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Bulk Viscous String Cosmological Model With and Without Magnetic Field

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Abstract – The present paper provides solution for Bianchi type-III string cosmological model is presence of bulk viscosity with and without magnetic field using different conditions like equation of state $\rho = A\epsilon^n + B$ and an assumption that the scalar of expansion is proportional to the shear scalar $\rho \propto \theta$ which leads to the relation between metric potentials $\beta = C + D\gamma^n$ where to A, B, C, D and n are constants. The physical and geometrical aspects of the models are also discussed in presence and absence of magnetic field.

Key Words – Viscosity, Expansion, Shear Scalar, Metric, Magnetic Field, Strings.

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1. INTRODUCTION:

Tikekar and Patel [12] and Chackraborty and Chackraborty [6] have presented the exact solutions of Bioanchi type-III and spherically symmetric cosmology respectively for a cloud string. It is well known that in an early stage of universe when neutrino decoupling occurs, the matter behaves like a viscous fluid. The cosmological models of a fluid with viscosity play a significant role in study the evolution of the universe. Recently, string cosmological models of Bianchi type I, II, III with bulk viscosity have been discussed by several authors [3, 13-17]. The magnetic field has an important role at the cosmological scale and is present in galactic and intergalactic spaces. The importance of the magnetic field for various astrophysical phenomena has been studied in many papers. Melvin [9] has pointed out that during the evolution of the universe, the matter was in a highly ionized state and its smoothly coupled with the field, subsequently forming neutral matter as a result of universe expansion. Therefore the possibility of the presence of magnetic field in the cloud string universe is not unrealistic and has been investigated by many authors [7, 10, 15]. Some other workers in this field are Bahera [2], Bali and Dave [4]. Ball et al. [5], Kibble [8], Takabayski [11], Yadav at. al [18], Zimadahl [22], Yadav and Kumar [19], Zeldovich et. al. [20, 21]. Tyagi and Sharma [23], Sharma et al. [24] and Roy et. al. [25].

In this paper we have studied Bianchi type-III string cosmological model in the presence of bulk viscosity with and without magnetic field. To obtain an exact solution (we have used different craditionsive and equation of state $\rho = A\epsilon^n + B$ and an assumption that the scalar of expansion is proportional to the shear scalar $\sigma \propto \theta$, which leads to the relation between metric potential $\beta = C + D\gamma^n$. The physical and geometric aspects of the models absence of magnetic field are also discussed in presence and absence of magnetic field.

2. THE FIELD EQUATIONS

The Bianchi type-III space-time metric we considered here is [16].

(2.1) $$ds^2 = -dt^2 + \alpha^2 dx^2 + \beta^2 e^{2x} dy^2 + \gamma^2 dz^2$$

where □, □, and □ are the functions of time t only.

The energy-momentum tensor for a cloud of string with bulk viscosity and magnetic field [15].

(2.2) $$J_{ij} - \rho u_i u_j - \lambda \chi_i \chi_j - \xi\theta(u_i u_j + g_{ji}) + E_{ij}$$

where $\rho = \rho_p + \in$, is the rest energy density of the cloud of strings with particles attached to them, ρ_p is the rest energy density of particle $\in$ is the tension density of the cloud of string, $\theta = u^i_{;i}$, is the scalar of expansion, and ξ is the coefficient of bulk viscosity. According to Letelier [8a] the energy density for the coupled system □ and $□_p$ is is restricted to be positive, while the tension density $\in$ may be positive or negative. The vector u^i describes the cloud four-velocity and χ^i represents a direction of anisotropy, i.e., the direction of string. They satisfy the standard relation [8(a)].

(2.3) $$u^i u_j = \chi^i \chi_j = -1,\ u^i \chi_i = 0$$

E_{ij} is the energy-momentum tensor for the magnetic field.

(2.4) $$E_{ij} = \frac{1}{4\pi} g^{hk} F_{ih} F_{jk} - \frac{1}{4} g_{ij} F_{hk} F^{hk}$$

Where F_{ij} is the electromagnetic field tensor, which satisfies the Maxwell equations.

