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Solution of Fractional Impulsive Problem under Power Law

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Abstract – This paper is basically concerned with finding the solution using formulae, existence and uniqueness of solutions of impulsive fractional differential equations with ABC (Atangana-Baleanu-Caputo) fractional derivative with non-singular Mittag-Leffler kernel. Our examination depends on non-singular fractional analysis and few techniques of fixed-point theory. An example is illustrated to clarify the proved concepts.

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INTRODUCTION

Fractional calculus has taken a notable sector of inspection on real life problems with its tremendous application in science and engineering. It has been extended because of global character of the fractional operator, which relates the reminiscence and it authorizes to learn an adjacent look at the vital behavior and inherited things of the relevant phenomena. We referred the monographs [1-3] to know about current development in such field.

The purpose of fractional derivative with power law base in the sense of Riemann Liouville was established. A modern fractional derivative is added has suggested by Caputo-Fabrizio [4] depending on the augmented kernel. To through away troubles related to Caputo-Fabrizio's, a new change in version of a fractional derivative with non-singular and global kernel of Mittag-Leffler function (MLF) have been introduced by Atangana and Baleanu (AB) in [5]. Generalization of MLF is noted and used as non-singular and global kernel in later on extension but it does not assure singularity. Also, the ABC derivatives comes up with a magnificent memory description [6-9]. Recently the authors [10-14] deliberated inquisitive and numerical solution for few fractional models by means of AB fractional derivative with global trouble-free kernel.

We display the nature and uniqueness of solutions to impulsive fractional differential equations with initial and global conditions in this article.

$$\left.\begin{aligned} &{}^{ABC}D^{a}_{[\rho]}\xi(\rho)=h(\rho,\xi(\rho)), \quad \rho\in\Omega=[0,S],\ \rho\neq\rho_m; \quad m=1,2,\ldots\ldots,n \\ &\Delta\xi|_{\rho=\rho_m}=I_m\xi(\rho_m^{-}), \quad m=1,2,\ldots\ldots,n \\ &\xi(0)=\xi_0 \end{aligned}\right\} \qquad (1.1)$$

and

$$\left.\begin{aligned} &{}^{ABC}D^{a}_{[\rho]}\xi(\rho)=h(\rho,\xi(\rho)), \quad \rho\in\Omega=[0,S],\ \rho\neq\rho_m; \quad m=1,2,\ldots\ldots,n \\ &\Delta\xi|_{\rho=\rho_m}=I_m\xi(\rho_m^{-}), \quad m=1,2,\ldots\ldots,n \\ &\xi(0)+f(\xi)=\xi_0 \end{aligned}\right\} \qquad (1.2)$$

Where $0<a\leq 1,\ {}^{ABC}D^{a}_{[\rho]}$ refers the Atangana-Baleanu-Caputo (ABC) fractional derivative of order $a,\ h:\Omega\times R\to R$ is a given non-discontinuous function. Moreover, $h(0,\xi(0))=0$ and also disappears at impulse points $\rho_m,\ m=1,2,\ldots.,n,\ I_m:R\to R,\ m=1,2,\ldots.,n,\ \xi_0\in R,\ \rho_m$ satisfy

$$\begin{aligned} &0=\rho_0<\rho_1<\rho_2<\ldots..<\rho_n<\rho_{n+1}=\rho \\ &\Delta\xi|_{\rho=\rho_m}=\xi(\rho_m^{+})-\xi(\rho_m^{-})=\xi(\rho_m^{+})-\xi(\rho_m) \\ &\xi(\rho_m^{+})=\lim_{h\to 0^{+}}\xi(\rho_m+h), \qquad \xi(\rho_m^{-})=\lim_{h\to 0^{-}}\xi(\rho_m+h) \end{aligned}$$

Represents the right and left limits of $\xi(\rho)$ at $\rho=\rho_m$ and $f:PC(\Omega,R)\to R$ is given function.

Also $[\rho]=\rho_m$ if $\rho\in(\rho_m,\rho_{m+1}],m=0,1,2,,,,,,\ \&\ \rho_0=0.$ By reference from Theorem 3.11 of [24], we should have standard condition $h(0,\xi(0))=0$ to fix the starting data for solution. By the previous referred articles, there are only few articles on Cauchy problems for ABC impulsive fractional differential equations.

The main part of this paper is to derive formula of solutions for types of impulsive fractional differential equations with ABC fractional operators. Already we proved existence and uniqueness theorem theorems by few fixed-point theorems of Banach space, Kransosekliskii, Schader and

Schafer for suggested problems taken in previous papers [13-31]. We grasped that the equality $h(0,\xi(0)) = h(\rho_m, \xi(\rho_m)) = 0 \ (m = 1,2,\dots\dots,n)$ is mandatory to assure a unique solution.

This paper is divided as follows:

Section 1 deals with survey of literature.

Section 2 comprises basic definitions, fundamental lemmas and theorems.

The suggested formulae for taken problem is established in section 3.

The existence of unique solution for Cauchy problem and global Cauchy problem are acquired in 4th and 5th section.

Finally, examples are stated and proved to the affirmation of our results.

