# A Study of Weighted Distribution and Transmuted power function Distribution

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Abstract - The distribution of weight in a vehicle, such as a car, an aircraft, or a train, is known as weight distribution. As a rule, it is stated in the form x/y, where x is the proportion of weight in the front and y is the percentage of weight in the rear. Weight distribution has a direct impact on a number of vehicle attributes, including handling, acceleration, traction, and component life, in vehicles that rely on gravity. So, the planned use of the vehicle has an effect on the weight distribution. Drag cars, for example, use rear-wheel drive to enhance traction while minimizing reactive pitch-up torque. By putting a little quantity of counterweight front of the rear axle, it provides this counter-torque. It is common to use the power function distribution to simulate failure processes and to analyses data on the distributions like the Weibull distribution, power function distributions are very simple to understand in terms of their mathematical structure. A primary focus of the research was on Characterizations. Characteristics based on notions of dependability, It is possible to calculate the probabilities of a weighted distribution based on the weighted distribution's size-biased distribution, as well as the maximum likelihood estimation of a simulated dataset.

Keyword - Distribution, Weighted

## INTRODUCTION

In general, a weighted probability is a mean. As a result, the business pipeline value of each contract may be determined and utilized to predict the revenue amount produced by each contract. It's a big part of sales forecasting reports. To better understand how ascertainment processes impact the distribution of recorded observations, Fisher (1934) introduced the idea of weighted distributions. When traditional distributions were found to be insufficient for the purpose, Rao (1965) devised and formalized it in general terms to represent statistical data. There are a variety of situations in which weighted distributions might be useful, as explained in Rao's study. Examples of when it's not possible to get a random sample from an initial distribution: Due to a lack of information, a lower value may be acquired or an unjustly biassed sampling technique may be used, depending on the condition of the original observation. On damage models, discrete distributions and the sampling method that produces numerous weighted distributions Rao garnered a lot of praise. When the recorded data cannot be regarded a random sampling from the original distributions, then these are the cases. As a consequence, the original observation may have a lower value than it should have because certain events aren't visible, or because damage was done to it, or because of a sampling technique that unfairly gave various units of the original an unfair chance. Distinctive distributions, sampling processes, and sampling procedures that create a wide variety of weighted distributions were the main emphasis of Rao's research.

"A mathematical definition of the weighted distribution is as follows. Let  $(\Omega, \Im, P)$  be a probability space,  $X:\Omega \to H$  be a random variable (rv) where H = (a, b) be an interval on real line with a > 0 and b(> a) can be finite or infinite. When the distribution function (df) F(t) of X is absolutely continuous with probability density function (pdf) f(t) and wet) be a non-negative weight function satisfying  $\mu_w = E(w(X)) < \infty$ , then the rv XII having pdf"

$$f^{w}(t) = \frac{w(t)f(t)}{\mu_{w}}, \quad a < t < b$$
 1.1)

It's said to have a weighted distribution since that's how X's distribution is weighted as well. Discrete and continuous definitions are equivalent.

One of the most important considerations when utilizing weighted distributions to find good models for observed data is the weight function to use. Our models may change based on the weight function we use w (t) we choose. A weight function that relies on unit lengths (i.e.w(t) =t) results in a length-biased distribution, for example. For length-biased rvs, X L's pdf is

$$f^{L}(t) = \frac{tf(t)}{\mu}; a < t < b$$
 (1.2)

Where  $\mu = E(X) < \infty$  For example, Cox (1962)started by looking at length biases in terms of renewal theory A length bias in clinical trials, reliability theory, survival analysis, and demographic research may all be caused by using the improper sample frame. In order to sample a bigger value more often, a sampling rate proportionate to the item's length is used. In the context of length-biased sampling, studies of family size and the sex ratio have been conducted in detail.

To put it another way, a probability proportional to a measure of unit size is used in the sampling process., i.e., when  $w(t) = t^a$ ; a > 0, When this occurs, the resulting distribution is said to be size-biased. The weight function is the primary focus of the great majority of research on length-biased sampling. Denoting  $\mu_a = E(X^a) < \infty$ , Thepdf specifies the distribution of the order a size-biased rv X S

$$f^{S}(t) = \frac{t^{a} f(t)}{\mu_{a}}; a < t < b$$
(1.3)

When a=1, (1.3) becomes the pdf of arv with a length bias. For example, in life-extension studies, Scheaffer (1972), Gupta (1984), etiological research, and studies of animal populations and human families (Rao (1965, 1977, 1985), size-biased distributions appear. Takahasi (1968), Bartlett (1969), and Zelen (1970) have all investigated the influence of size-biased sampling on cell kinetics issues and the distribution of cell populations (1974).