(2.5) $$F_{[ij;h]} = 0,\ \left(F^{ij}\sqrt{-g}\right)_{;j} = 0$$

Einstein's equation we consider here is

(2.6) $$R_{ij} - \frac{1}{2} R g_{ij} = T_{ij}$$

Where we have choose the units such that c = 1 and 8□G = 1. In the co-moving coordinates $u^i = \delta^i_0$ and $u^i = -\delta^0_i$, and the incident magnetic field taken along the z-axis, with the help of Maxwell equation [2.5], the only non-vanishing component of F_{ij} is [15].

(2.7) F_{12} = constant = H

The Einstein equation (2.6) for the metric (2.1) can be written as following system of equations [15, 16]

(2.8) $$\frac{\ddot\beta}{\beta} + \frac{\ddot\gamma}{\gamma} + \frac{\dot\beta\dot\gamma}{\beta\gamma} = \xi\theta - \frac{H^2}{8\pi\alpha^2\beta^2 e^{2x}}$$

(2.9) $$\frac{\ddot\alpha}{\alpha} + \frac{\ddot\gamma}{\gamma} + \frac{\dot\alpha\dot\gamma}{\alpha\gamma} = \xi\theta - \frac{H^2}{8\pi\alpha^2\beta^2 e^{2x}}$$

(2.10) $$\frac{\ddot\alpha}{\alpha} + \frac{\ddot\beta}{\beta} + \frac{\dot\alpha\dot\beta}{\alpha\beta} = -\frac{1}{\alpha^2} = \in + \xi\theta + \frac{H^2}{8\pi\alpha^2\beta^2 e^{2x}}$$

(2.11) $$\frac{\dot\alpha\dot\beta}{\alpha\beta} + \frac{\dot\beta\dot\gamma}{\alpha\gamma} + \frac{\dot\alpha\dot\gamma}{\alpha\gamma} - \frac{1}{\alpha^2} = \rho + \frac{H^2}{8\pi\alpha^2\beta^2 e^{2x}}$$

(2.12) $$\frac{\dot\alpha}{\alpha} - \frac{\dot\beta}{\beta} = 0$$

Where the dot denotes the differentiation with respect to time t. From Eq. (2.12), we have

(2.13) $$\alpha = \mu\beta$$

where □ is the constant of integration. In order to obtain a more general solution, we assume Takabayasl's equation of state [11].

(2.14) $$\rho = A \in^n + B$$

where A, B and n are +ve constants

The expression for scalar of expansion and shear scalar are

(2.15) $$\theta = u^i_{;i} = \frac{\dot\alpha}{\alpha} + \frac{\dot\beta}{\beta} + \frac{\dot\gamma}{\gamma}$$

(2.16) $$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$$

$$= \frac{1}{3}\left(\frac{\dot\alpha^2}{\alpha^2} + \frac{\dot\beta^2}{\beta^2} + \frac{\dot\gamma^2}{\gamma^2} - \frac{\dot\alpha\dot\beta}{\alpha\beta} - \frac{\dot\beta\dot\gamma}{\beta\gamma} - \frac{\dot\alpha\dot\gamma}{\alpha\gamma}\right)$$

3. SOLUTION OF FIELD EQUATIONS

We see that the five independent equation (2.9). (2.12) and (2.14) connecting six unknown variable $(\alpha,\beta,\gamma,\in,\rho,\xi)$. Thus, one more relation connecting these variables is needed to solve these equations. In order to obtain explicit solutions, one additional relation is needed and we adopt an assumption that the shear scalar of expansion is proportional to the shear scalar of expansion $\sigma \propto \theta$, which leads to

(3.1) $$\beta = C + Dr^N$$

where a, b and □ is a constant.

