A Study of Deviating Arguments in Differential-Difference Equations of Order

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Abstract - Ordinary differential equations (ODEs) are frequently employed in modeling research. If a system is modelled using ordinary differential equations (ODEs), we presume that the current state has no influence over what will happen in the future. Assumption is made that the system and its subsystems interact instantaneously and there is no delay between them in ODE models. Realistic models, on the other hand, incorporate a small bit of lag. As a result, in order to predict the future, it is necessary to take into account the present and the past, as well as derivatives of the former. Functional differential equations are used to model these models (FDEs).In this study, of the FDEs has developed considerably in past few decades, the reason being its wide application in physical and biological system. System inheritance is taken into account in both physical and biological model systems. The examples and applications of differential equations with diverging arguments have piqued my interest in studying them further. Many different types of problems can be solved using differential equations with diverging arguments. The findings of this study should be examined further: According to what has been said, there appears to be plenty of room to analyze these equations in terms of properties such as asymptotic behavior, periodicity, anti-periodicity, stability, and so on. There hasn't been enough discussion of boundary value limitations. Nonlinear systems' Observability and controllability can be investigated.

Keywords - Deviating Arguments, Differential-Difference Equations, Ordinary differential equations, Functional differential equations, Nonlinear systems

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INTRODUCTION

It is common in FDEs for the system's prior state to have a major impact on its present state. This field of study began with Euler's investigation of curves that are identical to their own evolutes in terms of general form. FDEs were studied by Bernoulli, Laplace, Poisson, Condorcet, and Cauchy in the eighteenth and nineteenth centuries respectively. A wide class of differential-difference equations was explored by Schmidt in 1911. It was in 1931 when Volterra published a book titled "The Interaction of Animal Species" that he discussed the role of previous history in the modeling of interactions between species. On the basis of his research in 1977, he concluded that there was an urgent need for a comprehensive theory of differential equations with trailing arguments that are either piecewise constant or continuously piecewise constant. In 1982, Busenberg and Cooke constructed the first piecewise constant argument mathematical model while addressing the biomedical issue of vertically transmitted illnesses. For many years since then, various researchers and references therein] have explored and expanded upon the theory of differential equations with piecewise constant arguments. Researchers working on topics including neural networks, lossless transmission, nuclear reactors, population dynamics, finance, stock markets, aerodynamics, and mechanics rely on it heavily.

One of the most common types of differential equations explored by the aforementioned writers is:

$$x'(t) = f(t, x(t), x(\gamma(t))),$$

with deviating arguments of the form

$$\gamma(t) = [t],\ 2[\frac{t+1}{2}],\ [t-n],\ [t+n],\ [t]+n,\ [t]-n,$$

which stands for the greatest-integer function and which has an integer value of at least one (n). If the argument has the form [t + n], [t] + n, it is classified as advanced type. Retarded or delay type equations have arguments in the form [t n], [t] n. If both arguments are present, it is referred to as mixed type.

Scalar initial-value problems, such as impulse response, were first investigated by Cooke and Wiener in 1984 and loaded equation:

$$x'(t) = ax(t) + a_0x([t]) + a_1x([t-1]),$$
$$x(-1) = c_{-1}, x(0) = c_0,$$

where a, a0, a1 are constants. In 1987, they consider the equation of the form:

$$x'(t) = ax(t) + a_0x(2[\frac{t+1}{2}]); \quad x(0) = c_0,$$

For- t ∈ [2n − 1, 2n), the deviating argument $\gamma(t) = t - 2[\frac{t+1}{2}]$ is negative while it is positive on t ∈ (2n, 2n + 1), where n is an integer. Equation (2.1) on (2n, 2n + 1) is retarded type and on [2n − 1, 2n) advanced type. That is on the interval [2n − 1, 2n + 1), it is alternatively retarded and advanced. In 1988 Wiener and Aftabizadeh [54] studied the system:

$$x'(t) \;=\; f(x(t), x(m[\frac{t+k}{m}])); \quad x(0) = c_0,$$

where [.] denotes the greatest integer function and k and m are positive integers such that k < m. The argument deviation $\gamma(t) = t - m[\frac{t+k}{m}]$ is positive for t ∈ (mn,m(n + 1) − k) and negative for t ∈ [mn − k,mn), where n is integer.

