Journal of Advances and Scholarly Researches in Allied Education
Vol. 17, Issue No. 1, April-2020, ISSN 2230-7540

Study of Some Useful Properties of a Matrix and its Characteristics

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Abstract - When we make an array with only the real constants of a linear equation or a system of linear equations, we get a matrix. There are various types of matrices that are identified and defined based on their structure, which includes elements in various ways and in various positions. This research paper will discuss a matrix that is distinct from other types of matrices described in books and papers. This paper discusses some of the matrix's most important properties and characteristics.

Keywords - Matrix; Perimeter Matrix; New type of matrix.

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1. INTRODUCTION

The importance of matrices in linear algebra and their applications is so great that they can be considered the soul of linear algebra. Matrix applications include not only mathematics but also other fields of science and real life, such as probability theory and statistics, electronics, engineering, computer science, cryptography, wireless communication [1], and so on. From ancient times to the present, mathematicians all over the world have used the matrix. There are some matrices reviews in this section.

Matrix Definition

A matrix is a m x n array of numbers, where m represents the number of rows and n represents the number of columns.

$$[A]=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

A matrix is a rectangular array that contains the real constants of a linear equation or a system of linear equations. All of the elements of a matrix are referred to as entries [2,3].

Let the following system of linear equations

x + 4y + 5z = 0

6x + 2y + 3z = 0

2x - 7z = 0

Then the matrix of the above system of linear equations is

$$\begin{bmatrix} 1 & 4 & 5 \\ 6 & 2 & 3 \\ 2 & 0 & 7 \end{bmatrix}$$

Order of Matrix

A matrix of m numbers of rows and n numbers of columns is said to be a matrix of order m * n [3].

Let A be a matrix of the following forms

$$\begin{bmatrix} 1 & 4 & 12 & 5 \\ 13 & 0 & 4 & 1 \\ 2 & 8 & 1 & 9 \end{bmatrix}$$

Here, matrix has 3 rows and 4 columns. So, matrix A is of 3*4 order .

Square Matrix

An m*n matrix A is called a square matrix if the numbers of rows is m equal to the number of the columns n(m= n).

Example of a square matrix is as following

$$\begin{bmatrix} 1 & 4 & 5 \\ 6 & 2 & 3 \\ 2 & 0 & 7 \end{bmatrix}$$

Diagonal Matrix

A diagonal matrix is a square matrix, where all the elements other than the diagonal are zero.

Example of a diagonal matrix is as following:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

Identity Matrix

A diagonal matrix is said to be an identity matrix if all the diagonal elements are 1 [4].

The following is an example of an identity matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Zero Matrix

The matrix in which all the elements are zero is said to be a zero matrix [5].

As an example of a zero matrix we can consider the following matrix A.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A square matrixA = (a_{ij}) of order N , is said to be

- □ Symmetric if a_{ij} = $a_{ji}$for all i and j , i.e. $A^T = A$, where A^T is the transpose of A Skew symmetric if ,$a_{ij} = - a_{ji}$, i.e. A = A^T .

Every square matrix can be expressed as the sum of a symmetric and skew symmetric matrix as:

$A = 1/2\ (A+ A^T) + 1/2\ (A- A^T)$

Where $1/2\ (A+ A^T)$ is a symmetric and $1/2\ (A- A^T)$ is a skew symmetric matrix.

- Singular if det(A)=0 and Nonsingular if det(A) ≠0 i.e. if there exists a unique matrix B such that AB=I=BA , where $B = A^{-1}$ is the inverse of A and I is the Identity Matrix.
- Null matrix if its every element is zero and is denoted by O.
- Diagonal Matrix when a_{ij}=0for i≠j and a_n≠ 0 for all *i*.
- Transposition matrix: If two rows (or columns) of an Identity matrix are interchanged, then the matrix so obtained is called a *Transposition Matrix*. If the transposition matrix, say P_1, is multiplied with a square matrix , then the product P_1*A* will be the matrix but with the same two rows (or columns) interchanged.

$$P1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} A = \begin{pmatrix} 2 & 3 & 1 \\ 9 & 6 & 4 \\ 5 & 2 & 7 \end{pmatrix}$$

$$P1 * A = \begin{pmatrix} 2 & 3 & 1 \\ 5 & 2 & 7 \\ 9 & 6 & 4 \end{pmatrix}$$

- Permutation matrix if it has exactly one non-zero element, namely, unity, in each row and each column and all other entries are zero.

e.g. $$P = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

is a permutation matrix.

- Sparse matrix if a large percentage of the entries of the matrix are zero.
- Dense if relatively large number of elements of the matrix are non-zero.
- Triangular matrix: If all the elements above the diagonal of a square matrix are zero, then the matrix is called a *Lower-Triangular Matrix.* If all the elements below the diagonal of the matrix are zero, then it is called an *Upper-Triangular Matrix.*

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 8 & 4 & 0 \\ -1 & -1 & 2 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & -6 & 3 \\ 0 & 7 & 0 \\ 0 & 0 & -5 \end{pmatrix}$$

then*L*iscalled a lower-triangular matrix, and*U* iscalled an Upper-triangularmatrix. If a matrix is triangular, its determinant is just the product of its diagonalelements.

