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A Study of Fuzzy Approach for problem solving using Mixed Intuitionistic Ranking Approach

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Abstract - Mathematical programming is the use of mathematical models, such as optimising models, to help people make decisions. The term "programming" means making a plan for what you're going to do. If you're working on a math problem, you're trying to find the best way to solve a problem. You have to keep certain things in mind when you do this: This is a type of decision-making problem called a "mathematical programming problem." In this type of problem, preferences between alternatives are described by an objective function that is defined on the set of alternatives in a way that greater (or smaller) values of this function correspond to more preferable alternatives. Values of the objective function show what happens if you choose one or another option over the other. In any case, the results of an analysis using a given formulation of the mathematical programming problem depend a lot on how well it is written. Some of the things that make up real systems or a process are shown in the description of the objective function and the constraints. In an intermediate and flexible way, experts' understanding of the nature of the parameters could be represented in the model as a fuzzy set of their possible values. This way, the model could be more flexible. New types of mathematical programming problems are created when fuzzy parameters are used. To solve these new types of problems, fuzzy set theory tools must be used in a consistent way when they are used. This is how we get a new type of mathematical programming problem with fuzzy parameters. Control theory and management sciences, mathematical modelling, operation research, and many industrial applications have used fuzzy set theory to make things easier to understand.

Keywords - Fuzzy Approach, Problem Solving, Mixed Intuitionistic Ranking Approach, Mathematical programming, operation research, fuzzy parameters.

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INTRODUCTION

The travelling salesman problem is one of the most important computational math problems. Information about real-world systems is often given in the form of vague words. Thus, fuzzy methods can handle vague terms, and they are best for finding the best solution to problems with vague parameters. The algorithm that was used to find the best solution to the travelling salesman problem can also be used to solve the assignment problem. However, because it has a lot of degeneracy, a special algorithm called the Hungarian method was created by Kuhn (1955) to solve it. However, in real life, the parameters of an assignment problem are not set in stone. This is because the time and cost of a job done by a facility (machine or person) may change for a variety of reasons. Zadeh (1965) came up with the idea of fuzzy sets to help people deal with uncertainty and vagueness in real life situations. In the last few years, the fuzzy travelling salesman problem has been getting a lot of attention, and different techniques have been used to solve the problems in the fuzzy travelling salesman problem. Fuzzy travelling salesman problem: Mukerjee and Basu (2010) came up with a new way to solve the problem. There were fuzzy costs in the travel salesman problem that Majumdar and his team of researchers tried to solve with a genetic algorithm in 2011. Sapideh (2011) used multi-objective linear programming to solve the problem. If you want to solve the fuzzy travelling salesman problem with LR-fuzzy parameters, Kumar and Gupta (2010) did it. Fischer and Richter (1982) came up with a way to solve a multi-objective travelling salesman problem by using dynamic programming, which is what they did. Zimmermann (1978) used fuzzy programming and linear programming with a lot of different goals.

EFINITATIONS AND NOTATIONS

In this section, some necessary definitions and notions of fuzzy set theory are reviewed.

Types of triangular fuzzy numbers

The triangular fuzzy number is based on three-value judgment: The minimum a_1, possible value a_2, the most possible value and the maximum possible value a_3.

- A triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ is said to be nonnegative fuzzy number iff $a_1 > 0$.
- In a triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ if $a_2 = a_3$ if than it is called symmetric triangular fuzzy number. It is denoted by $\tilde{a} = (a_1, a_2, a_2)$

α-cut

- Given a fuzzy set A defined on X and any number $\alpha \in [0,1]$, the α- the - cut, α_A is the crisp set $\alpha_A = \{x \,/\, A(x) \geq \alpha\}$.
- Given a fuzzy set A defined on X and any number $\alpha \in [0,1]$, the α- , the - cut, α_{A^+} is the crisp set $\alpha_{A^+} = \{x \,/\, A(x) > \alpha\}$.

