A Study on Development of Theory on Algebra

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Abstract – Graph Theory on Algebra is a newly emerging discipline in mathematics that has significant applications. Newline: Newline In recent decades, newline complex graphical theory of algebraic structures is being increasingly used in algebraic techniques. A newline with the interplay between algebra and the theory of graphics is a fascinating topic. Algebraic tools can be used to provide surprising newline and fine evidence of theoretical graphic facts, and numerous newline graphs are linked to interesting algebraic objects. The literature itself has grown dramatically in the field of algebraic graph theory. New line Thousands of research papers was literally published. There are currently several newline books dealing with this topic. Algebraic graph theory is a branch of newline mathematics which is used for graphic problem by algebraic methods. This contrasts with the combined or algorithmic approaches of geometry and newline. The newline graphic theory has three main branches: the use of linear algebra, the use of group theory and the investigation of newline graphic invariants. Newline: The first branch of the graphic theory of algebraics involves a newline study of graphs with linear algebra. It studies the adjacence matrix or the newline in particular A graph's laplacian matrix (this part of algebraic graph theory is also called spectral graph newline theory). Newline: The second branch of algebraic graph theory involves the investigation of graphs into group theory in connection with newline, especially auto-orophytic groups and geometric group theory. This new feature this category covers the dissertation. Newline: Finally, the third branch of algebraic graph theory deals with algebraic characteristics of newline graph invariants, and particularly the chromatic polynomials of the polynomial and the nodes of newlines. The newline number counts its proper peak colours, for example, in the chromatic polynomial of a diagram.

Keywords – Development, Theory on Algebra, Graph Theory, Mathematics, Algebraic Techniques, Geometric Group Theory

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INTRODUCTION

The literature of algebraic graph theory itself has grown enormously. Literally thousands of research papers have appeared. Several books now cover aspects of this topic. Norman Biggs is a normal book in this area for introduction to algebraic graph theory [1993] Biggs. The Godsil and Royle books [2001] and Cvetkovic and al. [1980] also contain huge amounts of data. Algebraic graphs are graphs made of algebraic structures, for example groups and rings. The newly developed algebraic diagrams include Order Prime Graph Rajendra et al. [2016], Coprime graph Ma et al. [2014]. Noncyclic Graph Abdollahi and Hassanabadi [2007]. Colored commutative rings Lucido [1999] Zero-dividing charts, regular rings by Neumann, and boolean algebras Beck (1988) Anderson et al. [2003], The Cyclic Graph of a Finite group, Non-commuting Graph of a Group of Abdollahi et al. Ma et al. [2013], Finite Abelian Groups Zelinka and Liberec intersection chart [1975], Cayley Graph and Center graph subgroup, Complementary chart. All these graphs are made up of groups and rings whose vertices are the group and ring elements and the rings they, and certain properties of the pair of elements have been taken into account. This approach to algebraic graphics simplifies the complex stuff. This is a more complicated group structure based on binary functions, which is converted into relatively simple graphs based on the element relationships. The group properties are discussed through the charts when the graphs are made from groups. This approach also focuses on how much information you can obtain from the built graphs about the original group without losing a great deal of data when converting.

A well-known algebraic graph is the Cayley graph. The Cayley Graph concept plays a key role in addressing certain optimization problems, in particular routing problems in parallel computer interconnection networks. In the development of the modern science and engineering processing and supercomputing continue to have a significant impact. In facilitating communication between processors in a computer system, the network of processors plays a vital role. The Rings, Torso and Hypercubes are among the popular connection systems. Their popularity derives from the availability of these architectures for commercial applications. These three graph families – ring, torus and hybercube – share the common characteristic of Cayley's graphic design. Cayley graphs have modeled many important network problems. Given the property of vertex transitivity, Cayley graphs allow routing and communication systems to be implemented at each network node. The identification of ideal dominant sets in Cayley graphs is one of the key problems regarding routing problems. It helps to identify optimum substructures so that communication between processors can be facilitated.