The following is a list of the most essential weight functions that may be utilised in both discrete and continuous setups.

$$w(t) = t$$
  

$$w(t) = t^{a}; a > 0$$
  

$$w(t) = (1 - (1 - \beta)^{t})0 < \beta < 1$$
  

$$w(t) = (t + 1)$$
  

$$w(t) = t(t - 1) \dots (t - r + 1); r > 0$$
  

$$w(t) = \phi^{t}; 0 < \phi < 1$$
  

$$w(t) = \exp(\phi t)$$

These weight functions may also be used to model data using the IDs that connect the original and weighted random variables. Theories based on various hypotheses regarding the link between the original and weighted distributions may provide intriguing and valuable insights.

## Characterizations

Physical features that influence data pattern may be used to build an accurate model based on a challenge with characterizing a distribution. Original and weighted distributions with different weight functions are studied for their moment qualities, form invariance and reliability attributes by researchers. In this part, you will learn about weighted distributions in general.

It was Krumbein and Pettijohn (1938) who first noted that the log nominal distribution closely matched the particle sizes that had been measured. Gupta (1975) used the length-biased distribution features to describe distinct distributions. Gupta (1976) used the restriction that the mean of length-biased rv is twice the original rv's value to first characterize the exponential distribution. By comparing the variances of the logarithms of the original and weighted distributions, Mahfoud and Patil (1982) established the log normal distribution. This method was used to investigate the log-normal, Poisson, and gamma distributions. As part of their investigation of the distribution's Inverse Gaussian characteristic functions, Kirmani and Ahsanullah (1987) examined the link between its characteristic functions in its length-biased and undistorted forms.

## Characterizations based on reliability concepts

In reliability and survival analysis, length-biased models are often used. Longer distributions are critical to renewal theory, according to Cox (1962). Imagine that, in the event of one component in a system failing, another one with the same life distributions is used to replace it. A renewal process is then formed by the progression of component life spans. Let L(t) = U(t) + V(t)U(t) and V(t) represent the age and remaining life of the component at any

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given time t, respectively. The limiting pdf of L shows a length-biased distribution (t). The equilibrium distribution of recurrence times is shown in the limiting condition.

## **Bivariate weighted distributions**

According to Mahfoud and Patil, the weighted distribution may be extended in two dimensions (1982). For a pair of non-negative random variables (X 1,X 2) with a joint density function  $f(f_1, f_2)$  and a non $w(t_1, t_2)$ negative weight function that  $E(w(X_1, X_2) < \infty)$ , the random vector  $(X_{1_w}, X_{2_w})$ the density function

$$f^{w}(t_{1}, t_{2}) = \frac{w(t_{1}, t_{2})}{E(w(X_{1}, X_{2}))} f(t_{1}, t_{2}); a_{i} < t_{i} < b_{i}; i = 1, 2$$
(1.4)

A bivariate weighted distribution has X 1 and X 2 as the weights, and is called such. The p-variety case is a simple expansion. Mahfoud and Patil reached findings concerning certain probability distributions based on the nature of particular weight functions.

## Weighted Probabilities Calculate

Probabilities are measures of the likelihood that certain occurrences will take place. There is an equal chance of rolling a one on a six-sided die as there is of rolling any other number since each number appears once in six. However, not all outcomes are equally weighted in all cases. If you throw in a second die, the chances of the dice adding up to two are far lower than the odds of the dice adding up to seven. One dice combination (1, 1) yields two whereas various other die combinations (such as 3, 4, 4, 3, 2, 5 and 5) yield seven.) Are available, Calculate the total number of potential outcomes. Rolling two dice, for example, yields 36 potential results since each die has six possible outcomes, resulting from multiplying six by six. Check to see how many possibilities there are for achieving your goal. How many ways an eight may be rolled? There are five: 2,6, 3,5, 4,4, and 5,3. (6,2). To compute the weighted probability, divide the number of methods to reach the desired result by the total number of potential outcomes. Divide five by 36 to get at a chance of 13.89 percent, or 0.01399.

## Weighted Distribution

Weighted distribution may be traced back to the work of Fisher (1934), in which it is examined how ascertainment procedures might affect how recorded observations are distributed. Rao (1965) used it to statistical data when conventional represent distributions were determined to be inappropriate for the task. Rao then refined and formalized it in generic terms. Consequently, Fisher (1934) and Rao (1965) are credited with first pondering non-experimental, non-replicated, and non-random observation conditions before introducing the notion of weighted distribution. Observations are only recorded when they

are encountered, and this may be shown best via encounter sampling. Conditions where observations are collected with probability proportional to some weight function w may be described more broadly (x)..