Now we consider B = C and D = 1 = n, the equation (2.14) and (3.1) reduces to

(3.2) $$\rho = A \in$$

(3.3) $$\beta = \gamma^N$$

From equation (2.9), (2.10), (2.11), with the help of eq. (3.2) eliminating □, □ and $\xi\theta$, we obtain

(3.4) $$A\frac{\ddot\alpha}{\alpha} - A\frac{\ddot\gamma}{\gamma} + (A-1)\frac{\dot\alpha\dot\beta}{\alpha\beta} - (A+1)\frac{\dot\beta\dot\gamma}{\beta\gamma} - \frac{\dot\alpha\dot\gamma}{\alpha\gamma} = (A-1)$$

$$\frac{1}{\alpha^2} + \frac{H^2}{8\pi}\frac{(2K^A - 1)}{\alpha^2\beta^2 e^{2x}}$$

Substituting Equation (2.13) and (2.3) into eq. (2.4), we have

(3.5) $$\ddot{\gamma}+\frac{N2A(N-1)(N+2)}{A(N-1)}\frac{\dot{\gamma}^2}{\gamma^2}-\frac{(A-1)}{A(N-1)}\gamma^{-(2N-1)}$$

$$+\frac{H^2}{8\pi}\frac{(2A-1)}{A(N-1)\mu^2e^{2x}}\gamma^{-(4N-1)}$$

To solve Eq. (3.5), we denote $\ddot{\gamma}=\phi$, then $\ddot{\gamma}=\phi\frac{d\phi}{d\gamma}$ and the eq. (3.5) can be reduced to the following form

(3.6) $$\phi\frac{d\phi}{d\gamma}+\frac{\eta\phi^2}{\gamma}=\frac{(A-1)}{A(N-1)\mu^2}\gamma^{-(N-1)}+\frac{(2A-1)M}{A(N-D\mu^2e^{2x}}\gamma^{-(4N-1)}$$

where

(3.7) $$\eta=\frac{N[2A(N-1)-(N+2)]}{a(N-1)}$$

(3.8) $$M=\frac{H^2}{8\pi}$$

Equation (3.6) can be written as

(3.9) $$\frac{d}{d\gamma}(\phi^2\gamma^{2\eta})=\frac{2(A-1)}{A(N-1)\mu^2}\gamma^{2\eta-(2N-1)}$$

$$+\frac{2(2a-1)M}{A(N-1)\mu^2e^{2x}}\gamma^{2\eta-(4N-1)}$$

Therefore, we get

(3.10) $$dt=\left[\frac{(A-1)}{\{A(N^2-1)-(N+1)^2-1\}\mu^2}\right.$$

$$\left.+\frac{(2A-1)M}{\{A(N-1)-(N+1)^2-1\}\mu^2e^{2x}}\gamma^{-(4N-2)}+K\gamma^{-2\eta}\right]^{-\frac{1}{2}}d\gamma$$

where K is the constant of integration. For this solution, the geometry of the universe is described by the metric.

(3.11) $$dt^2=-\left[\frac{(A-1)}{\{A(N^2-1)-(N+1)^2-1\}\mu^2}\gamma^{-2(N-1)}\right.$$

$$\left.+\frac{(2A-1)M}{\{A(N-1)-(N+1)^2-1\}\mu^2e^{2x}}\gamma^{-(4N-2)}+K\gamma^{-2\eta}\right]^{-1}d\gamma^2$$

$$+\mu^2\gamma^{2N}dx^2+\gamma^{2N}e^{2x}dy^2+\gamma^2dz^2$$

Under suitable transformation of coordinates, Eq. (3.11) reduces to

(3.12) $$dt^2=-\left[\frac{(A-1)}{\{A(N^2-1)-(N+1)^2-1\}\mu^2}T^{-2(N-1)}\right.$$

$$\left.+\frac{(2A-1)M}{\{A(N-1)-(N+1)^2-1\}\mu^2e^{2x}}T^{-(4N-2)}+KT^{-2\eta}\right]^{-1}dT^2$$

$$+\mu^2T^{2N}dx^2+T^{2N}e^{2x}dy^2+T^2dz^2$$

The expressions for the energy density □, the string tension density ∈, the particle density □p the coefficient of bulk viscosity ξ, the scalar of expression □ and the shear scalar □^{2} for (3.12) are given by