2. PRELIMINARIES

Consider the space

$$PC(\Omega,R)=\left\{\begin{array}{l}\xi:\Omega\to R;\quad \xi\in C(\rho_m,\rho_{m+1}],R;\quad m=0,1,2,\dots.,n+1\\ \text{and }\ \xi(\rho_m^+)\ \&\ \xi(\rho_m^-)\text{ exist for } m=1,2,\dots,n \text{ with } \xi(\rho_m^-)=\xi(\rho_m^+)\end{array}\right\}.$$

The space $PC(\Omega,R)$ is a complete normed linear space with the norm $\|\xi\|_{PC}=\max_{\rho\in\Omega}|\xi(\rho)|.$ Let $\Omega=[0,S]$ and $\Omega' := \Omega\setminus\{\rho_1,\rho_2,\dots\dots,\rho_m\}.$

Definition 2.1. [5,27] Let $a\in[0,1]$ and $\eta\in H^1(\alpha,\beta)\ \alpha<\beta.$ Then the left AB Caputo and AB Riemann-Liouville fractional derivatives of order a for a function η are defined by

$${}^{ABC}D_\alpha^a\eta(\rho)=\frac{M(a)}{1-a}\int_\alpha^\rho E_a\left(\frac{-a}{a-1}(\rho-\varphi)^a\right)\eta'(\varphi)\,d\varphi,\ \rho>\alpha$$

and

$${}^{ABR}D_\alpha^a\eta(\rho)=\frac{M(a)}{1-a}\frac{d}{d\rho}\int_\alpha^\rho E_a\left(\frac{-a}{a-1}(\rho-\varphi)^a\right)\eta'(\varphi)\,d\varphi,\ \rho>\alpha,$$

respectively, where $M(a)$ is a function with normalization convinces the results $M(0)=M(1)=1$ and E_a is called the MLF defined by

$$E_a(x)=\sum_{j=0}^{\infty}\frac{x^j}{\Gamma(ja+1)},\ \operatorname{Re}(a)>0,\ x\in C. \tag{2.3}$$

Definition 2.2. [5,27] Let $a\in(0,1]$ and $\eta\in L^1(\alpha,\beta).$ Then the left AB fractional integral of order a for a function η is denoted by

$${}^{AB}I_\alpha^a\eta(\rho)=\frac{1-a}{M(a)}\eta(\rho)+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\int_\alpha^\rho(\rho-\varphi)^{a-1}\eta(\varphi)\,d\varphi,\ \rho>\alpha.$$

Definition 2.3. [5] The ABC fractional derivative with its Laplace transformation is defined by

$$L\left[{}^{ABC}D_\alpha^a\eta(\rho)\right]=\frac{M(a)}{t^a(1-a)+a}\left[t^aL[\eta(\rho)]-t^{a-1}\eta(\alpha)\right]$$

where L is the Laplace transform initiating from α defined by

$$L\{h(s)\}(t)=\int_\alpha^\infty e^{-t(s-a)}h(s)\,ds$$

Definition 2.4. [29] Let X be a complete normed linear space. Then the operator $\Pi: X\to X$ is a contraction if $\|\Pi x_1-\Pi x_2\|\le\kappa\|x_1-x_2\|$ for all $x_1,x_2\in X,\ 0<\kappa<1.$

Definition 2.5. A function $\xi\in PC(\Omega,R)$ is a solution of (1.1) if ξ convinces the equation ${}^{ABC}D_{[\rho]}^a\xi(\rho)=h(\rho,\xi(\rho))$ on Ω' and conditions

$$\Delta\xi\big|_{\rho=\rho_m}=I_m\xi(\rho^-),\ m=1,2,\dots.,n \text{ and } \xi(0)=\xi_0.$$

To prove the main concept of this paper, we need to remember the following lemmas and theorems which is proved already.

Lemma 2.1. [28] Let $\eta\in H'(\alpha,\beta),\ \beta>\alpha,$ such that the ABC fractional derivative exists. Then we have ${}^{ABC}D_\alpha^a\ {}^{AB}I_\alpha^a\ \eta(\rho)=\eta(\rho)$ and ${}^{AB}I_\alpha^a\ {}^{ABC}D_\alpha^a\ \eta(\rho)=\eta(\rho)-\eta(\alpha)$ for $0<a\le1.$ Also, ${}^{ABC}D_\alpha^a\ \eta(\rho)=0$ if $\eta(\rho)$ is a constant.

Lemma 2.2. [24,28] For $a\in[0,1],$ the solution of the following problem

$${}^{ABC}D_\alpha^a\ \eta(\rho)=\omega(\rho);\ \eta(\alpha)=\eta_0$$

is given by

$$\eta(\rho)=\eta_0+\frac{1-a}{M(a)}\omega(\rho)+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\int_\alpha^\rho(\rho-\varphi)^{a-1}\omega(\varphi)\,d\varphi. \tag{2.4}$$

Theorem 2.1. [29] Let *X* be a complete normed linear space and *A* be a non-empty closed subset of *X*. If $\Pi: A\to A$ is a contraction, then there exists a unique fixed point of Π.

Theorem 2.2. [29] Let *X* be a complete normed linear space and $\Pi: X\to X$ be a non-discontinuous and compact mapping (completely continuous). Suppose $T=\{y\in X: y=\lambda\Pi y \text{ for } \lambda\in(0,1)\}$ be a bounded set. Then Π has at least one fixed point in *X*.

Theorem 2.3. [29] Let B be a non-empty, convex, closed subset of a complete normed linear space X and let o_1, o_2 be two operators, such that

(i) $o_1 a + o_2 b \in B \ \forall \ a, b \in B$

(ii) o_1 is compact and continuous

(iii) o_2 is a contraction mapping. Then there exist $\omega \in B$ such that $o_1\omega + o_2\omega = \omega.$

3. FORMULAE DERIVATION FOR REPRESENTED PROBLEM

Theorem 3.1. Let $a \in (0,1]$ and let $v : \Omega \to R$ be continuous with $v(0) = 0$ and also disappears at hasty points ρ_m, for $m = 1, 2, ..., n.$ A function $\xi \in PC(\Omega, R)$ is a solution of the fractional integral equation

$$\xi(\rho) = \begin{cases} \xi_0 + \frac{1-a}{M(a)} v(\rho) + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_0^{\rho} (\rho - \varphi)^{a-1} v(\varphi) d\varphi, \text{ if } \rho \in [0, \rho_1], \\ \xi_0 + \frac{1-a}{M(a)} \sum_{i=1}^{m+1} v(\rho_i) + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \sum_{i=1}^{m} \int_{\rho_{i-1}}^{\rho_i} (\rho_i - \varphi)^{a-1} v(\varphi) d\varphi \\ + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_m}^{\rho} (\rho - \varphi)^{a-1} v(\varphi) d\varphi + \sum_{i=1}^{m} I_i \xi(\rho_i^-) \text{ if } \rho \in (\rho_m, \rho_{m+1}], m = 1, ..., n \end{cases} \tag{3.5}$$