Cayley graph is a discrete structure created out of groups, more specifically from a finite group G and its generating set Ω. A non-empty subset Ω ⊂ G is called a generating set for G, denoted by G =< Ω >, if every element of G can be expressed as a product of elements in Ω. For a generating set Ω of G, we assume that: C1: The identity element e /∈ Ω and C2: If a ∈ Ω, then a −1 ∈ Ω. Given the pair (G, Ω), such that G =< Ω > and Ω satisfies the two conditions C1 and C2, define a Cayley graph Γ = (V, E) corresponding to (G, Ω), as V (Γ) = G and E(Γ) = {(x, y)a|x, y ∈ G, a ∈ Ω and y = xa} and it is denoted by Cay(G, Ω). When G = Zn =< Ω >, it is called as circulant graph and denoted by Cir(n, Ω).

In the last three decades, the study of algebraic structures using the graphic properties has become an interesting research topic that leads to many fascinating results and questions. There are many papers on the algebraic properties of ring or group by assigning a graph to the ring or group. In Sattanathan and Kala[2009], the concept of the order primary graph has been introduced. Relative primary integrals play an important role, especially in algebras and numbers theory, in various branches of mathematics. The primary order graph is a graph built from groups that take into account the relative primary property of its elements. The order prime graph of a group Γ is a graph with V (op(Γ))= Γ and two vertices a and b are adjacent in order prime graph of a group if and only if (o(a), o(b))= 1.

Complementary concept of subgroup Chelvam and Sattanathan [2010] introduced the Cayley graph. That all is a group, and all are sub-groups — all of which then are known as a cayley graph (all of which is − all of which are H). Another concept is the center graph of a group where a graph is assigned to the group and the algebraic properties are investigated using a corresponding graph. In P. Balakrishnan [2011] and Balakrishnan et al. [2011a], the concept of the center graph of a group is presented. It is a diagram with vertices that adjacent to group elements and two distinct vertices only when their product is in the middle of the group.

Definition 1. The cyclic subgroup graph Γz(G) for a finite group G is a simple undirected graph in which the cyclic subgroups are vertices and two distinct subgroups are adjacent if one of them is a subset of the other. Few examples for cyclic subgroup graph are discussed in the following paragraphs. Consider the finite group $Z_2 \oplus Z_2$. The cyclic subgroups of this group $Z_2 \oplus Z_2$ are

$$v_1 = \langle(0,0)\rangle = \{(0,0)\}$$

$$v_2 = \langle(1,0)\rangle = \{(1,0),(0,0)\}$$

$$v_3 = \langle(1,1)\rangle = \{(1,1),(0,0)\}$$

$$v_4 = \langle(0,1)\rangle = \{(0,1),(0,0)\}$$

From the set relations $\langle(0,0)\rangle$ is a subset of all other cyclic groups and other three cyclic groups $\langle(1,0)\rangle, \langle(1,1)\rangle$ and $\langle(0,1)\rangle$ are not comparable. So v_1 is adjacent to v_2, v_3 and v_4 as shown in figure 1

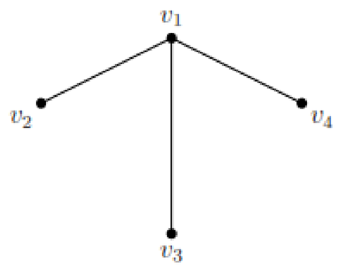

Figure 1: The Cyclic Subgroup Graph of $Z_2 \oplus Z_2$

The Figure 2 illustrates the cyclic subgroup graph of $Z_3 \oplus Z_3$.

$$Z_3 \oplus Z_3 = \{0,1,2\} \oplus \{0,1,2\}$$
$$= \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)\}$$