Qualitative theory of differential equations relies heavily on the oscillation theory. The occurrence of oscillatory and nonoscillatory features in solutions is a major focus of this research. Sturm's work on the oscillation theorem for ordinary differential equations in 1840 marked the beginning of oscillation theory. This topic is explored in depth in the monographs by Bainov and Mishev, Erbe, as well as Gyochev and Ladas.

In 1997, Das and Misra studied oscillation properties of a nonlinear differential equation of the type

$$(x(t) - px(t-\tau))' + Q(t)G(x(t-\sigma)) = f(t),$$

where $f, Q \in C([T,\infty),(0,\infty))$, $\sigma, \tau \in (0,\infty)$, $0 \le p < 1$, $G : \mathbb{R} \to \mathbb{R}$ such that $xG(x) > 0$ for $x \neq 0$,, G is nondecreasing, Lipschitzian, and satisfy a sublinear condition

$$\int_0^{\pm k} \frac{dx}{G(x)} < \infty, \text{ and } \int^{\infty} f(s)\, ds < \infty$$

is either oscillatory or tends to zero asymptotically iff $\int_T^{\infty} Q(s)\, ds = \infty.$ As of 2000, Shen introduced new criteria for determining whether a solution to the autonomous delays differential equation with piecewise constant argument exhibits oscillations or not.

CLASSIFICATION

According to G.A. Kamenskii, we divide differential equations with deviating arguments into three groups.

• Delay Differential Equations (DDEs) and Retarded Functional Differential Equations (RFDEs)

DDEs are equations in which the unknown function's past and current values are included. In this case, the previous dependence is derived from the state variable itself, rather than from its derivative. First-order DDEs in a generic form is

$$x'(t) = f(t, x(t), x(t-\tau)), \tag{1.1}$$

where $\tau > 0$ and $f : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$.
Example: $x'(t) = -ax(t-2)$.

• Neutral Delay Differential Equations (NDDEs)

NDDEs are equations involving an unknown function and its derivatives at current and previous values of the argument.... First-order NDDE in a generic form is

$$x'(t) = g(t, x(t), x(t-\tau), x'(t-\tau)), \tag{1.2}$$

where $g : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and $\tau > 0$.
Example: $x'(t) = -ax(t) + x(t-1) - x'(t-1)$.

• Advanced Differential Equations(ADEs)

Unknown function equations at present and future values of the arguments are known as ADE equations An initial ADE order is of the form:

$$x'(t) = h(t, x(t), x(t+\tau)), \tag{1.3}$$

where $h(t) : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and $\tau > 0$.
Example: $x'(t) = x^2(t) + x(t+10)$.

As with ordinary differential equations, a common approach to understanding these equations is to prove the existence and uniqueness of their solutions. Qualitative features of solutions are examined in the absence of a method for finding explicit solutions. Such studies are critical in developing the theory of differential equations because they focus on features such as oscillations, periodicity, and stability.

APPLICATIONS

Following are few of the applications.

(1) Engineering

An engine's combustion chamber is filled with liquid propellant, which finally becomes a gas. Tsien [48] analyzed the time lag between injection and hot gas transformation and came up with the FDE listed below:

$$\frac{dp}{dt} + (1-n)p(t) + np(t-a) = 0, \tag{1.4}$$

the dimensionless departure from steady pressure, t is the dimensionless time variable, an is the dimensionless constant time lag of combustion, and n is an invariant constant variable.

(2) Biology

Predation:

When it comes to interactions between predators and prey, predation is king. Biological control was described in Chen [12] by using a model of a delayed predator-prey relationship with predator migration. The model is given by

$$\begin{cases} x'(t) = x(t)[r - ay(t)], \\ y'(t) = y(t)[-d + bx(t-\tau) - cy(t)] + m[x(t) - py(t)], \end{cases} \tag{1.5}$$

The prey biomass (x) and the predator biomass (y) are equal. There are no predators to keep a predatorless species in check, hence it grows at an exponential rate, as in r = rx(t). In the presence of predators, the hunting phrase ay(t) is used. It takes time for prey biomass to be converted into predator biomass when using bx(t) positive feedback, and this time is represented by the positive delay. Predator consumption rates are given by the formula p/m/c/d, where p is the rate of prey consumption per unit time, m is the rate of predator migration, and c is the predator's self-limitation constant.