If and only if ξ is a solution of the impulsive ABC-fractional FDE

$${}^{ABC}D^a_{[\rho]} \xi(\rho) = v(\rho), \ \rho \in \Omega' := \Omega \setminus \{\rho_1, \rho_2, ..., \rho_m\} \tag{3.6}$$

$$\Delta \xi|_{\rho = \rho_m} = I_m \xi(\rho_m^-), \ m = 1, 2, ..., n \text{ and} \tag{3.7}$$

$$\xi(0) = \xi_0. \tag{3.8}$$

where

$$[\rho] = \rho_m \text{ if } \rho \in (\rho_m, \rho_{m+1}], \ m = 0, 1, 2, ... \text{ and } \rho_0 = 0.$$

Proof. By lemma 2.1, we can prove this again by assuming ξ satisfies (3.6)-(3.8).

If $\rho \in [0, \rho_1]$, then ${}^{ABC}D^a_{[\rho]} \xi(\rho) = v(\rho), [\rho] = 0.$

Using lemma (2.1), we have

$$\xi(\rho) = \xi_0 + \frac{1-a}{M(a)} v(\rho) + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_0^{\rho} (\rho - \varphi)^{a-1} v(\varphi) d\varphi.$$

This implies

$$\xi(\rho_1^-) = \xi_0 + \frac{1-a}{M(a)} v(\rho_1) + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_0^{\rho_1} (\rho_1 - \varphi)^{a-1} v(\varphi) d\varphi$$

By impulse $\xi(\rho_1^-) = \xi(\rho_1^+) - I_1 \xi(\rho_1^-)$, we get

$$\xi(\rho_1^+) = \xi_0 + \frac{1-a}{M(a)} v(\rho_1) + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_0^{\rho_1} (\rho_1 - \varphi)^{a-1} v(\varphi) d\varphi + I_1 \xi(\rho_1^-)$$

If $\rho \in (\rho_1, \rho_2]$, then v vanishes at ρ_1 implies

$$\begin{aligned} \xi(\rho) &= \xi(\rho_1^+) + \frac{1-a}{M(a)} v(\rho) + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_1}^{\rho} (\rho - \varphi)^{a-1} v(\varphi) d\varphi \\ &= \xi_0 + \frac{1-a}{M(a)} [v(\rho_1) + v(\rho)] + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_0^{\rho_1} (\rho_1 - \varphi)^{a-1} v(\varphi) d\varphi \\ &\quad + I_1 \xi(\rho_1^-) + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_1}^{\rho} (\rho - \varphi)^{a-1} v(\varphi) d\varphi \end{aligned}$$

$$\begin{aligned} \xi(\rho_2^-) &= \xi_0 + \frac{1-a}{M(a)} [v(\rho_1) + v(\rho_2)] + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_0^{\rho_1} (\rho_1 - \varphi)^{a-1} v(\varphi) d\varphi \\ &\quad + I_1 \xi(\rho_1^-) + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_1}^{\rho_2} (\rho_2 - \varphi)^{a-1} v(\varphi) d\varphi \end{aligned}$$

By impulse $\xi(\rho_2^-) = \xi(\rho_2^+) - I_2 \xi(\rho_2^-)$, we get

$$\begin{aligned} \xi(\rho_2^+) &= \xi_0 + \frac{1-a}{M(a)} [v(\rho_1) + v(\rho_2)] + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_0^{\rho_1} (\rho_1 - \varphi)^{a-1} v(\varphi) d\varphi \\ &\quad + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_1}^{\rho_2} (\rho_2 - \varphi)^{a-1} v(\varphi) d\varphi + I_1 \xi(\rho_1^-) + I_2 \xi(\rho_2^-) \end{aligned}$$

If $\rho \in (\rho_2, \rho_3]$, then v vanishes at ρ_2 implies

$$\begin{aligned} \xi(\rho) &= \xi(\rho_2^+) + \frac{1-a}{M(a)} v(\rho) + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_2}^{\rho} (\rho - \varphi)^{a-1} v(\varphi) d\varphi \\ &= \xi_0 + \frac{1-a}{M(a)} [v(\rho_2) + v(\rho_1) + v(\rho)] + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_0^{\rho_1} (\rho_1 - \varphi)^{a-1} v(\varphi) d\varphi \\ &\quad + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_1}^{\rho_2} (\rho_2 - \varphi)^{a-1} v(\varphi) d\varphi + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_1}^{\rho} (\rho - \varphi)^{a-1} v(\varphi) d\varphi \\ &\quad + I_1 \xi(\rho_1^-) + I_2 \xi(\rho_2^-) \end{aligned}$$

This implies that

$$\begin{aligned} \xi(\rho_3^-) &= \xi_0 + \frac{1-a}{M(a)} [v(\rho_1) + v(\rho_2) + v(\rho_3)] + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_0^{\rho_1} (\rho_1 - \varphi)^{a-1} v(\varphi) d\varphi \\ &\quad + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_1}^{\rho_2} (\rho_2 - \varphi)^{a-1} v(\varphi) d\varphi + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_2}^{\rho_3} (\rho_3 - \varphi)^{a-1} v(\varphi) d\varphi \\ &\quad + I_1 \xi(\rho_1^-) + I_2 \xi(\rho_2^-) \end{aligned}$$