It's cyclic subgroups are as follows

$$\langle(0,0)\rangle = \{(0,0)\}$$

$$\langle(0,1)\rangle = \{(0,1),(0,2),(0,0)\}$$

$$\langle(0,2)\rangle = \{(0,2),(0,1),(0,0)\}$$

$$\langle(1,0)\rangle = \{(1,0),(2,0),(0,0)\}$$

$$\langle(1,1)\rangle = \{(1,1),(2,2),(0,0)\}$$

$$\langle(1,2)\rangle = \{(1,2),(2,1),(0,0)\}$$

$$\langle(2,0)\rangle = \{(2,0),(1,0),(0,0)\}$$

$$\langle(2,1)\rangle = \{(2,1),(1,2),(0,0)\}$$

$$\langle(2,2)\rangle = \{(2,2),(1,1),(0,0)\}$$

For simplicity of notation take

$$\langle(0,1)\rangle = \langle(0,2)\rangle = v_1$$
$$\langle(1,0)\rangle = \langle(2,0)\rangle = v_2$$
$$\langle(1,1)\rangle = \langle(2,2)\rangle = v_3$$
$$\langle(1,2)\rangle = \langle(2,1)\rangle = v_4$$
$$\langle(0,0)\rangle = v_5$$

In this example the subgroup generated by (0, 0) is a subset of every other subgroups. Therefore the subgroup generated by (0, 0) is adjacent with the subgroups generated by (0, 1), (1, 0), (1, 1), (1, 2).

So v5 is adjacent with v1, v2, v3, v4.

Then from the set theory relations we get the following graph shown in figure 2 as the cyclic subgroup graph of Z3 ⊕ Z3.

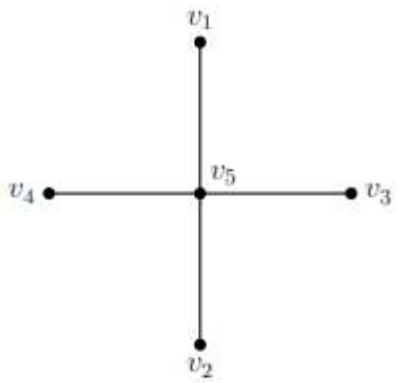

Figure 2: The Cyclic Subgroup Graph of Z3 ⊕ Z3

In this chapter few parameters in cyclic subgroup graph of a finite group such as independence number Allan and Lasker [1978], chromatic number, matching number, restrained domination number, restrained triple connected domination number, strong domination number, strong triple connected domination number, 2-domination number, triple connected 2-domination number, perfect domination number and 2-connected domination number are studied. Also the independence number of a power graph Chakrabarty et al. [2009] Cameron and Ghosh [2011] and the independence number of cyclic subgroup graph are found to be same.

PERCEPTUAL LEVEL IN THE HISTORY OF ALGEBRA

Perceptional representation related with pre-conscious and conscious levels of thinking are taken into consideration in this study. The first level could be considered as having both a pre-conscious and a conscious part to it. Vision gives the idea of chief sensory apparatus in mathematics. As the senses analyses the discrete signals going into the brain, the brain in turn integrates or synthesises the sensations into percepts or unified images of the images acquired. Consequently one unified percept (for example, a triangle) is consciously seen. Perception can define as an act of identification or categorisation, that is, an event in which a simple or complex array is identified as a member of some meaningful category on the basis of characteristics it shares with other members of that category. These concepts or conceptual categories represent the common characteristics of objects be in the first level could be considered as having both a pre-conscious and a conscious part to it. Vision gives the idea of chief sensory apparatus in mathematics. As the senses analyses the discrete signals going into the brain, the brain in turn integrates or synthesises the sensations into percepts or unified images of the images acquired. Consequently one unified percept (for example, a triangle) is consciously seen. Perception can define as an act of identification or categorisation, that is, an event in which a simple or complex array is identified as a member of some meaningful category on the basis of characteristics it shares with other members of that category. The simplest concepts, such as the concept of a triangle, lie very close to a percept. However, the more complex ones, such as a torsion-free abelian group, are not. This suggests the helical idea of concept formation. As one proceeds up the helical, more and more complex concepts are encountered. Concepts lie between percepts and linguistic representations or abstractions. The latter involves axioms and definitions. These in turn lead to theorems at different stages of the helical. The lack of concept acquisition linking percepts to abstractions can often lead to problems of understanding. Moving from level 1 to 2 and level 2 to 3 involves moving from iconic to symbolic, concrete to abstract, picture-like to linguistic or simple to complex. Concepts form the link between iconic representations or percepts and linguistic representations or abstractions. The rise from level 2 to level 3 is important because while a student is still at the level of images or intuitive notions, he/she does not consult definitions which are necessary for further advancement up the helical. The word “percept” means the product of being made aware of something by one of the senses and “concept” means an “idea of class of objects; general notion”. The noun “abstraction” means “withdrawal” and is a derivation of the word “abstract” meaning an “essence” or “summary”. In Latin “ab” means “from” and “tracto” means “I haul” and so literally abstraction means what has been hauled, pulled or drawn out of something. Since this gives a concise description of activities associated with the third level, the terms percepts, concepts and abstractions are being used here to describe the three levels of the helical.