(3) Economics

Finance

Everyone in the modern society is concerned about their financial well-being. This information is available to investors and how they choose to use it is their decision. One can look back on the price history of assets to make educated guesses about future asset prices. As a result, future asset prices are influenced by both the current situation and the past. Calculating option prices was made easier with Black-Scholes (also known as BSM or BS) formulas. For European-style options, this formula estimates their value. A number of factors are taken into account when constructing the model, such as the expiration date, volatility projections, current stock prices, interest rates, and dividends to be received. Fischer Black, Myron Scholes, and Robert Merton published "The Pricing of Options and Corporate Liabilities," a study in the Journal of Political Economy in 1973 that established options price models. According to Chang et al [11], they investigated a stock price model in which drift and volatility are both dependent on the stock price's past. For a set of Black Scholes Equations with delays, they devised an approximation approach. Hereditary market models are the source of the infinite-dimensional Black-Scholes equation {S(t),t ≥ −h} satisfies the following nonlinear stochastic functional differential equation

$$\frac{dS(t)}{S(t)} = f(S_t)dt + g(S_t)dZ(t),\ t \geq 0, \tag{1.6}$$

where Z(t) is a 1-dimensional standard Brownian motion. It is notable that the stock appreciation f(St) and stock volatility g(St) are given non-linear functions of the stock price throughout the time interval [[t-h,t]] instead of only stock price S(t) at the time of the stock price S(t) at time of the stock price.

When it comes to our everyday routines, time lags are a must. It's a fact that delays in time are a need in certain circumstances. We'll look at two instances where time delays have been beneficial.

(1) Electric Bell:

An electromagnet drives a mechanical bell called an electric bell. When electricity flows through the electromagnet, it creates a magnetic field that, once interrupted, vanishes. However, the magnetic force that causes the bell to ring has a lag time between when it appears and when it disappears. The hammer would have struck the gong weakly if this hadn't been the case. As a result, delay is critical when it comes to an electric bell ringing. The motion of the hammer in the electric bell as studied by Norkin [41] is given by:

$$mx''(t) + rx'(t) + kx(t) + cx(t-a) = 0. \tag{1.7}$$

where x(t) is the displacement of the hammer at time t. m,r, k, c are constants and cx(t − a) is an approximation to the force acting on the hammer.

(2) Auditory Feedback

The sound we hear when we complete a task, such as closing a door or hanging up a phone, is called auditory feedback. Auditory perception, as we all know, has a lag time. Setting up an experiment where the delay is altered is possible [See Lee[38]]. Stuttering therapy technologies like Delay Auditory Feedback (DAF) make use of this phenomenon. A directional microphone and a speaker typically make up an acoustic feedback system. Hearing one's own voice through headphones takes some time after speaking into a microphone. Stuttering can be treated with DAF devices because they can increase the delay, which reduces mental stress.

ODD-ORDER DIFFERENTIAL EQUATIONS WITH DEVIATING ARGUMENTS

This study investigates the oscillatory and asymptotic behavior of delay differential equations (DDEs) of odd-order

$$\left(a(\eta)\left(\psi^{(n-1)}(\eta)\right)^{\kappa}\right)' + q(\eta) f(\psi(\phi(\eta))) = 0 \tag{1.1}$$

and

$$\left(a(\eta)\left(\upsilon^{(n-1)}(\eta)\right)^{\kappa}\right)' + q(\eta) f(\psi(\phi(\eta))) = 0, \tag{1.2}$$

where $n \geq 3$ is an odd integer and $\upsilon(\eta) = \psi(\eta) + p(\eta)\psi(\tau(\eta))$. Further, we assume that:

(i) κ is a ratio of odd natural numbers;
(ii) $q, p \in C([\eta_0, \infty), (0, \infty))$ and $0 \leq p(\eta) < 1$;
(iii) $a, \tau, \phi \in C^1([\eta_0, \infty))$, $a(\eta) > 0$, $a'(\eta) \geq 0$, $\phi(\eta) \geq \eta \geq \tau(\eta)$, $\lim_{\eta\to\infty} \tau(\eta) = \infty$;
(iv) $f \in C(\mathbb{R}, \mathbb{R})$, $f(\psi) \geq k\psi^{\kappa}$ and

$$\pi(\eta) = \int_{\eta_0}^{\eta} \frac{1}{a^{1/\kappa}(s)} \mathrm{d}s \to \infty \ \text{ as } \eta \to \infty. \tag{1.3}$$

If there exists a $\eta_\psi \geq \eta_0$ with a continuous function ψ satisfies (1.1), $a(\eta)\left(\psi^{(n-1)}\right)^{\kappa}(\eta) \in C^1\left(\left[\eta_\psi, \infty\right), \mathbb{R}\right)$, and $\sup\{|\psi(\eta)| : \eta_1 \leq \eta\} > 0$ for every $\eta_1, \eta \in \left[\eta_\psi, \infty\right)$, then ψ is said to be a proper solution of (1.1). A solution ψ of (1.1) is said to be non-oscillatory if it is positive or negative, ultimately; otherwise, it is said to be oscillatory.

Oscillation theory in DDEs has recently received a lot of attention. The oscillations of second-order DDEs are studied in the works [1–10]. Studies of the oscillation of even-order DDEs had to reflect this trend, as can be observed in studies like [11–18]. Odd-order DDEs, on the other hand, have gotten little study. Papers [19–29] and the references listed in them provide a history of the evolution of the study of these equations.

Baculikova and Dzurina [30] studied the asymptotic properties of neutral DDE

$$(a(\eta)((\psi(\eta) \pm p(\eta)\psi(\delta(\eta)))'')^{\kappa})' + q(\eta)\psi^{\kappa}(\phi(\eta)) = 0.$$

Li and Rogovchenko [31] investigated the oscillation of neutral DDE

$$(a(\eta)(\upsilon''(\eta))^{\kappa})' + q(\eta)\psi^{\kappa}(\phi(\eta)) = 0,$$

where $\upsilon(\eta) = \psi(y) + p_0\psi(\eta - \delta_0)$ and $\delta_0 \geq 0$ (delayed argument) or $\delta_0 \leq 0$ (advanced argument). Lackova [32] deduced oscillatory and asymptotic behavior of neutral DDE

$$(\psi(\eta) + p(\eta)\psi(\tau(\eta)))^{(n)} + q(\eta) f(\psi(\phi(\eta))) = 0,$$

where $\phi(\eta)$ is a delayed argument and $n \geq 2$, $f(\varrho)\operatorname{sgn}\varrho \geq k|\varrho|^{\kappa}$, $\kappa \geq 1$, $k > 0$.

> 0. P Laplace equations, porous medium difficulties, chemotaxis models, and so on are all examples of half-linear diffraction equations. For further information, see [33–35]. For the half-linear differential equations (1.1) and (1.2), we analyze the oscillatory and asymptotic features of solutions under the circumstances indicated above, and offer some new conclusions that are complimentary and expand to [30–32]. We will support the results obtained with two example.

2. Auxiliary lemmas

We start with some lemmas that we will need to use later. The next result is a well-known result; see [36, Lemma 2], also see [37, Lemma 2.2.1].

Lemma 2.1. If ψ is a solution of (1.1) and positive eventually, then ψ (k) (η) , 1 ≤ k ≤ n − 1, are of constant signs, a (η) ψ (n−1) (η) κ is decreasing. Moreover, ψ satisfies either

$$\psi'(\eta) > 0,\ \psi''(\eta) > 0,\ \psi^{(n-1)}(\eta) > 0,\ \psi^{(n)}(\eta) < 0 \tag{2.1}$$

or

$$(-1)^m \psi^{(m)} > 0,\ m = 1, 2, \ldots, n. \tag{2.2}$$

Lemma 2.2. *[36] Let $\psi \in C^n([\eta_0, \infty), (0, \infty))$, $\psi^{(n-1)}(\eta)\psi^{(n)}(\eta) \leq 0$ for $\eta \geq \eta_\psi$ and assume that $\lim_{\eta\to\infty}\psi(\eta) \neq 0$, then there exists an $\eta_\theta \in \left[\eta_\psi, \infty\right)$ with*

$$\psi(\eta) \geq \frac{\theta}{(n-1)!}\eta^{n-1}\left|\psi^{(n-1)}(\eta)\right| \ \text{for all } \eta \in [\eta_\theta, \infty) \text{ and } \theta \in (0, 1).$$