- Tridiagonal matrix if *A*hasnon-zero elements only on the diagonal and inpositionsadjacentto the diagonal.

e.g. $$A = \begin{pmatrix} 3 & 8 & 0 & 0 \\ 1 & 2 & -2 & 0 \\ 0 & -4 & 6 & 5 \\ 0 & 0 & 7 & 9 \end{pmatrix}$$

is a tridiagonal of order 4*4.

- $$A = \begin{pmatrix} B_1 & & & O \\ & B_2 & & \\ & & \ddots & \\ O & & & B_s \end{pmatrix}$$

where B_i's , i = 1,2,3,……,s are sqaure sub-matrices . not necessarily all of the order.

- Non- Negative (positive) if A≥O(>0) , where o is null matrix.

- Hermitian if A^H = A and Skew - Hermitian if A^H = - A, where A^H denoted the conjugate transpose of A.

- Positive Define if v^H Av > 0 where v^H = (v complememt $)^T$, and Positive Semi-definite if v^H Av ≥ 0 for any vector v≠0.

Positive Definite Matrices have the following important properties:

If is non-singular and positive definite, then is Hermitian and positiveB = A^HA definite.

If A is symmetric and positive definite, then its eigenvalues are all positive. A

All leading minors of A are positive. A

- Similar to a square matrix of the same order if a non-singular matrix can be determined such that

$B = S^{-1}AS$

Similar matrices have same rank and same eignvalues.

- Monotone matrix if det(A) ≠ 0 and A^{-1}≥ 0
- Orthogonal if$A^{-1} = A^T$ i.e. A^TA = I= A A^T
- Unitary if $A^{-1} = A^H$ i.e.A^HA = I = AA^H
- Normal if $A^HA = AA^H$

2. RESULT

In this section, we will define a matrix that is distinct from the matrices defined previously in numerous books and papers.

2.1 Definitions

Let, A_{ij}be a square matrix of order m*m . Let, i = 1,….,m and j = 1,….,m. Then all the element of A except those whose i-th or j-th element has at least one 1 or m, are zero; construct a new type of matrix.

Consider the following matrix of order 4*4

$$A = \begin{bmatrix} 2 & 6 & 1 & 4 \\ 8 & 0 & 0 & 5 \\ 9 & 0 & 0 & 1 \\ 4 & 5 & 3 & 2 \end{bmatrix}$$

Here, the matrix A $_{4*4}$ is a square matrix where , i = 1,….,4 and j = 1,…,4.

All the elements of A $_{4*4}$ which does not have i=1 or 4 or j = 1 or 4 are

a_{22}, a_{23}, a_{32}, a_{33}

And $a_{22} = a_{23} = a_{32} = a_{33}=0$

This matrix resembles the perimeter of a square. So, for the sake of convenience, we can call it a Perimeter matrix.

2.2. A Perimeter Matrix's Properties

- ❖ A square matrix of the form A_{m*m} must be used.
- ❖ The smallest value of m is 3.
- ❖ All elements in the first and last rows, as well as the first and last columns, must have a value other than 0.
- ❖ Except for the third property, all of the elements are 0.

2.3 Theorem

When all other elements of a Perimeter matrix of order m*m where m = 1, 2, …., n except the zero element are equal, the determinant of the matrix is zero.

Example:

Consider the following matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Then , determinant of A ,

det(A) = 0

2.4. Theorem

If A be a perimeter matrix and I be an identity matrix of same order then

det(A) - det(I) ≠det(A - I)

2.5 Theorem

Eigenvalues of a tridiagonal matrix: This class of matrices arises commonly in the study of stability of the finite difference processes, and a knowledge of its eigenvalues leads immediately into useful stability conditions. Let ,

$$A = \begin{bmatrix} a & b & & & & \\ c & a & b & & O & \\ & c & a & b & & \\ & & \ddots & \ddots & \ddots & \\ & O & & c & a & b \\ & & & & c & a \end{bmatrix}_{NXN} = [c \quad a \quad b]_{NXN}$$

Then its eigenvalues are given by ; where and may be real or complex $\lambda_s = a + 2\sqrt{bc}\cos(s\Pi / N+1)$, s = 1(1)N ; where a,b and c may be real or complex.

Let A be N*N cyclic tridiagonal matrix given by

$$A = \begin{bmatrix} a & b & & & & c \\ c & a & b & & O & \\ & c & a & b & & \\ & & \ddots & \ddots & \ddots & \\ & O & & c & a & b \\ b & & & & c & a \end{bmatrix}$$

Then the eigenvalues of A are given by

$\lambda_s = a + 2\sqrt{bc}\cos(2s\Pi / N+1)$, s =0,1,2 ,……..,N-1.

2.6 Theorem

A square matrix A = (a_{ij}) of order N is irreducible if N = 1 or if in case N>1 , then given any two non-empty disjoint subsets S and T and of w , the set of first *N* positive integers i.e. w = {1,2,3,….,N such that , there exists and such that . ( ) ij a □ A N 1N □ 1N □ S T w{1,2,...,} w N□ S T w□ □ iS □ jT □ 0 ij a □

3. CONCLUSIONS

Matrix theory has numerous applications in mathematics and real-world sciences. This research paper begins by providing a brief overview of various types of matrices. The paper then introduces a new type of matrix in addition to the known matrices. A few properties and theorems are included here.

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ISBN-10 : 0486788539

ISBN-13 : 0486788531-978

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