By impulse $\xi(\rho_3^-) = \xi(\rho_3^+) - I_3 \xi(\rho_3^-)$, we get

$$\begin{aligned} \xi(\rho_3^+) &= \xi_0 + \frac{1-a}{M(a)} [v(\rho_1) + v(\rho_2) + v(\rho_3)] + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_0^{\rho_1} (\rho_1 - \varphi)^{a-1} v(\varphi) d\varphi \\ &\quad + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_1}^{\rho_2} (\rho_2 - \varphi)^{a-1} v(\varphi) d\varphi + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_2}^{\rho_3} (\rho_3 - \varphi)^{a-1} v(\varphi) d\varphi \\ &\quad + I_1 \xi(\rho_1^-) + I_2 \xi(\rho_2^-) + I_3 \xi(\rho_3^-) \end{aligned}$$

Proceeding like this, we get the general expression as given below:

$$\begin{aligned} \xi(\rho_m^+) &= \xi_0 + \frac{1-a}{M(a)} [v(\rho_1) + v(\rho_2) + v(\rho_3) + ... + v(\rho_m)] \\ &\quad + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \left[\int_0^{\rho_1} (\rho_1 - \varphi)^{a-1} v(\varphi) d\varphi + ... + \int_{\rho_{m-1}}^{\rho_m} (\rho_m - \varphi)^{a-1} v(\varphi) d\varphi \right] \\ &\quad + I_1 \xi(\rho_1^-) + I_2 \xi(\rho_2^-) + I_3 \xi(\rho_3^-) + ... + I_m \xi(\rho_m^-) \end{aligned}$$

For $\rho \in (\rho_m, \rho_{m+1}], [\rho] = \rho_m$ and hence we get the solution

$$\xi(\rho)=\xi(\rho_m^+)+\frac{1-a}{M(a)}[v(\rho)]+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\int_{\rho_m}^{\rho}(\rho-\varphi)^{a-1}v(\varphi)d\varphi+$$
$$=\xi_0+\frac{1-a}{M(a)}\sum_{i=1}^{m+1}v(\rho_i)+\sum_{i=1}^{m}I_i\xi(\rho_i^-)$$
$$+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\sum_{i=1}^{m}\int_{\rho_{i-1}}^{\rho_i}(\rho_i-\varphi)^{a-1}v(\varphi)d\varphi+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\int_{\rho_m}^{\rho}(\rho-\varphi)^{a-1}v(\varphi)d\varphi$$

Hence (2.5) is satisfied.

Conversely assume that ξ satisfies the equation (1.1).

If $\rho\in[0,\rho_1]$, then $v(0)=0;\ \xi(0)=\xi_0$. By the concept that ${}^{ABC}D_\alpha^a$ is the left inverse of ${}^{AB}I_\alpha^a$. By using lemma 2.1, we get ${}^{ABC}D_0^a\,\xi(\rho)=v(\rho),\ \rho\in[0,\rho_1]$.

If $\rho\in[\rho_m,\rho_{m+1}),\ m=1,2,\ldots\ldots,n$ and by the fact that ${}^{ABC}D_{[\rho]}^a\,\eta(\cdot)=0,$ where $\eta(\cdot)$ is a constant function, we obtain ${}^{ABC}D_{[\rho]}^a\,\xi(\rho)=v(\rho),$ for each $\rho\in[\rho_m,\rho_{m+1}),\ m=1,2,\ldots\ldots,n.$

Hence $\xi(\rho_m^+)-\xi(\rho_m^-)=I_m\xi(\rho_m^-),\ m=1,2,\ldots\ldots,n.$

4. CAUCHY PROBLEM

We need the following hypothesis to prove our results,

(A1) $\left|h(\rho,\theta)-h(\rho,\theta^*)\right|\le L_h\left|\theta-\theta^*\right|,$ for each $\rho\in\Omega,\ \ \theta,\theta^*\in R$ and a constant $L_h>0.$

(A2) The function $I_m:R\to R$ are continuous and there exist a constant $L_J>0$ such that

$$\left|J_m(\theta)-J_m(\theta^*)\right|\le L_J\left|\theta-\theta^*\right|,\ \ m=1,2,\ldots.,n\ \ \&\ \ \theta,\theta^*\in R.$$

(A3) There exist $\lambda\in C(\Omega,R)$ such that $\left|h(\rho,\theta)\right|\le\lambda(\rho)$, for each $(\rho,\xi)\in\Omega\times R.$

(A4) There exist $N>0$ such that $\left|I_m(\theta)\right|\le N,\ m=1,2,\ldots.,n,\ \theta\in R.$

Now let us prove main results.

Theorem 4.1.

Assume $h:\Omega\times R\to R$ is continuous. If (A1) and (A2) hold with

$$\mu L_h+nL_I<1, \tag{4.9}$$

then the impulsive ABC – fractional FDE (1.1) has a solution which is unique on $\Omega,$ where

$$\mu:=\frac{(1-a)n}{M(a)}+\frac{\rho^a(n+1)}{M(a)\Gamma(a)}. \tag{4.10}$$

Proof: By lemma 2.3, define the mapping $B:PC(\Omega,R)\to PC(\Omega,R)$ by

$$B\xi(\rho)=\xi_0+\frac{1-a}{M(a)}\sum_{0<\rho_m<\rho}h(\rho_m,\xi(\rho_m))+\sum_{0<\rho_m<\rho}I_m\xi(\rho_m^-)$$
$$+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\sum_{0<\rho_m<\rho}\int_{\rho_{m-1}}^{\rho_m}(\rho_m-\varphi)^{a-1}h(\varphi,\xi(\varphi))d\varphi \tag{4.11}$$
$$+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\int_{\rho_m}^{\rho}(\rho-\varphi)^{a-1}h(\varphi,\xi(\varphi))d\varphi$$

Let

$$A_l=\left\{\xi\in PC(\Omega,R):\|\xi\|_{PC}\le l\right\}\ \text{with}\ l\ge\frac{|\xi_0+\mu\Lambda_h|}{1-[\mu L_h+nL_I]}.$$