SOME PROPERTIES

Let F(R) be a set of triangular fuzzy numbers and $(a_1, a_2, a_3), (b_1, b_2, b_3)$ are two triangular fuzzy numbers, then some mathematical notation/operations are as follows

(i) $(a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

(ii) $(a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$

(iii) $k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)\ for\ k \geq 0$

(iv) $k(a_1, a_2, a_3) = (ka_3, ka_2, ka_1)\ for\ k < 0$

(v) $(a_1, a_2, a_3) \times (b_1, b_2, b_3) = (a_1b_1, a_2b_2, a_3b_3)\ for\ a_1 > 0$

$= (a_1b_3, a_2b_2, a_3b_3)\ for\ a_1 < 0, a_3 \geq 0$

$= (a_1b_3, a_2b_2, a_3b_1)\ for\ a_3 > 0$

Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be are fuzzy triangular numbers, then some relational notation/operations are as follows

(i) $\tilde{a} < \tilde{b}$ $iff\ R(\tilde{a}) < R(\tilde{b})$

(ii) $\tilde{a} > \tilde{b}$ $iff\ R(\tilde{a}) > R(\tilde{b})$

(iii) $\tilde{a} = \tilde{b}$ $iff\ R(\tilde{a}) = R(\tilde{b})$

(iv) $\tilde{a} - \tilde{b} = 0$ $iff\ R(\tilde{a}) - R(\tilde{b}) = 0$

A triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) \in F(R)$ is said to be positive if $R(\tilde{a}) > 0$ and denoted by $\tilde{a} > 0$ and if $R(\tilde{a}) = 0$ then $\tilde{a} = 0$, if then the triangular numbers $\tilde{a}$ and $\tilde{b}$ are said to be equivalent and is denoted by $\tilde{a} = \tilde{b}$.

RANKING FUNCTION OF TRIANGULAR FUZZY NUMBERS

A good way to compare fuzzy numbers is to use a ranking function to do so. Rank functions are maps from F(R) to the real line, like a road map. Because there are many ways to rank fuzzy numbers, we used Yager's ranking method because it meets the criteria for compensating, linearity, and adding. This gives us results that are in line with human intuition.

Given a convex fuzzy number $\tilde{a}$, then the Yager's ranking index is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^l, a_\alpha^u)\,d\alpha$$

Where (a_α^l, a_α^u) is the α the level cut of the fuzzy number and

$$a_\alpha^l = (a_2 - a_1)\alpha + a_1,$$

$$a_\alpha^u = -(a_3 - a_2)\alpha + a_3.$$

FUZZY TRANSPORTATION PROBLEM

Transportation is one of the most well-known linear programming problems. It is a special case of it, in which a commodity must be transported from different sources of supply to different destinations of demand in such a way that the total transportation cost is the least. But in reality, the supply, demand, and cost of transportation for each unit of time aren't certain because of a lot of things. There may be better ways to show these vague data than with precise numbers. The transportation problem is called a "fuzzy transportation problem" when the costs of transportation, the amount of supply and demand, and other things are represented by fuzzy numbers. Many researchers have come up with ways to solve this kind of fuzzy transportation problem, like fuzzy and interval programming. Ishibuchi and Tanka (1990), Harrera and Verdegay (1995), Pandian and Natarajan (1990, 1995, 2010) have all come up with ways to do this. Zadeh (1965) came up with the idea of fuzzy numbers. Buckley and Feuring (2000) came up with a way to solve a fully fuzzified linear programming problem. They changed the objective function into a multi-objective linear programming problem. It was suggested by Liu and Kao in 2006, Chanas et al. (1984), and Chanas and Kuchta (1996). When Samual and Gani (2011) tried to solve a fuzzy transportation problem, they used Arshamrhan's method. For a fuzzy transportation problem where all of the parameters are trapezoidal fuzzy numbers, Pandian and Natarajan (2010) came up with a fuzzy zero-point method. Rani et al. (2014) came up with a new way to find the best solution for a transportation problem that is completely fuzzy and unbalanced. All of the numbers in a fuzzy transportation problem are fuzzy numbers. Fuzzy numbers can be normal or weird, triangular or trapezoidal, and they can also be hard to read. That makes it hard to compare some fuzzy numbers with the numbers you have in your head. One of the most important things to learn about is how to compare two or more fuzzy numbers and rank them. How to set the rank of fuzzy numbers has been one of the main issues. In this section we will solve fuzzy transportation problem by using Yager's ranking method, mathematically, a fuzzy transportation problem (FTP) can be stated as:

$$\text{Minimize } Z=\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\tilde{x}_{ij}$$

subject to constraints

$$\sum_{j=1}^{n}\tilde{x}_{ij}=\tilde{a}_i,\quad i=1,2,\dots m$$

$$\sum_{i=1}^{m}\tilde{x}_{ij}=\tilde{b}_j,\quad j=1,2,\dots n$$

$$\tilde{x}_{ij}\geq 0$$

Where,

m = total number of origins

n = total number of destinations

$\tilde{a}_i$ = fuzzy availability of commodity at i^{th} origin

$\tilde{b}_j$ = fuzzy commodity needed at the j^{th} destination

$\tilde{c}_{ij}$ = fuzzy transportation cost of one unit of product from i^{th} origin to j^{th} destination