For sublevels of first level of thought, we may take the learning of complete mathematical induction as an illustration. Initially, at the perceptual level, students should be provided with examples which compel them to invent complete induction. As a result of these examples, they come to recognise the common principle. The examples provided should be non-trivial or at least non-trivial looking types.

For example

$$1^2 = 1$$
$$2^2 = 1 + 3$$
$$3^2 = 1 + 3 + 5$$
$$n^2 = \sum_{k=1}^{n} (2k - 1)$$

It would serve as an example of the initial perceptual level. Further examples would be suitable at the conceptual level. Binomial coefficients and the binomial theorem would be examples suggesting structure at the abstract level. However, textbooks often present the binomial theorem as a consequence of complete induction which forms a "vicious circle" in mathematical invention21. Furthermore, to deduce complete induction from Peano's axioms and then apply it to various examples would be the decisive experience that leads to this principle.

Conceptual level as an attribute of mathematical concepts

Representations and symbol systems are fundamental to mathematics as a discipline since mathematics is "inherently representational in its intentions and methods". Vergnaud (1997) suggested viewing representation as an attribute of mathematical concepts, which are defined by three variables:

i) The situation that makes the concept useful and meaningful,

ii) The operation that can be used to deal with the situation, and

iii) The set of symbolic, linguistic, and graphic representation that can be used to represent situations and procedures.

Several ideas related to the concept of representation are pertinent to this research. The first and the foremost is that external systems of representation and internal systems of representation and their interaction are essential to mathematics teaching and learning. Internal representations are usually associated with mental images individuals create in their minds. Bruner (1966) proposed to distinguish three different modes of mental representation

i) The sensory- motor (physical action upon objects),

ii) The iconic (creating mental images) and

iii) The symbolic (mathematical language and symbols).

Estes (1996) posited that internal representation and categorization are the attributes of high-order human cognitive processes; both involve abstraction to represent the entity of the object of communication. Matsuka and Sakamoto (2007) suggested that "By compressing the vast amount of available information, a cognitive process called categorization allows us to process, understand, and communicate complex thoughts and ideas by efficiently utilizing salient and relevant information while ignoring other types". Pape and Tchoshanov (2001) described mathematics representation as an internal abstraction of mathematical ideas or cognitive schemata, that according to Hiebert and Carpenter (1992) the learner constructs to establish internal mental network or representational system. Thus, one can assert that internal representation, categorization and abstraction are closely related mental constructs. External representations are usually associated with the "knowledge and structure in the environment, as physical symbols, objects, or dimensions" as well as "external rules, constraints, or relations embedded in physical configurations". Goldin and Shteingold (2001) suggested that an external representation "is typically a sign or a configuration or signs, characters, or objects" and that external representation can symbolize "something other than itself. Most of the external representations in mathematics are conventional; they are objectively determined, defined and accepted.

THE ABSTRACT LEVEL IN THE HISTORY OF ALGEBRA

The term 'abstraction' has been used to describe both the cognitive process of isolating, or 'abstracting' a common feature or relationship observed in a number of things, and the product of such a process. The distinction in meanings is usually ensured by the situated context in which the word 'abstraction' is used. Czemecka (2006) uses the term abstraction to describe the method for constructing the object of intellectual cognition in general. Abstraction as a mental action separates a property or a characteristic of an object from the object to which it belongs or is linked to and forms a cognitive image or a concept (an abstraction) of the object. Thus, abstraction can be understood as a mental process that promotes the basis of thoughts that allow one to reason. Abstract objects are defined as those that lack certain features possessed by concrete things. The Oxford Desk Dictionary and Thesaurus (1997) suggests that "abstract is what exists in thought or in theory and not in matter or in practice." Turchin (1997) posits that 'abstractness' is a concept in which one does not take into account a specific value or characteristic of the object in consideration, but any of all possible values and characteristics related to the object. The concept of abstraction in the field of mathematics education research has been examined from different perspectives. There is an agreement that mathematics students are continuously involved in the process of abstraction because they are engaged in transformation of their perceptions into mental images by means of different representations. The following notions are essential to examine the processes of transforming prior mental images and developing conceptual understanding:

i) The notion of the degree of abstraction

ii) The notion of adaptation to abstraction and

iii) The notion of reducing level of abstraction

Cifarelli's (1988) proposed the levels of reflective abstraction to describe a college students' learning process while they solving algebra word problems. These levels include recognition, representation, structural abstraction, and structural awareness. At the highest level, structural awareness, the student is able to consider problem structures and operate upon the mas objects. Skemp (1986), Heibert and Lefevre (1986), and Wilensky (1991) argued that the degree of abstraction is a variable that depends on the student's prior knowledge and subjective way of integrating past experience with new information. If conceptual understanding is defined by the degree of abstraction, then the idea of adaptation to abstraction becomes critical, and the process of building mathematics conceptual understanding can be viewed as a transition between the levels of abstraction from lower to higher. Hazzan and Zazkis (2005) assert that certain types of concepts are more abstract than others, and that the ability to abstract is an important skill for a meaningful learning of mathematics. Hazzan (1991) posits that the growth in conceptual understanding is manifested by the increased ability to "cope with" a higher degree of abstraction. To describe learners' behaviors in terms of coping with levels of abstraction, Hazzan (1999) introduced a theoretical framework of reducing level of abstraction. It refers to situations in which students are unable to manipulate the concepts at the level presented in a given problem and therefore, they reduce the level of abstraction of the concepts involved to make these concepts mentally accessible. The transition between the levels of abstractions can be illustrated by the following example: during the process of counting physical objects one abstracts from the properties of these objects and uses linguistic objects or linguistic abstraction, i.e., words to represent the quantity of the physical objects. Next, one uses the number symbol to represent the word that in turn represents the quantity of the physical objects. In algebra, one abstracts from number symbols and uses x to represent all the possible numbers. The following levels of abstraction provide the means to view the process of developing conceptual understanding in algebra. Assume that operating on 'number words' which represent certain quantities of real objects is a first level of abstraction. Operating with 'number symbols' can be thought as the second level of abstraction, and operating on letters that's t and for 'number symbols' can be viewed as the third level of abstraction. Thus, one can assert that abstraction in mathematics is an activity of integrating pieces of information of previously constructed mathematics knowledge and reorganizing them into anew mathematics structure or a new hierarchy. For example, a number line can be viewed as a set of one-unit segments joined together by their ends sequentially. It is also a visual representation of the one-to-one correspondence concept where each point on a number line corresponds to a unique real number and vice-versa. Thus, a single segment can be used to represent 1 (fixed number or quantity), as well as ' 1 ' can be used to represent a line segment with the length of one unit. Two segments jointed together can represent number 2, etc.1 unit one unit (or just 1) Two units (or just 2) Then the sum of two numbers ' 1 ' and'2' can be represented as a line segment which consists of three unit segments2 units Moving to the next level of abstraction, a single segment can represent a fixed quantity which is unknown then two segments of the same length joined together will represent the sum of the two fixed quantities or two unknowns. Then the sum of one unknown and two unknowns can be described as 3 unknowns. This example shows the transition from concrete (number system, pictorial aids) to abstract (algebraic symbols). On the other end, when the students are facing with the problem of collecting like terms (e.g., X and 2x) where x represents a variable, they might experience a need to reduce the level of abstraction and to think in numbers and in pictures (e.g., line segments).They can be encouraged to manipulate line segments and numbers (act upon the objects) to find the sum of x and 2x as the length of the integral segment and then translate the length of the segment into symbols. Any schematization has its natural limitations. As Raymond Nickerson (1986) noted, "Taxonomies are, at best, convenient ways of organizing ideas and should never be taken very seriously. The world seldom is quite as simply divisible into neat compartments as our penchant for partitioning it conceptually would suggest". Nevertheless, it is useful to organize ideas and use pictorial representations for communication purposes. In this connection, it is important to recognize that a line segment representation of a number or variable as any external representation defined by Zhang (1997), provides only certain information, and "stresses some aspects and hide others"29, thus limited in certain ways. Yet, this representation might be sufficient to use in the process of building the concept of operations with unknown and variable. The need for pictorial representation might become obsolete as the 'object conception' of unknown is formed and developed to the degree of abstraction that (lower level) images are no longer needed to consciously manipulate letters. Learning algebra, students develop the mental abilities that in helder and Piaget (1958) cuddled formal operations. These mental abilities enable the student to deal with highest level of abstraction, i.e., algebraic symbol system. Those students who have not developed formal operations and struggle when dealing with the algebra symbol systems, try to 'reduce' the level of abstraction given by the problem (for example, to solve the equation 2x + 4 =15) to the lower level on which they can operate, i.e., 'number symbols.' To cope with the problem, these students use a trial and error method replacing a letter (x) with numbers until they find the solution. Drawn from Piaget's (1970) idea that children first learn about an object by acting upon it and through interaction they eventually understand its nature, theories30distinguish between a process conception or operational conception and an object conception or structural conception of mathematical principles and notions, and agree that when a mathematical concept is learned, its conception as a process precedes its conception as an object. These theories also suggest that the process conception is less abstract than an objectonception. One may conclude that the process conception of a mathematical concept can be interpreted as being on a lower (reduced) level of abstraction than its conception as an object. When students show a tendency to reduce the level of abstraction and work on a lower level of abstraction, it might be hypothesized that although they demonstrate a certain level of process conception, they have not yet developed conceptual understanding. It seems plausible to assume that every algebra student goes through the process of familiarization with and adaptation to different levels of abstraction. It also seems credible to believe that students are familiarizing and adapting to abstraction at different rate. Wilensky (1991) suggested that the higher the rate of adaptation to abstraction the less the need for reducing the level of abstraction. In this sense, the process of adaptation to abstraction might involve certain behavior manifested in coping with level of abstraction. In other words, when students are unable to manipulate with the level of abstraction (words, numbers, symbols) presented in a given problem, they consciously or unconsciously reduce the level of abstraction of the concepts involved to make these concepts within the reach of their actual mental stage of development. The above overview of the ideas and assumptions about representations, abstraction and conceptual understanding provided reasonable and sufficient basis for developing the study that offered another perspective on the process of assessing algebra students' conceptual understanding of linear relationship with one unknown.

