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Some Charged Fluid Spheres in General Relativity

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Abstract - The present paper provides some solutions of Einstein Maxwell field equations for Some Charged Fluid Spheres by using a judicious choice of metric potential g_{11} and g_{44}. The central and boundary conditions have been also discussed,

Keywords - Metric, Potential, Boundary Conditions, Charged Fluid Spheres.

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INTRODUCTION

A various authors have already studied the charged fluid distribution in equilibrium. Bonner [4], Effinger [6] and Kyle and Martin [11] have considered the interior solution for a static charged sphere. As the field equations do not completely determine the system different solutions were obtained by Effinger [6], Wilson [16] and Kyle and Martin [11] by using different conditions. Some exact static solutions of Einstein-Maxwell equations representing a charged fluid sphere were obtained by Singh and Yadav [14]. Shi-Chang [15] found some conformal flat interior solutions of the Einstein-Maxwell equations for a charged stable static sphere. These solutions satisfy physical conditions inside the sphere. Xingxiang [18] obtained an exact solution by specifying matter distribution and charge distribution. The metric is regular and can be matched to the Reissner-Nordstrom metric and pressure is finite. In the limit of vanishing charge, the solution reduces to the interior solution of an uncharged sphere. Buchdahl [5] has also considered some regular general relativistic charged fluid spheres. Some other cases of the interior solutions for charged fluid sphere have been presented by Bekenstein[3], Bailyn[2], Whiman and Burch [17], Kramer and Neugebauer [9], Krori and Barua [10], Junevicous[8], Florides [7], Noluka[12, 13] and Yadav et. al. [19, 20]. Some other researchers in this field are Pradhan [21], Yilmaz (22) and Saha & Visinescu [23].

In this paper, we have solved Einstein-Maxwell field equations for static changed fluid spheres by using different assumptions. These solutions satisfy physical conditions. The central and boundary conditions have been also discussed. The pressure and density have been found for the distribution.

THE FIELD EQUATIONS

We take the metric in the form

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 (d\theta^2 + \sin^2\theta\, d\phi^2) \qquad (2.1)$$

where □ □ and v are function of r only.

Thus the Einstein-Maxwell field equations are (Adler et. al. [1]].

$$e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{1}{r^2} = -8\pi p - E \qquad (2.2)$$

$$\frac{1}{r^2} - e^{-\lambda}\left(\frac{1}{r^2} + \frac{\upsilon'}{r}\right) = -8\pi p + E, \qquad (2.3)$$

$$e^{-\lambda}\left[\frac{1}{4}\upsilon'\lambda' - \frac{1}{4}\upsilon'^2 - \frac{1}{2}\upsilon'' - \frac{1}{2}\left(\frac{\upsilon' - \lambda'}{r}\right)\right] = -8\pi p - E \qquad (2.4)$$

where

$$E = -F^{41} F_{41} \qquad (2.5)$$

and

$$4\pi\sigma = \left(\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + \frac{\lambda' + \upsilon'}{2}F^{41}\right)e^{\upsilon/2} \qquad (2.6)$$

By the use of equations (2.2) – (2.4), we get the expressions for p, □ and E as

$$8\pi p = \frac{e^{-\lambda}}{2}\left(\frac{3\upsilon'}{2r} + \frac{\upsilon''}{2} - \frac{\lambda'\upsilon'}{4} + \frac{\upsilon'^2}{4} - \frac{\lambda'}{2r} + \frac{1}{r^2}\right) - \frac{1}{2r^2} \qquad (2.7)$$

$$8\pi p = e^{-\lambda}\left(\frac{5\lambda}{4r} - \frac{\upsilon''}{4} - \frac{\lambda'\upsilon'}{8} + \frac{\upsilon'^2}{8} + \frac{\upsilon}{4r} - \frac{1}{2r^2}\right) + \frac{1}{2r^2} \qquad (2.8)$$

and

$$2E = e^{-\lambda}\left(\frac{\upsilon''}{2} - \frac{\lambda'\upsilon'}{4} + \frac{\upsilon'^2}{4} - \frac{\upsilon}{2r} - \frac{\lambda'}{2r} - \frac{1}{r^2}\right) + \frac{1}{2r^2} \qquad (2.9)$$

SOLUTION OF THE FIELD EQUATIONS

We have four equations (2.2) – (2.4) and (2.6) in six variables $(\rho, E, p, \lambda, \upsilon, \sigma)$. Hence the two variables are free. We take u and v as the two free variables. Here we choose.

$$u = a_1 r^4 + a_2 r^2 + c \qquad (3.1)$$

and

$$\upsilon = dr^2 + gr + k \qquad (3.2)$$

where a_1, a_2, c, d, g and k are constant and n positive integer $(n \neq 0)$.