Let $\max_{\varphi\in\Omega}|h(\varphi,0)|=\Lambda_h.$ From (A1), we get

$$\left|h(\varphi,\xi(\varphi))\right|\le\left|h(\varphi,\xi(\varphi))-h(\varphi,0)\right|+h(\varphi,0)$$
$$\le L_h|\xi|+\Lambda_h\le L_hl+\Lambda_h$$

To prove B has a fixed point, for that we need to show that $BA_l\subset A_l.$ For $\xi\in A_l$ we have

$$\|B_\xi\|=\max_{\rho\in\Omega}|B\xi(\rho)|\le|\xi_0|+\frac{1-a}{M(a)}\max_{\rho\in\Omega}\sum_{0<\rho_m<\rho}h(\rho_m,\xi(\rho_m))+\max_{\rho\in\Omega}\sum_{0<\rho_m<\rho}I_m\xi(\rho_m^-)$$
$$+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\max_{\rho\in\Omega}\sum_{0<\rho_m<\rho}\int_{\rho_{m-1}}^{\rho_m}(\rho_m-\varphi)^{a-1}h(\varphi,\xi(\varphi))d\varphi$$
$$+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\max_{\rho\in\Omega}\int_{\rho_m}^{\rho}(\rho-\varphi)^{a-1}h(\varphi,\xi(\varphi))d\varphi$$

$$\|B_\xi\|\le|\xi_0|+\frac{1-a}{M(a)}n[L_hl+\Lambda_h]+nL_Il+\frac{\rho^a}{M(a)\Gamma(a)}(n+1)[L_hl+\Lambda_h]$$
$$=|\xi_0|+\mu\Lambda_h+[\mu L_h+nL_I]l$$
$$\le l|1-(\mu L_h+nL_I)|+l(\mu L_h+nL_I)=l$$

Thus B maps A_l into itself.

Next, we have to show that B is a contraction on $PC(\Omega,R)$. Let $\xi,\xi^*\in PC(\Omega,R)$ and $\rho\in\Omega$.

Then we get

$$\|B_\xi-B_\xi^*\|=\max_{\rho\in\Omega}|B\xi(\rho)-B\xi^*(\rho)|$$

$$\|B_\xi-B_\xi^*\|\le\frac{1-a}{M(a)}\max_{\rho\in\Omega}\sum_{0<\rho_m<\rho}\left|h(\rho_m,\xi(\rho_m))-h(\rho_m,\xi^*(\rho_m))\right|$$
$$+\max_{\rho\in\Omega}\sum_{0<\rho_m<\rho}\left|I_m\xi(\rho_m^-)-I_m\xi^*(\rho_m^-)\right|$$
$$+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\max_{\rho\in\Omega}\sum_{0<\rho_m<\rho}\int_{\rho_{m-1}}^{\rho_m}(\rho_m-\varphi)^{a-1}\left|h(\varphi,\xi(\varphi))-h(\varphi,\xi^*(\varphi))\right|d\varphi$$
$$+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\max_{\rho\in\Omega}\int_{\rho_m}^{\rho}(\rho-\varphi)^{a-1}\left|h(\varphi,\xi(\varphi))-h(\varphi,\xi^*(\varphi))\right|d\varphi$$

$$\|B_\xi - B_\xi^*\| \le \frac{1-a}{M(a)} \max_{\rho\in\Omega} nL_h |\xi(\rho)-\xi^*(\rho)| + \max_{\rho\in\Omega} nL_I |\xi(\rho)-\xi^*(\rho)|$$
$$+\frac{a}{M(a)}\frac{1}{\Gamma(a)} \max_{\rho\in\Omega} \sum_{0<\rho_m<\rho} \int_{\rho_{m-1}}^{\rho_m} (\rho_m-\varphi)^{a-1} L_h |\xi(\varphi)-\xi^*(\varphi)| d\varphi$$
$$+\frac{a}{M(a)}\frac{1}{\Gamma(a)} \max_{\rho\in\Omega} \int_{\rho_m}^{\rho} (\rho-\varphi)^{a-1} L_h |\xi(\varphi)-\xi^*(\varphi)| d\varphi$$

$$\|B_\xi - B_\xi^*\| \le \frac{(1-a)^n}{M(a)} L_h \|\xi-\xi^*\| + nL_I\|\xi-\xi^*\| + \frac{\rho^a(n+1)}{M(a)}\frac{1}{\Gamma(a)} L_h \|\xi-\xi^*\|$$
$$=(\mu L_h + nL_I)\|\xi-\xi^*\|$$

The inequality (4.9) shows that B is contraction on $PC(\Omega,R)$. It is proved that the impulsive fractional differential equation with ABC (1.1) has a solution which is unique.

Theorem 4.2. Suppose $h:\Omega\times R\to R$ is continuous and assume (A3) and (A4) hold. Then there exists at least one solution for fractional differential equations with ABC fractional derivative and impulse condition on Ω.

Step 1: To show that $B:PC(\Omega,R)\to PC(\Omega,R)$ is compact.

Since h and I_m are continuous, we have to verify that B is continuous.

Let $A_{l_1}=\{\xi\in PC(\Omega,R):\|\xi\|_{PC}\le l_1\}$ be a set(ball) with $l_1=|\xi_0|+\mu\lambda_0+nN+1$.