$\tilde{x}_{ij}$ = fuzzy quantity transported from i^{th} origin to j^{th} destination

$$\tilde{a}_i=\left[a_i^{(1)},a_i^{(2)},a_i^{(3)}\right],$$

$$\tilde{b}_j=\left[b_j^{(1)},b_j^{(2)},b_j^{(3)}\right],\ \tilde{c}_{ij}=\left[c_{ij}^{(1)},c_{ij}^{(2)},c_{ij}^{(3)}\right],\ \tilde{x}_{ij}=\left[x_{ij}^{(1)},x_{ij}^{(2)},x_{ij}^{(3)}\right]$$

The above fuzzy transportation problem is said to be balanced if , $\sum_{i=1}^{m}\tilde{a}_i=\sum_{j=1}^{n}\tilde{b}_j$, otherwise it is called unbalanced.

The tabular representation of transportation problem is as follows

Table1

	1	……………….	n	Supply
1	$\tilde{c}_{11}$	……………….	$\tilde{c}_{1n}$	$\tilde{a}_1$
2	$\tilde{c}_{21}$	……………….	$\tilde{c}_{2n}$	$\tilde{a}_2$
. . . .	. . . .	. . . .	. . . .	. . . .
M	$\tilde{c}_{m1}$	……………….	$\tilde{c}_{mn}$	$\tilde{a}_m$
Demand	$\tilde{b}_1$	……………….	$\tilde{b}_n$	

ALGORITHM

This proposed method in algorithm form for finding the optimal basic feasible solution in a symmetric triangular fuzzy environment and step by step procedure is as follows:

Step 1 From the given data, construct the transportation table whose cost matrix, supplies and demands are fuzzy symmetric triangular numbers.

Step 2 Calculate Yager's ranking index for each cell of transportation table.

Step 3 Replace symmetric triangular numbers by their respective ranking indices.

Step 4 Solve the resulting transportation problem by using existing Vogel's method to find the optimal solution.

The all step by step procedure is explained in next section of numerical example.

NUMERICAL EXAMPLE

Consider the following fuzzy transportation problem

Table 2 (a)

	Destination				Supply
Source	(2,3,3)	(2,3,3)	(2,3,3)	(1,4,4)	(0,3,3)
	(4,9,9)	(4,8,8)	(2,5,5)	(1,4,4)	(2,13,13)
	(2,7,7)	(0,5,5)	(0,5,5)	(4,8,8)	(2,8,8)
Demand	(1,4,4)	(0,9,9)	(1,4,4)	(2,7,7)	

Convert the given fuzzy problem into a crisp value problem by using the measure.

$$Y(2, 3, 3) = \int_0^1 0.5(2+\alpha+3)d\alpha$$

$$=\int_0^1 0.5(5+\alpha)d\alpha$$

$$= 2.75$$

Similarly

$Y(1, 4, 4) = 3.25$ $Y(2, 5, 5) = 4.25$

$Y(4, 9, 9) = 7.75$ $Y(4, 8, 8) = 7$

$Y(2, 7, 7) = 5.75$ $Y(0, 3, 3) = 2.25$

$Y(0, 5, 5) = 3.75$ $Y(2, 8, 8) = 6.5$

$Y(0, 9, 9) = 6.75$ $Y(2, 13, 13) = 10.2$

Table2 (b)

	Destination				Supply
Source	2.75	2.75	2.75	3.25	2.25
	7.75	7	4.25	3.25	10.25
	5.75	3.75	3.75	7	6.5
Demand	3.25	6.75	3.25	5.75	

Using Vogel's procedure, we obtain the initial solution as

Table 2 (c)

	Destination				Supply
Source	**2.25**(2.75)				2.25
	1(7.75)	**0.25**(7)	**3.25**(4.25)	**5.75**(3.25)	10.25
		6.5(3.75)			6.5
Demand	3.25	6.75	3.25	5.75	

Now using the allotment rules, the solution of the problem can be obtained in the form of symmetric triangular fuzzy numbers.