Lemma 2.3. *Assume that $\psi^{(i)}(\eta) > 0$ for $i = 0, 1, 2$, eventually. Then, for all $\delta_0 \in (0, 1)$,*

$$\psi'(\eta) \geq \frac{\delta_0}{\eta}\psi(\eta)$$

and

$$\psi(\phi(\eta)) \geq \left(\frac{\phi(\eta)}{\eta}\right)^{\delta_0}\psi(\eta). \tag{2.3}$$

Proof. Assume that $\psi^{(i)}(\eta) > 0$ for $i = 0, 1, 2$ and for all $\eta \geq \eta_1 \geq \eta_0$, η_1 large enough. Then, we get

$$\psi(\phi(\eta)) - \psi(\eta) = \int_{\eta}^{\phi(\eta)} \psi'(s)\,\mathrm{d}s \geq \psi'(\eta)(\phi(\eta) - \eta). \tag{2.4}$$

It is easy to notice that $\lim_{\eta\to\infty}\psi(\eta) = \infty$. Hence, there exists $\eta_2 \geq \eta_1$ large enough such that

$$\delta_0\psi(\eta) \leq \psi(\eta) - \psi(\eta_2) = \int_{\eta_2}^{\eta} \psi'(s)\,\mathrm{d}s \leq \psi'(\eta)(\eta - \eta_2) \leq \eta\psi'(\eta), \tag{2.5}$$

for all $\delta_0 \in (0, 1)$. By integrating this inequality from η to $\phi(\eta)$, we find

$$\psi(\phi(\eta)) \geq \left(\frac{\phi(\eta)}{\eta}\right)^{\delta_0}\psi(\eta).$$

This completes the proof of this Lemma

Lemma 2.4. *Suppose that ψ is a positive solution of (1.2). Then, $\left(a(\eta)\left(\upsilon^{(n-1)}(\eta)\right)^{\kappa}\right)' < 0$, $\upsilon^{(i)}(\eta)$, $1 \leq i \leq n-1$, are of constant signs, and $\upsilon(\eta)$ satisfies either*

$$\textit{Case } (1): \upsilon(\eta) > 0,\ \upsilon'(\eta) > 0,\ \upsilon''(\eta) > 0,\ \upsilon^{(n-1)}(\eta) > 0 \text{ and } \upsilon^{(n)}(\eta) \leq 0 \tag{2.6}$$

or

$$\textit{Case } (2): (-1)^k \upsilon^{(k)}(\eta) > 0, \text{ for } k = 1, 2, \ldots, n. \tag{2.7}$$

Proof. Suppose that ψ is a solution of (1.2) and positive eventually. Produces directly from (1.2) that

$$\left(a(\eta)\left(\upsilon^{(n-1)}(\eta)\right)^{\kappa}\right)' \leq -kq(\eta)\psi^{\kappa}(\phi(\eta)) < 0. \tag{2.8}$$

Now, from the above inequality we find either $\upsilon^{(n-1)}(\eta) > 0$ or $\upsilon^{(n-1)}(\eta) < 0$.
If $\upsilon^{(n-1)}(\eta) < 0$, then

$$a(\eta)\left(\upsilon^{(n-1)}(\eta)\right)^{\kappa} < -c < 0,$$

integration from η_1 to η, we have

$$\upsilon^{(n-2)}(\eta) < \upsilon^{(n-2)}(\eta_1) - c^{1/\kappa}\int_{\eta_1}^{\eta} \frac{1}{a^{1/\kappa}(s)}\mathrm{d}s,$$