Where $\lambda_0=\sup_{\rho\in\Omega}|\lambda(\rho)|$ and μ is given by (4.10). For $\xi\in A_l$ and $\rho\in\Omega$, we have

$$\|B_\xi\| = \max_{\rho\in\Omega}|B\xi(\rho)| \le |\xi_0| + \frac{1-a}{M(a)}\max_{\rho\in\Omega}\sum_{0<\rho_m<\rho}|h(\rho_m,\xi(\rho_m))|$$
$$+\max_{\rho\in\Omega}\sum_{0<\rho_m<\rho}|I_m\xi(\rho_m^-)| + \frac{a}{M(a)}\frac{1}{\Gamma(a)}\max_{\rho\in\Omega}\sum_{0<\rho_m<\rho}\int_{\rho_{m-1}}^{\rho_m}(\rho_m-\varphi)^{a-1}|h(\varphi,\xi(\varphi))|d\varphi$$
$$+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\max_{\rho\in\Omega}\int_{\rho_m}^{\rho}(\rho-\varphi)^{a-1}|h(\varphi,\xi(\varphi))|d\varphi$$
$$\le|\xi_0|+\mu\lambda_0+nN\le l_1.$$

Hence $B(Al_1)$ is uniformly bounded.

Next, we have to prove B maps bounded sets into equicontinuous set of $PC(\Omega,R)$.

By (A3), Let us fix $\sup_{(\rho,\xi)\in\Omega\times A_{l_1}}|h(\rho,\xi)|=h_0.$ Now

$$\|B\xi(\rho_2)-B\xi(\rho_1)\| \le \frac{a}{M(a)}\frac{1}{\Gamma(a)}\left|\int_{\rho_m}^{\rho_2}(\rho_2-\varphi)^{a-1}h(\varphi,\xi(\varphi))d\varphi - \int_{\rho_m}^{\rho_1}(\rho_1-\varphi)^{a-1}h(\varphi,\xi(\varphi))d\varphi\right|$$
$$\le\frac{a}{M(a)}\frac{1}{\Gamma(a)}\int_{\rho_1}^{\rho_2}(\rho_2-\varphi)^{a-1}|h(\varphi,\xi(\varphi))|d\varphi$$
$$+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\int_{\rho_m}^{\rho_1}\left[(\rho_1-\varphi)^{a-1}-(\rho_2-\varphi)^{a-1}\right]|h(\varphi,\xi(\varphi))|d\varphi$$
$$\le\frac{a}{M(a)}\frac{h_0}{\Gamma(a)}\left[2(\rho_2-\rho_1)^a+(\rho_1-\rho_m)^a-(\rho_2-\rho_m)^a\right]$$
$$\le\frac{2}{M(a)}\frac{h_0}{\Gamma(a)}(\rho_2-\rho_1)^a$$

As $\rho_1\to\rho_2, \|B\xi(\rho_2)-B\xi(\rho_1)\|\to 0,$ that is $B(A_{l_1})$ relatively compact for $\rho\in\Omega.$

The operator B is compact on A_{l_1} by Arzela Ascoli theorem.

Step 2: To prove that the set $T=\{\xi\in PC(\Omega,R):\xi=\gamma B_\xi \text{ for some } \gamma\in(0,1)\}$ is bounded.

Let $\xi\in T.$ Then $\xi=\gamma B_\xi$ for some $\gamma\in(0,1)$. Hence for $\rho\in\Omega,$ we obtain

$$|\xi(\rho)|<|B\xi(\rho)|\le|\xi_0|+\frac{1-a}{M(a)}\sum_{0<\rho_m<\rho}|h(\rho_m,\xi(\rho_m))|$$
$$+\sum_{0<\rho_m<\rho}|I_m\xi(\rho_m^-)|+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\sum_{0<\rho_m<\rho}\int_{\rho_{m-1}}^{\rho_m}(\rho_m-\varphi)^{a-1}|h(\varphi,\xi(\varphi))|d\varphi$$
$$+\frac{a}{M(a)}\frac{1}{\Gamma(a)}\int_{\rho_m}^{\rho}(\rho-\varphi)^{a-1}|h(\varphi,\xi(\varphi))|d\varphi$$
$$\le|\xi_0|+\frac{1-a}{N(a)}n\lambda_0+nN+\frac{\rho^a}{M(a)}\frac{1}{\Gamma(a)}(n+1)\lambda_0$$
$$=|\xi_0|+\mu\lambda_0+nN$$

For every $\rho\in\Omega,$ we get $\|\xi\|_{PC}\le|\xi_0|+\mu\lambda_0+nN:=R.$ Hence T is bounded.

That is B has a fixed point which is a solution of ABC-fractional differential equation (1.1)

5. NON-LOCAL CAUCHY PROBLEM OF ABC-FDE:

Finally, we have to prove existence of solution for the impulsive FDE with ABC fractional derivative.

Let f satisfying the following condition

(A5) $f:PC(\Omega,R)\to R$ is continuous and there exists a constant $0<L_f<1$ such that

$|f(\theta)-f(\theta^*)|\le L_f|\theta-\theta^*|$ for all $\theta,\theta^*\in PC(\Omega,R).$

Theorem 5.1. Suppose $g:\Omega\times R\to R$ is continuous and assume (A1), (A2) and (A5) hold. If

$$(L_f+\mu L_h+nL_I)<1 \quad (5.12)$$

where μ is given by (4.10), then ABC-FDE with impulse condition has a unique solution on Ω.

Proof: Define the non-linear mapping $B^*:PC(\Omega,R)\to PC(\Omega,R)$ as follows

$$B^*\xi(\rho) \le \xi_0 - f(\xi) + \frac{1-a}{M(a)} \sum_{0<\rho_m<\rho} h(\rho_m, \xi(\rho_m)) + \sum_{0<\rho_m<\rho} I_m \xi(\rho_m^-) + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \sum_{0<\rho_m<\rho} \int_{\rho_{m-1}}^{\rho_m} (\rho_m - \varphi)^{a-1} h(\varphi, \xi(\varphi)) d\varphi + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_m}^{\rho} (\rho - \varphi)^{a-1} |h(\varphi, \xi(\varphi))| d\varphi \quad (5.13)$$

Then B^* has a fixed point if and only if the ABC-FDE with impulse condition (1.2) has a solution. Choose

$$l^* \ge \frac{|\xi_0| + |f(0)| + \mu\Lambda_h}{1 - (L_f + \mu L_h + nL_I)}$$

From hypothesis (A5), it is simple to verify that $B^* A_{l^*} \subset A_{l^*}$,

where

$$A_{l^*} = \{\xi \in PC(\Omega, R) : \|\xi\|_{PC} \le l^*\}.$$

Next, we have to prove that B^* is a contraction.