CONCLUSION

In this study algebraic graphs are constructed and their algebraic properties are studied in terms of graph theoretic parameters. The algebraic graph is cyclic subgroup graph which is constructed from a finite group. In a finite group finite numbers of subgroups are there. These subgroups are taken as vertices and are made adjacent if one of them is a subset of the other. For each finite group a graph is obtained but these graphs may not be distinct. Some non-isomorphic finite groups give isomorphic cyclic subgroup graphs. For example both Z4 and Z9 has K3 as cyclic subgroup graph. This cyclic subgroup graph is related with various parameters especially power graph. For power graph and cyclic subgroup graph constructed from a same group has their independent numbers are same. The problem of finding chromatic number of this graph for finite cyclic group is settled completely. For finite cyclic groups this graph is found to be Hamiltonian always. The matching number of cyclic subgroup for finite cyclic group is determined. Some domination parameters are determined from this graph. These domination parameters including strong domination, strong triple conected domination, perfect domination, triple connected domination, restrained domination, restrained triple connected domination, two domination, two connected domination, two triple connected domination are all determined completely for cyclic subgroup graph of any finite groups. There are lot of open problems available in this research area. Chromatic number, Hamiltonicity and matching number can all be found for other non-cyclic groups such as abelian, dihedral and symmetric groups as an extension work. Another algebraic graph is formed from finite group named as cyclic graph. In cyclic subgroup graph subgroups are taken as vertices but in cyclic graph each group elements are graph vertices and adjacancy is made for elements of same cyclic subgroup. For finite cyclic groups obtained cyclic graph is complete. And for non-cyclic groups it’s never complete.

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