{-r(s-t)} (\rho(S_s - A_s))^+ \mid S_t = S, A_t = A \right],$$

where $\mathcal{T}_{[t, T]}$ signifies the set of all discontinuing periods in the intermission $[t, T]$ and $\mathbb{E}_t^P[X] = \mathbb{E}^P[X | \mathcal{F}_t]$ is the accustomed expectative through available material at time t (the filtration \mathcal{F}_t of \mathbb{S} -algebra \mathbb{F} reflects the set of data supporting the Brownian motion).

We change the chance measure by the martingale to simplify the formula

$$\eta_t = e^{-(r-q)t} \frac{S_t}{S_0} = e^{-\frac{1}{2}\sigma^2 t + \sigma W_t^P}$$

The new possibility quantity Q actuality definite as $dQ = \eta T dP$. According to Girsanov's theorem (Revuz and Yor 2005), the process

$$W_t^Q = W_t^P - \sigma t$$

Is a Brownian standard motion regarding measure Q . The estimation of the fundamental resource cost under this measure is controlled by

$$S_t = S_0 e^{(r-q + \frac{1}{2}\sigma^2)t + \sigma W_t^Q}$$

Q -martingales when the underlying price is discounted for all assets priced under this measure. Accordingly, the number of stochastic variables can be decreased. A new variable is introduced

$$x_t = \frac{A_t}{S_t}. \text{ We obtain:}$$

$$\begin{aligned} V(t, S, A) &= \text{esssup}_{s \in \mathcal{T}_{[t, T]}} \mathbb{E}_t^P \left[e^{-r(s-t)} (\rho(S_s - A_s))^+ \mid S_t = S, A_t = A \right] \\ &= \text{esssup}_{s \in \mathcal{T}_{[t, T]}} \mathbb{E}_t^Q \left[\frac{\rho}{\eta} e^{-r(s-t)} (\rho(S_s - A_s))^+ \mid S_t = S, A_t = A \right] \\ &= \text{esssup}_{s \in \mathcal{T}_{[t, T]}} \mathbb{E}_t^Q \left[e^{-r(s-t)} S_t \left(\rho \left(1 - \frac{A_t}{S_t} \right) \right)^+ \mid S_t = S, A_t = A \right] \\ &= \text{esssup}_{s \in \mathcal{T}_{[t, T]}} e^{-r(s-t)} S \mathbb{E}_t^Q \left[\left(\rho \left(1 - x_s \right) \right)^+ \mid S_t = S, A_t = A \right]. \end{aligned}$$

The last articulation can be reworked as far as the new variable $x = \frac{A}{S}$ as follows:

$$\bar{V}(t, x) = e^{-rt} \frac{V(t, S, A)}{S} = e^{-rt} \mathbb{E}_t^Q \left[\left(\rho \left(1 - x_{TT} \right) \right)^+ \right], \quad (3)$$

CONCLUSION

The obvious issues of breaking down American Asian price models come from the way that the estimation of the choice relies upon the stochastic developments of the benefit cost and its normal worth. Right now, show that the estimation of the alternative possibly relies upon a solitary stochastic variable when the advantage cost is standardized. This stochastic variable is the normal cost to-resource proportion. The American Asian alternatives' early exercise approach can be assessed by finding a full condition for the activity top whose multifaceted nature looks like an American vanilla other choice. In the present examination, some noteworthy features of the early exercise framework are uncommon to the averaging alternatives. The extent of early exercise premium for American Asian choices differentiated and American vanilla choices. One may investigate that this assessment deals with the cost of a skimming strike American Asian alternative, which is seen as one of the least mind boggling choice model among the entire class of American Asian choices. The systems inspected here at any rate can be connected with costs of various types of Asian American alternatives, for instance, models whose typical is wisely tried, and other terminal outcome structures. Upon a singular asset, the GBM model offers an American alternative the dynamic programming, joined with an area express polynomial worth gauge after a commendable factor modification. We have examined American-style Asian price alternatives with normal drifting strike right now. We have been focusing on estimating the cost of skimming strike mathematically, geometrically, and numerically. We got an essential portrayal of call and set choice costs in the initial segment of the examination and given a necessary condition for the free fringe position. We investigated the lead of the early-preparing limit close to the termination date. We proposed a general strategy for deciding the early preparing position at termination. We additionally acquired the early exercise top asymptotic equation close to the expiry. The other piece of the examination comprised of building up a dependable numerical framework to ascertain the

early exercise top. Utilizing the front securing strategy, for the combined portfolio and the free limit position a non-neighborhood fractional allegorical distinction was inferred. We have built up a numerical framework to take care of the issue with a view to the administrator parting strategy. Asian choices are normal alternatives where terminal adjustments are needed for a section or an entire existence of some type of normal resource costs. Normal choices are particularly helpful for organizations occupied with the exchange of meagerly exchanged merchandise. This sort of choice is utilized by dealers who need to cover the normal cost of the item over a period, not the cost toward the finish of the period, for instance.

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