## Size - Biased Distribution

The weight function w(x)=xc in the equation results in a size-biased distribution (1). The probability density function for the size-biased distribution may be found here.

$$f(x) = \frac{x^c f(x)}{\mu_c}, \quad x > 0.$$

Research on Poisson-Lindley distribution parameters was conducted using the moment technique and maximum likelihood estimation. It was shown by Hassan et al. that a misclassified, size-biased modified power series distribution may be used (2008). Misclassification of the size-biased power series distribution results in x = 2 being misclassified as x =1 in certain cases. Size-biased generalized negative binomial and generalized Poisson distributions were also included in its recurrence relations, as were certain particular instances. New methods were devised by Mir (2009a) in order to estimate the generalized logarithmic series distribution, which is size-biased.

## LITERATURE REVIEW

Brijesh P. Singh and UtpalDhar Das (2020) the weighted distribution is largely employed in different real world domains such as ecology, reliability engineering, medical research etc. The hazard function, the moment generating function, the characteristic function, the cumulative generating function, the Renyi entropy, and the cumulative residual entropy are some of the statistical features that are calculated. The approach of maximum estimation is likelihood used for parameter estimation. The studied weighted probability distribution is used to two actual data sets of waiting time to test the appropriateness and applicability.

TasfalemEyob and Rama Shanker (2019) in this research, a two-parameter weighted Rama distribution which incorporates one parameter Rama distribution established by Shanker has been presented for modelling actual lifespan data. Statistical features of the distribution including forms of a probability density function, moments and moment related measures, hazard rate function, mean residual life function, and stochastic orderings have been examined. The estimate of its parameters has been studied using the approach of maximum likelihood. There has been some discussion over the potential use of this distribution.

M. Mahdy (2018) an important dynamic uncertainty metric in reliability and survivability research is the weighted entropy measure. There are fresh results

on weighted entropies with particular descriptions. Using specific dependability systems, such as a series structure and a parallel structure, we have also offered some results for weighted entropy residuals and weighted past residuals of order statistics. We also introduced the weighted residual (past) entropy lower limit. Moreover, the stochastic ordering based on weighted entropy is provided. A real-world application of these non-parametric weighted entropy estimates completes our proof of concept.

Aijaz Ahmad Dar, A. Ahmed and J. A. Reshi (2018) A weighted Maxwell-Boltzmann distribution, or WMD, was presented in this study. Various characteristics of the new distribution have been investigated in great depth. There are no closed-form estimators, but parameters are computed by fitting WMD to specific data sets using the MLE method. Accordingly, we have used a WMD statistical model to four distinct data sets to demonstrate its validity and potential. We next compared the special instances of WMD to see which had the lowest values of BIC, AIC, and AICC after fitting WMD to the relevant data sets. The Inverse Cdf technique is used to generate random numbers from WMD. Programming language R has been used for the simulation.

AamirSaghir, G. G. Hamedani (2017) Weighed distributions are commonly utilized to construct accurate statistical models in numerous disciplines like medicine, ecology and trustworthiness to mention a few. Using weighted distributions instead of standard distributions may significantly improve the accuracy of statistical data modelling and forecasting. There have been several studies done on the distribution of body weight. An overview of various distributions is provided in this article. Future research and various tactics are considered in light of the different weight models. We next show how these distributions may be described by looking at the correlation between two truncated moments.

AshkanNikeghbali Dirk Zeindlerb,(2013) and extended Weighted and weighted random permutations, which are important models in physics and which expand Ewes' measure, are studied for their asymptotic behavior and total number of cycles in this study. Combining combinatory and complex analysis (such as a singularity analysis of generating functions) is used to illustrate that under certain analytic circumstances (on relevant generating functions) the cycle process is converging toward an independently-variable vector of independent Poisson variables. It is possible for us to regulate the pace of convergence by obtaining an asymptotic estimate and an error estimate for the characteristic functions of the various random vectors of interest. Much for the whole number of cycles, we may demonstrate even tighter convergence, specifically mod-Poisson convergence. To get a Poisson approximation of the total number of cycles and estimates of big deviations, we first use earlier work on mod-Poisson convergence.