Then equations (2.6) . (2.9) yield

$$8\pi p = \frac{e^{(a_1 r^4 + a_2 r^2 + c)}}{2}\left[\frac{(6d - 2a_2)r + 3g - 2na_1 r^3}{2r} + \right.$$

$$\left.\frac{4d + g^2 - 4a_1 r^3(2dr + g) - 4dr^2(a_2 - d) + 2gr(2d - a_2)}{4} + \frac{1}{r^2}\right] - \frac{1}{2r^2}. \qquad (3.3)$$

$$8\pi p = \frac{e^{(a_1 r^4 + a_2 r^2 + c)}}{2}\left[\frac{20a_1 r^3 + 10a_2 r + 2dr + g}{2r} + \right.$$
$$\left.\frac{1}{4}\left\{8a_1 dr^4 + 4a_1 gr^3 + 4dr^2(a_2 - d) + 2gr)(a_2 - 2d) - 4d - g^2\right\} - \frac{1}{r^2}\right] + \frac{1}{2r^2}, \qquad (3.4)$$

$$\frac{4d + g^2 - 4a_1 r^3(2dr + g) - 4dr^2(a_2 - d) + 2gr(2d - a_2)}{4} - \frac{1}{r^2}] + \frac{1}{2r^2} \qquad (3.5)$$

$$4\pi\sigma = \left[\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + \frac{1}{2}\left\{4a_1 r^3 + 2r(a_2 - d) + g\right\}F^{41}\right]e^{\frac{dr^2 + gr + k}{2}} \qquad (3.6)$$

Now matching the solution with Reissner-Nordstron metric at the boundary $r = r_0$, we have

$$e^{-(a_1 r_b^4 + a_2 r_b^2 + c)} = \left(1 - \frac{2M}{r_b} + \frac{Q_b^2}{r_b^2}\right) \qquad (3.7)$$

$$e^{(dr_b^2 + gr_b + k)} = \left(1 - \frac{2M}{r_b} + \frac{Q_b^2}{r_b^2}\right) \qquad (3.8)$$

$$(2dr_b + g)e^{(dr_b^2 + gr_b + k)} = 2\left(\frac{M}{r_b^2} + \frac{Q_b^2}{r_b^3}\right) \qquad (3.9)$$

In particular, if we take $a_1 = g = 0$, then we get

$$16\pi p = e^{-(a_2 r^2 + c)}\left[\frac{1}{r^2} - a_2 - a_2 dr^2 + d(dr^2 + 4)\right] - \frac{1}{r^2}, \qquad (3.10)$$

$$16\pi p = e^{-(a_2 r^2 + c)}\left[-d^2 r^2 + a_2 dr^2 + 5a_2 - \frac{1}{r^2}\right] + \frac{1}{r^2}, \qquad (3.11)$$

$$2E = e^{-(a_2 r^2 + c)}\left[-a_2(dr^2 + 1) + d^2 r^2 - \frac{1}{r^2}\right] + \frac{1}{r^2} \qquad (3.12)$$

and

$$4\pi\sigma = \left[\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + (a_2 + d)rF^{41}\right]e^{(dr^2 + k)/2} \qquad (3.13)$$

At r = 0, these result give

$$16\pi p_0 = e^{-c}(4d - a_2), \qquad (3.14)$$

$$16\pi p_0 = 5a_2 e^{-c} \qquad (3.15)$$

and

$$E_0 = \frac{e^{-c}(-a_2)}{2} \qquad (3.16)$$

For p_0 and □$_0$ to be positive we must have

$$4d \geq a_2, a_2 \geq 0, \qquad (3.17)$$

Further for $\rho_0 \geq 3p_0$

$$2a_2 \geq 3d \qquad (3.18)$$

From conditions (3.17) and (3.18), we have

$$8d \geq 2a_2 \geq 3d. \qquad (3.19)$$

Again matching the solution with Reissner-Nordstrom metric at boundary $r = r_b$, we get

$$e^{-1(a_2 r_b^2 + c)} = \left(1 - \frac{2M}{r_b} + \frac{Q_b^2}{r_b^2}\right) \qquad (3.20)$$

$$e^{(dr_b^2 + k)} = \left(1 - \frac{2M}{r_b} + \frac{Q_b^2}{r_b^2}\right) \qquad (3.21)$$

and

$$dr_b e^{(dr_b^2+k)} = \left(\frac{M}{r_b^2} - \frac{Q_b^2}{r_b^3}\right) \qquad (3.22)$$

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