Table 2 (d)

	Destination				Supply
Source	(0,3,3)				(0,3,3)
	(1,1,1)	(-2,1,1)	(1,4,4)	(2,7,7)	(2,13,13)
		(2,8,8)			(2,8,8)
Demand	(1,4,4)	(0,9,9)	(1,4,4)	(2,7,7)	

Hence the crisp optimal solution is 72.5625 and the fuzzy optimal solution for the given transportation problem is

$$x_{11} = (0,3,3),\ x_{21} = (1,1,1),\ x_{22} = (-2,1,1),$$

$$x_{23} = (1,4,4),\ x_{24} = (2,7,7),\ x_{32} = (2,8,8).$$

COMPARISON

In the proposed method we obtain the optimal solution for the above fuzzy transportation problem is 72.5625 and

$$x_{11} = (0,3,3), x_{21} = (1,1,1), x_{22} = (-2,1,1), x_{23} = (1,4,4),$$

$$x_{24} = (2,7,7), x_{32} = (2,8,8).$$

By Kumar and Ghuru (2014) method the optimal solution for the above fuzzy transportation problem is 84.33

$$\text{and} x_{14} = (0,3,3), x_{21} = (1,4,4), x_{22} = (-2,1,1),$$

$$x_{23} = (1,4,4), x_{24} = (2,4,4,), x_{32} = (2,8,8).$$

Comparison of Results

Table 3

Method	Optimal solution
Kumar and Ghuru method	84.33
Proposed method	72.5625

In this section, a new ranking method for solving the fully fuzzy transportation problem (FFTP) is explained and shown with a good example. People who have written about how to do this have also given us a method to compare to (2014). All the fuzzy numbers in this paper are thought of as symmetric triangular fuzzy numbers, and the decision variables are also thought of as symmetric triangular fuzzy numbers Kumar and Ghuru used the same example to show how our method works, so we used that to show how it works. Table 1 shows that by using the proposed method, the ranking value and the best value are less than with the old method. There is both a fuzzy number and a ranked fuzzy number for the answer. The proposed method is very simple to understand and can be used to solve a fully fuzzy transportation problem in a real-world setting.

FUZZY TRANSSHIPMENT PROBLEM

In a transportation problem, only direct shipments are allowed. A transportation problem in which a commodity moves from one source to another source or location before it reaches its final destination is called a transshipment problem. In the transshipment problem, all the sources and destinations can work in any direction. When there is no transshipment, the transportation costs go up. Thus, transshipment is also a very good way to cut down on the cost of transportation. Garg and Prakash (1985) looked at a time-minimize transshipment problem. Ozemir et al. then looked at a multilocation transshipment problem with a capacity problem and lost sales (2006). Orden (1956) has added the case where transshipment is also allowed to the transportation problem. Gani et al. (2011) found a way to solve the transshipment problem in a fuzzy world. Kumar and Ghuru (2014) used symmetric triangular fuzzy numbers to solve a transportation problem. Jat et al. (2014) came up with a new way to solve a fuzzy transportation problem. To solve transportation problems that are fuzzy, we come up with a way to rank fuzzy triangular numbers like Yager's ranking method. Then, we can use the same method to solve the transportation problems. This method is very simple to understand and use. At the end, the best way to solve the problem can be in the form of a fuzzy number or a clear one. To solve the fuzzy transshipment problem, we came up with a way to rank fuzzy triangular numbers, like Yager's ranking method does. This way, the conventional method can still be used to solve the problem. This method is very simple to understand and use. A fuzzy number or a clear number can be used to get the best answer at the end of the problem.