Lyman L. McDonald (2010) Using weighted distribution theory and its applications in observational research; I examine the problem of bias. If every observation is given an equal chance of being recorded, the recorded observations will be skewed and will not reflect the original distribution. Despite the fact that weighted distributions appear often in numerous domains, the fundamental notion of weighted distributions as a significant stochastic concept does not seem to have been well understood," wrote G. P. Patil and C. R. Rao in 1977. To this day, the adage holds true. An opportunity to organize and comprehend a collection of independent statistical approaches for analyzing data from observational studies is being overlooked by our profession. Students who are interested in environmental concerns and statistics can find examples of how these concepts might be used to the study of animal and fish populations in this article.

# **RESEARCH METHODOLOGY**

The term "Research Methodology" refers to the study of how a study's methodologies were selected and implemented. Theories that support the selection and use of techniques are included in this debate as well. Data gathering procedures are also explained in your methodology. Experiments on samples, surveys, or interviews of research, 'A contextual framework' for research is a coherent and logical system based on ideas, attitudes and values that influences the decisions researchers make,' or so the term "methodology" would have it. Based on their historical history, different disciplines may use different procedures in the same field, but they are all theoretically analyzed in the same manner. In this way, there is a continuum of techniques that spans different views of how knowledge and reality should be interpreted. Methodologies are placed within broader philosophical frameworks and approaches.

It may be seen as a continuum from a mostly quantitative to a largely qualitative approach. It's fairly uncommon for researchers to mix and match ways in order to achieve their research goals, and hence have methodologies that are multi-method or multidisciplinary. As a general rule, methodologies are similar to methods in that they provide solutions. Instead, a methodology provides a theoretical framework for determining which technique, collection of methodologies, or best practices may be applied to the research questions at hand.

Examine the adequacy of weighted distributions in phenomenon random will modeling а be get probabilities recorded with observations proportional to some weight function w(x). For illustration purpose, weighted Maxwell distribution, weighted transmuted power function distribution, transmuted version of weighted exponential distribution and weighted gamma-Pareto distribution will be studied in detail. It will be shown that

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introduced versions prove comparatively to be more adequate than the parent distributions for modeling some considered data sets.

## DATA ANALYSIS

## Maximum Likelihood Estimation

Let  $0 \le x_{(1)} \le x_{(2)} \le ... \le x_{(n)} \le \theta$  be an ordered sample of size *n* from WTPFD ( $\alpha, \theta, \beta, \omega$ ). As a result, the logarithmic likelihood function is equal to

$$\log[l(\alpha, \theta, \beta, \omega | \underline{x})] = \sum_{i=1}^{n} \log((\beta + 1)\theta^{\alpha} - 2\beta x_{i}^{\beta}) + (\alpha + \omega - 1) \sum_{i=1}^{n} \log x_{i} + n \log(\alpha + \omega)$$
$$-n \log(2\alpha - \beta\omega + \omega) - n(2\alpha + \omega) \log \theta + n \log(2\alpha + \omega)$$
(3.28)

Differentiating (3.28) w.r.t.  $\theta$  we get the following gradient:



## Simulated data

A sample of size n = 100 from WTPD with  $\alpha = 5$ ,  $\theta = 10$ ,  $\beta = -0.7$  and  $\omega = 2$  is generated by using the algorithm given Section 3.8. This generated data set is named as "simulated\_data" and is given as:

> simulated\_data<-DataWTPFD (100, 1, 10, 2, -0.5, 4)

> head(simulated\_data)