(3.13) $$\rho=\frac{A(2N+1)}{\{A(N^2-1)-(N+1)^2-1\}\mu^2}T^{-2N}$$

$$+\frac{\{2AN^2+3A(N-1)+3A+3\}M}{\{A(N-1)-(N+1)^2-1\}\mu^2e^{2x}}T^{-4N}$$

$$+N(N+2)KT^{-2(\eta+1)}$$

(3.14) $$\in=\frac{\rho}{A}$$

(3.15) $$\rho_p=\rho-\in=\left(1-\frac{1}{A}\right)\rho$$

(3.16) $$\xi\theta=\frac{(A-1)}{\{A(N^2-1)-(NH)^2-1\}\mu^2}$$

$$+\frac{(N^2+N-1)(1-2A)}{\{A(N-1)-(N+1)^2-1\}\mu^2e^{2x}}$$

$$+\frac{\{N(N+1)-AN(N-1)\}(N+2)}{A(N-1)}KT^{-(2\eta+2)}$$

(3.17)

$$\theta = (2N+1).\left[\frac{(A-1)}{\{A(N^2-1)-(NH)^2-1\}\mu^2} T^{-2N} + \frac{(2A-1)M}{\{A(N-1)-(N+1)^2-1\}\mu^2 e^{2x}}\right]$$

(3.18)

$$\sigma^2 = \frac{(N-1)^2}{3}.\left[\frac{(A-1)}{\{A(N^2-1)-(NH)^2-1\}\mu^2} T^{-2N} + \frac{(2A-1)M}{\{A(N-1)-(N+1)^2-1\}\mu^2 e^{2x}}\right]$$

4. CONCLUSION AND DISCUSSION

In this paper, we have studied Bianchi type-III string cosmological model in the presence of bulk viscosity and magnetic field. To obtain an exact solution, an equation of state □ = AE + B and an assumption that the scalar of expansion is proportional to the shear scalar $\sigma \propto \theta$, which leads to the relation between metric potential $\beta = C + D\gamma^{2N}$. Then the cosmological model for a cosmic string with bulk viscosity and magnetic field is obtained. The physical and geometric aspects of the model in the presence and absence of magnetic field are also discussed. Out model describes a shearing non-rotating continuously expanding universe with a big-bang start. In the absence of magnetic field it reduces to the string model with bulk viscosity.

From Equations (3.13) and (3.15) it is observed that the energy condition □ ≥ 0 and □ > 0 are fulfilled, provided.

$$K \geq 0, N > 1 \text{ and } A > N(N+2)/(N-1)$$

Or $K \geq 0, N > 1 \text{ and } A < -1/(N(2N+3)$

When K ≥ 0, N > 1 and A > N(N+2)/(N–1), the string tension density $\in > 0$; however, $\in < 0$ when K ≥ 0, N > 1 and A < –1/N(2N+3)

The above expression (3.13) – (3.16) indicate that the magnetic field is related with ρ, λ, ρ_p and ξ. Here a term of M is involved in the expression for $\rho, \lambda, \rho_p, \xi, \theta$ and σ^2 respectively, and it represents the effect of magnetic field on the model.

It is seen that in the case N > 1, whether A > N(N+2)/(N–1) or A < –1/(N(2N+3), we have □ + 1 > 0. Hence equation (3.17) shows that the scalar of expansion □ tends to infinitely large with $J \to 0$, but $\theta \to 0$ or tends to finite when $T \to \infty$. The energy density □ tends to finite when $T \to \infty$ and $\rho \to \infty$ when $T \to 0$, therefore the model describes a shearing non rotating expanding universe with the big-bang start. We can see from the above discussion that the bulk viscosity plays a significant role in the evolution of universe [1, 8(b)].

Furthermore, since $\lim_{J\to\infty} \frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large values of T.

In the special case A = 1, the model represents a geometric string model [17]. In the absence of magnetic field M = 0, the metric (3.12) reduces to the string model with bulk viscosity i.e.,

(4.1)

$$ds^2 = -\left[\frac{(A-1)}{\{A(N^2-1)-(N+1)^2-1\}\mu^2} T^{-2(N-1)+KJ^{-2\eta}}\right]^{-1} dT^2$$

$$+\mu^2 T^{2N} dx^2 + T^{2N} e^{2x} dy^2 + T^2 dz^2$$

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