MATHEMATICAL FORMULATION OF TRANSSHIPMENT PROBLEM

The transportation problem assumes that direct routs exist from each source to each destination. However, there are situations in which units may be shipped from one source to another or to other destinations before reaching their final destinations. This is called a fuzzy transshipment problem. The purpose of transshipment the distinction between a source and destination is dropped so that a transportation problem with m source and n destinations gives rise to a transshipment problem with m + n source and m + n destinations. The basic feasible solution to such a problem will involve [(m+n) + (m+n)-1] or 2m + 2n - 1 basic variables and if we omit the variables appearing in the (m+n) diagonal cells, we are left with m + n-1 basic variables. Thus the transshipment problem may be written as:

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^{m+n} \sum_{j=1, j\neq i}^{m+n} \tilde{c}_{ij}\tilde{x}_{ij}$$

$$\text{subject to } \sum_{j=1, j\neq i}^{m+n} \tilde{x}_{ij} - \sum_{j=1, j\neq i}^{m+n} \tilde{x}_{ji} = \tilde{a}_i, \quad i = 1,2,\dots m$$

$$\sum_{i=1, i\neq j}^{m+n} \tilde{x}_{ij} - \sum_{i=1, i\neq j}^{m+n} \tilde{x}_{ji} = \tilde{b}_i, \quad j = m+1, m+2, \dots m+n$$

$$\text{where } \tilde{x}_{ij} \geq 0, \quad i, j = 1,2,\dots m+n, j \neq i$$

When $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$ then the problem is balanced otherwise unbalanced.

The above formulation is a transshipment model, the transshipment model is reduced to transportation form as:

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^{m+n} \sum_{j=1, j\neq i}^{m+n} \tilde{c}_{ij}\tilde{x}_{ij}$$

$$\text{subject to } \sum_{j=1}^{m+n} \tilde{x}_{ij} = \tilde{a}_i + T, \quad i = 1,2,\dots m$$

$$\sum_{j=1}^{m+n} \tilde{x}_{ij} = T, \; i = m+1, m+2, m+3, \dots m+n$$

$$\sum_{i=1}^{m+n} \tilde{x}_{ij} = T \quad j = 1,2,3\dots m$$

$$\sum_{i=1}^{m+n} \tilde{x}_{ij} = \tilde{b}_j + T \quad j = m+1, m+2, m+3, \dots m+n$$

$$\text{where } \tilde{x}_{ij} \geq 0, \; i, j = 1,2,3,\dots m+n, j \neq i$$

The above mathematical model represents a standard balanced transportation problem with (m+ n) origins and (m+ n) destinations. T can be interpreted as a buffer stock at each origin and destination. Since we assume that any amount of goods can be transshipped at each point, T should be large enough to take care of all transshipments. It is clear that the volume of goods transshipped at any point cannot exceed the amount produced or received and hence we take $T = \sum_{i=1}^{m} \tilde{a}_i \text{ or } \sum_{j=1}^{m} \tilde{b}_j$.

ALGORITHM

This proposed method in algorithm form for finding the optimal basic feasible solution in a symmetric triangular fuzzy environment and step by step procedure is as follows:

Step 1 First transforms the transshipment problem into standard transportation problem.

Step 2 Calculate Yager's ranking index for each cell of transportation table.

Step 3 Replace symmetric triangular numbers by their respective ranking indices.

Step 4 Solve the resulting transportation problem by using existing Vogel's method to find the optimal solution. The all step by step procedure is explained in next section of numerical example.

CONCLUSION

The fuzzy transshipment problem has been converted into a crisp general transportation problem using Yager's ranking index. The cost at the origins and destinations are all symmetric triangular fuzzy numbers and the solution to the problem is given both as a fuzzy number and also as a ranked fuzzy number. The proposed method is very easy to understand and can be applied for fuzzy transshipment problem occurring in real life situation. A new algorithm has been proposed to solve the fuzzy travelling salesman problems occurring in real life situation. To illustrate the algorithm a numerical example has been solved in which approximate cost is represented as different type of numbers. So, the method can be applied to solve real world travelling salesman problem where the data is not in symmetric numerical values. The proposed method is very simple and easy to understand. The assignment cost has been considered as trapezoidal fuzzy numbers which are more realistic and general in nature. Here the fuzzy assignment problem has been converted into crisp assignment problem using the proposed algorithm with ranking function and branch and bound method has been applied to find an optimal solution. Numerical example has been shown that the total cost obtained is optimal. This method is systematic procedure, easy to apply and can be utilized for all type of assignment problem whether maximize or minimize objective function. An alternative method to the solution of special case of fuzzy quadratic fractional programming problem has suggested. A number of algorithm have been developed to solve such type of QFFPP, each applicable to specific type our approach is general purpose to solve QFFPP and reduce number of iteration by selecting pivot element also it gives more efficiency.

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