Let $\xi, \xi^* \in PC(\Omega, R)$ and $\rho \in \Omega,$ then we have

$$\begin{aligned}\|B^*(\xi) - B^*(\xi^*)\| &= \max_{\rho\in\Omega} |B^*\xi(\rho) - B^*\xi^*(\rho)| \\ &\le \max_{\rho\in\Omega} |f(\xi(\rho)) - f(\xi^*(\rho))| + \frac{1-a}{M(a)} \max_{\rho\in\Omega} \sum_{0<\rho_m<\rho} |h(\rho_m, \xi(\rho_m)) - h(\rho_m, \xi^*(\rho_m))| \\ &+ \max_{\rho\in\Omega} \sum_{0<\rho_m<\rho} |I_m \xi(\rho_m^-) - I_m \xi^*(\rho_m^-)| \\ &+ \frac{a}{M(a)} \frac{1}{\Gamma(a)} \max_{\rho\in\Omega} \sum_{0<\rho_m<\rho} \int_{\rho_{m-1}}^{\rho_m} (\rho_m - \varphi)^{a-1} |h(\varphi, \xi(\varphi)) - h(\varphi, \xi^*(\varphi))| d\varphi\end{aligned}$$

$$\begin{aligned}\|B^*(\xi) - B^*(\xi^*)\| &\le L_f \|\xi - \xi^*\| + \frac{(1-a)^n}{M(a)} L_h \|\xi - \xi^*\| + nL_I \|\xi - \xi^*\| + \frac{\rho^a (n+1)}{M(a)} \frac{1}{\Gamma(a)} L_h \|\xi - \xi^*\| \\ &= (L_f + \mu L_h + nL_I) \|\xi - \xi^*\|\end{aligned}$$

The inequality (5.12) proves that B^* is contraction on $PC(\Omega, R)$. Then the ABC-FDE with impulse condition has a unique solution.

Theorem 5.2. Suppose $h : \Omega \times R \to R$ is continuous and let (A1) to (A5) hold. If

$$\left(L_f + nL_I + \frac{(1-a)n}{M(a)} L_h \right) < 1, \quad (5.14)$$

Then the global ABC-FDE (1.2) with impulse condition has a solution on given domain.

Proof: Let the operator $B^* : PC(\Omega, R) \to PC(\Omega, R)$ defined by (5.13).

Define the operator $B_1^* \,\&\, B_2^*$ on A_l as

$$B_1^*(\xi(\rho)) = \xi_0 - f(\xi) + \sum_{0<\rho_m<\rho} I_m \xi(\rho_m^-) + \frac{1-a}{M(a)} \sum_{0<\rho_m<\rho} h(\rho_m, \xi(\rho_m))$$

$$B_2^*(\xi(\rho)) = \frac{a}{M(a)} \frac{1}{\Gamma(a)} \sum_{0<\rho_m<\rho} \int_{\rho_{m-1}}^{\rho_m} (\rho_m - \varphi)^{a-1} h(\varphi, \xi(\varphi)) d\varphi + \frac{a}{M(a)} \frac{1}{\Gamma(a)} \int_{\rho_m}^{\rho} (\rho - \varphi)^{a-1} h(\varphi, \xi(\varphi)) d\varphi$$

where $B^* = B_1^* + B_2^*$. Let the ball $Al_0 = \{\xi \in PC(\Omega, R) : \|\xi\|_{PC} \le l_0\}$ and

$$l_0^* \ge \frac{|\xi_0| + |f(0)| + \mu\lambda_0 + nN}{1 - L_f}$$

For $\xi_1, \xi_2 \in Al_0,$ we obtain $\|B_1^*\xi_1 + B_2^*\xi_2\| \le \|B_1^*\xi_1\| + \|B_2^*\xi_2\|$

$$\begin{aligned}\|B_1^*\xi_1 + B_2^*\xi_2\| &= \max_{\rho\in\Omega} |B_1^*\xi_1(\rho)| + \max_{\rho\in\Omega} B_2^*\xi_2(\rho) \\ &\le |\xi_0| + |f(0)| + L_f \max_{\rho\in\Omega} |\xi_1(\rho)| + \max_{\rho\in\Omega} \sum_{0<\rho_m<\rho} |I_m \xi_1(\rho_m^-)| \\ &+ \frac{1-a}{M(a)} \max_{\rho\in\Omega} \sum_{0<\rho_m<\rho} |h(\rho_m, \xi_1(\rho_m))| \\ &+ \frac{a}{M(a)} \frac{1}{\Gamma(a)} \max_{\rho\in\Omega} \left\{ \sum_{0<\rho_m<\rho} \int_{\rho_{m-1}}^{\rho_m} (\rho_m - \varphi)^{a-1} h(\varphi, \xi_2(\varphi)) d\varphi + \int_{\rho_m}^{\rho} (\rho - \varphi)^{a-1} h(\varphi, \xi_2(\varphi)) d\varphi \right\}\end{aligned}$$

$$\begin{aligned}\|B_1^*\xi_1 + B_2^*\xi_2\| &\le |\xi_0| + |f(0)| + L_f l_0 + \frac{(1-a)n}{M(a)} \lambda_0 + nN + \frac{\rho^a (n+1)}{M(a)\Gamma(a)} \lambda_0 \\ &= |\xi_0| + |f(0)| + L_f l_0 + \mu\lambda_0 + nN \le l_0\end{aligned}$$