by using (1.3) we have $\upsilon^{(n-2)}(\eta) \to -\infty$ at $\eta \to \infty$, and by doing this process several times we get $\upsilon(\eta) \to -\infty$. This contradicts the positive $\upsilon(\eta)$, then $\upsilon^{(n-1)}(\eta) > 0$. Since $\upsilon^{(n-1)}(\eta) > 0$, we have that either $\upsilon^{(n-2)}(\eta) > 0$ or $\upsilon^{(n-2)}(\eta) < 0$. But, $\upsilon^{(n-2)}(\eta) > 0$ leads to $\upsilon^{(i)}(\eta) > 0$ for $0 \leq i \leq n-2$. Repeating these considerations, we verify that $\upsilon(\eta)$ satisfies either (2.6) or (2.7).
Now since $\upsilon^{(n-1)}(\eta) > 0$ and $a' \geq 0$. Then we have

$$0 > \left(a(\eta)\left(\upsilon^{(n-1)}(\eta)\right)^{\kappa}\right)' = a'(\eta)\left(\upsilon^{(n-1)}(\eta)\right)^{\kappa} + \kappa a(\eta)\left(\upsilon^{(n-1)}(\eta)\right)^{\kappa-1}\upsilon^{(n)}(\eta),$$

which shows us that $\upsilon^{(n)}(\eta) < 0$. This completes the proof of this lemma.

Lemma 2.5. *Let Case (1) hold. Then*

$$\frac{\upsilon(\phi(\eta))}{\upsilon(\eta)} \geq \left(\frac{\phi(\eta)}{\eta}\right)^{\delta_0}, \tag{2.9}$$

for all $\delta_0 \in (0, 1)$.

Proof. The proof of the above lemma is similar to that of Lemma 2.3 and so it is omitted.

Next, we will present the basic definitions and notations that we will use in our results. $\{h_m(\eta)\}_{m=0}^{\infty}$ is a sequence of continuous functions defined as follows

$$h_0(\eta) = k\Psi(\eta), \; k \in (0,1) \text{ fixed},$$

$$h_{m+1}(\eta) = h_0(\eta) + \frac{\kappa k}{(n-2)!} \int_\eta^\infty h_m^{(\kappa+1)/\kappa}(s) \frac{s^{n-2}}{a^{1/\kappa}(s)} \mathrm{d}s, \; m = 0, 1, \ldots, \tag{2.10}$$

$$\Psi(\eta) = \int_\eta^\infty q(s) \left(\frac{\phi(s)}{s} \right)^{\kappa\delta_0} (1 - p(\phi(s)))^\kappa \, \mathrm{d}s$$

and

$$\Theta(\eta) = \frac{k^{1/\kappa}}{a^{1/\kappa}(\eta)} \left(\int_\eta^\infty q(s) \, \mathrm{d}s \right)^{1/\kappa},$$

where $\delta_0 \in (0,1)$.

CONCLUSION

The oscillation theory plays an important role in qualitative differential equations theory. Non-frequencies and solution oscillations pose significant difficulties in this discipline. The development of oscillation theory was launched in 1840 by Sturm's discovery of the oscillation theorem for ordinary differential equations. Other monographs on the history of oscillation theory include those by Bainov and Mishev, Erbe, Gyori and Ladas, and others.Natural phenomena are described using mathematical equations that include derivatives and integrals. These equations can be categorized as either differential or integral equations, depending on the case. Differential equations, both linear and nonlinear, appear often in the physical, biological, social, and engineering sciences. When we discover a differential equation when researching a system, we are referring to the system's differential equation modeling. In ODE models, it is assumed that all interactions between the system and its subsystems are immediate and without delay. However, realistic models have some lag built in. If you want a better idea of what will happen in the future, you need to take into account what is happening now and what has happened before. These models are modeled using functional differential equations (FDEs). A system's past effects its current state implicitly in many FDEs, making them superior to ODEs. Delay differential equations are the simplest kind of FDEs (DDE). This is why they are known as DARs, or differential equations with a retarded argument. It is possible to have a neutral delay differential equation if an unknown function has a derivative at the delayed argument (NDDE).

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