Thus $B_1^*\xi_1 + B_2^*\xi_2 \in Al_0$. For any $\rho \in \Omega \,\&\, \xi_1, \xi_2 \in PC(\Omega, R)$, we have

$$\begin{aligned}\|B_1^*\xi_1 - B_1^*\xi_2\| &= \max_{\rho\in\Omega} |B_1^*\xi_1(\rho) - B_1^*\xi_2(\rho)| \\ &\le \max_{\rho\in\Omega} |f(\xi_1(\rho)) - f(\xi_2(\rho))| + \max_{\rho\in\Omega} \sum_{0<\rho_m<\rho} |I_m \xi_1(\rho_m^-) - I_m \xi_2(\rho_m^-)| \\ &+ \frac{1-a}{M(a)} \max_{\rho\in\Omega} \sum_{0<\rho_m<\rho} |h(\rho_m, \xi_1(\rho_m)) - h(\rho_m, \xi_2(\rho_m))| \\ &\le L_f \|\xi_1 - \xi_2\| + nL_I \|\xi_1 - \xi_2\| + \frac{(1-a)n}{M(a)} L_h \|\xi_1 - \xi_2\| + \left(L_f + \frac{(1-a)n}{M(a)} L_h + nL_I \right) \|\xi_1 - \xi_2\|\end{aligned}$$

From (5.14), B_1^* is a contraction mapping.

Now let us show that B_2^* is continuous and compact. h is continuous $\Rightarrow$ B_2^* is continuous.

Also B_2^* is uniformly bounded on Al_0 because for $\xi \in Al_0 \,\&\, \rho \in \Omega$ we get

$$\begin{aligned}\|B_2^*(\xi)\| &= \max_{\rho\in\Omega} |B_2^*\xi(\rho)| \\ &\le \frac{a}{M(a)} \frac{1}{\Gamma(a)} \max_{\rho\in\Omega} \left\{ \sum_{0<\rho_m<\rho} \int_{\rho_{m-1}}^{\rho_m} (\rho_m - \varphi)^{a-1} |h(\varphi, \xi(\varphi))| d\varphi + \int_{\rho_m}^{\rho} (\rho - \varphi)^{a-1} |h(\varphi, \xi(\varphi))| d\varphi \right\} \\ &\le \frac{\rho^a}{M(a)} \frac{(n+1)}{\Gamma(a)} \lambda_0\end{aligned}$$

Now the operator B_2^* is compactness on $Al_{0,}$ since $B_2^* \subset B.$

Hence, the global ABC-FDE with impulse condition possesses a solution Ω.

6. ILLUSTRATION

For $a \in (0,1]$, consider the following impulsive ABC-FDE

$$\left.\begin{aligned} {}^{ABC}D^{a}_{[\rho]}\xi(\rho) &= \frac{\rho\left(\rho-\frac{1}{2}\right)}{9}\frac{|\xi(\rho)|}{1+|\xi(\rho)|}, \rho\in\Omega=[0,1], \rho\neq\frac{1}{2} \\ \Delta\xi|_{\rho=0.5} &= \frac{|\xi(0.5^{-})|}{10+|\xi(0.5^{-})|} \quad \& \quad \xi(0)=\xi_0 \end{aligned}\right\} \qquad (6.15)$$

Set $h(\rho,\xi)=\frac{\rho(\rho-0.5)}{9}\frac{\xi}{1+\xi}$, for $\rho\in\Omega,\ \xi\in R^{+}$ and

$\mathrm{I}_m(\xi)=\frac{\xi}{10+\xi}$ for $\xi\in R^{+}$

Clearly $h(0,\xi(\rho))=h(0.5,\xi(0.5))=0.$ Let $\rho\in\Omega$ and $\xi,\bar{\xi}\in R^{+}$. Then

$$|h(\rho,\xi)-h(\rho,\bar{\xi})| = \frac{\rho\left(\rho-\frac{1}{2}\right)}{9}\left|\frac{\xi}{1+\xi}-\frac{\bar{\xi}}{1+\bar{\xi}}\right| \le \frac{2}{36}\frac{|\xi-\bar{\xi}|}{(1+\xi)(1+\bar{\xi})} \le \frac{1}{18}|\xi-\bar{\xi}|$$

$$|I_m(\xi)-I_m(\bar{\xi})| = \left|\frac{\xi}{10+\xi}-\frac{\bar{\xi}}{10+\bar{\xi}}\right| \le \frac{10|\xi-\bar{\xi}|}{(10+\xi)(10+\bar{\xi})} \le \frac{|\xi-\bar{\xi}|}{10}$$

Also, hypothesis (A1) and (A2) hold with

$$L_h=\frac{1}{18} \text{ and } L_I=\frac{1}{10}.$$

Also verified that the condition (4.9) holds with

$$\rho=1,\ n=1,\ a=\frac{1}{2} \text{ and } \mathrm{M}(a)=1.$$

Thus, we get

$$\mu=\frac{\sqrt{3.14}+4}{2\sqrt{3.14}} \ \ and\ \ \mu L_h+nL_I\approx\frac{2}{10}.$$

By theorem 4.1, the problem (6.15) has a one-of-a-kind approach [0,1]. Also it is noted that for each $\xi\in R^{+}$ and $\rho\in[0,1]$, we have $|h(\rho,\xi)|\le\frac{\rho}{9}(\rho-0.5)$ and $|I_m(\xi)|\le 0.1.$

Hence, condition (A3) is verified with

$$\lambda(\rho)=\frac{\rho}{9}(\rho-0.5)\in C([0,1],R^{+}) \text{ and } N=0.1$$

Thus, all conditions of theorem (4.2) are satisfied. Hence theorem (4.2) implies that the given problem has at least one solution on $[0,1]$.

7. CONCLUSION

In this paper, we'll look at, we have been derived the formulae for solution for the Mittag – ABC for an impulse state fractional differential equation. Also, we established existence and uniqueness for that problem. Hence, the prevailed results take part a remarkable role in expanding fractional calculus principle analysis.

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