[1] 8.298195 8.705786 9.251606 9.867045 7.978410 9.852132 9.921221

Table 1: M.L.E.'s, AIC and KS-test statistic

\$\bar{\theta_{mte}}\$           D         9.969           D         9.969           D         9.969           D         9.969           9.969         9.969           9.969         9.969           9.969         9.969           9.989         9.989           9.989         9.989           9.989         9.989           D         6.29           CD         6.29	\$\hfill\$mile\$           5.503         4.264           4.888         5.502           4.239         8.025           1*         1*           0.014         4.495	<i>\$\bar{\bar{\mathcal{P}}}\$</i> mle -0.664 -0.555 -0.611 -0.664 0* 0* 0* 0* 0* 0* 0* 0* 0* 0* 0* 0* 0*	<i>ω<sub>mie</sub></i> 2.07e-6 2* 1* 0* 3.7858 0* 7.0250 0*	214.48 215.86 215.15 214.48 218.87 218.87 218.87 265.39	222.48 221.86 221.15 220.48 222.87 222.87 222.87 267.43	0.0683 0.0680 0.0645 0.0607 0.0904 0.0904 0.0904 0.6559	0.7415 0.7435 0.8002 0.8551 0.3875 0.3874 0.3874 2.2e-16 0.2431
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9.989       9.989       9.989       9.989       9.989       9.989       0.989       0.989       0.989       0.989	4.239 8.025 1* 1* 0.014 4.495	0* 0* 0* 0*	3.7858 0* 7.0250 0* 8.2718	218.87 218.87 218.87 265.39 90.472	222.87 222.87 222.87 267.43 98.472	0.0904 0.0904 0.0904 0.6559 0.1483	0.3875 0.3874 0.3874 2.2e-16 0.2431
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			2	91.642	97.642	0.1437	0.2865
D 6.29	5.290	0.921	1*	92.031	98.031	0.1456	0.2723
6.29	6.107	0.911	0*	92.463	98.463	0.1478	0.2557
6.29	4.282	0*	0*	108.19	111.00	0.2603	0.0034
6.29	1*	0*	0*	109.09	111.09	0.6264	2.22e-16
D 4001.89	0.012	0.997	1.0698	799.18	806.18	0.1293	0.3437
D 4001.89	1.62e-8	1	2*	839.90	845.90	0.3712	4.97e-7
D 4001.89	0.057	0.987	1*	799.18	805.18	0.1286	0.3497
4001.89	0.809	0.859	0*	800.61	806.61	0.1331	0.3104
4001.89	0.574	0*	0*	810.74	814.74	0.1833	0.0609
	) 1*	0*	0*	1171.7	1173.7	0.3824	4.38e-7
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## **Transmuted Weighted Exponential Distribution**

The constant hazard rate and memory less feature of the exponential distribution make it a popular choice for modelling life expectancy distributions. Because of this, exponential distributions are often used in survival analysis, where the lack of memory is crucial. The basic exponential distribution as well as a number of additional expansions has been employed in a range of fields. W was the first person to extend the exponential. Which is now often referred to as the Weibull distribution? Generalized exponential and beta distributions, weighted exponential distributions, Kumaraswamy exponential distributions. and exponentiated exponential distributions have all been developed as a result of attempts to generalize the exponential distributions. we have introduced a three parameter extension of exponential distribution by employing quadratic rank transmutation map (QRTM) and the concept of weighted distribution. The weight function used is  $w(x) = x^{\omega}$ . The developed extension is named as transmuted weighted exponential distribution and abbreviated as TWED. The derivation of TWED is given below.

## Table 2 : Some special cases of TWED

β	ω	Distn.	$f_t(x)$	$F_t(x)$
0	0	ED	$\lambda e^{-\lambda x}$	$\gamma(1, \lambda x)$
	1	LBED	$\lambda^2 x e^{-\lambda x}$	$\gamma(2,\lambda x) \big( 1 + \beta \Gamma(2,\lambda x) \big)$
	2	ABED	$\lambda^3 x^2 e^{-\lambda x}/2$	$\gamma(3,\lambda x)(2+\beta\Gamma(3,\lambda x))/4$
≠ 0	0	TED	$\lambda e^{-\lambda x} (1 - \beta + 2\beta e^{-\lambda x})$	$\gamma(1, \lambda x)(1 + \beta \Gamma(1, \lambda x))$
	1	TLBED	$\lambda^2 e^{-\lambda x} x (1 - \beta + 2\beta \Gamma(2, \lambda x))$	$\gamma(2,\lambda x) \big(1 + \beta \Gamma(2,\lambda x)\big)$
	2	TABED	$\lambda^3 e^{-\lambda x} x^2 \big( 2(1-\beta) + 2\beta \Gamma(3,\lambda x) \big) / 4$	$\gamma(3,\lambda x)(2+\beta\Gamma(3,\lambda x))/4$

Table 3: Characteristics of TWED at different value of  $\omega$ ,  $\lambda$  and  $\beta$ .

Distn.	ω	β	λ	μ	$\sigma^2$	γ1	<i>Y</i> <sub>2</sub>	cv	$H_R(\delta)$	
									<i>δ</i> =0.5	δ=0.9999
TED	0	0.5	5	0.1500	0.0275	2.467	9.471	1.106	-0.4288	-0.9052
		0	10	0.0750	0.0069	2.467	9.471	1.106	-1.1324	-1.5922
		-0.5	5	0.2000	0.0400	2.000	6.000	1.000	-0.2196	-0.6068
			10	0.1000	0.0100	2.000	6.000	1.000	-0.9210	-1.2967
			5	0.2500	0.0475	1.715	4.504	0.872	-0.0880	-0.4016
			10	0.1250	0.0119	1.715	4.504	0.872	-0.7784	-1.0962
TLBED	1	0.5	5	0.3250	0.0594	1.713	4.658	0.749	0.0761	-0.2308
		0	10	0.1625	0.0148	1.713	4.658	0.749	-0.5722	-0.9197
		-0.5	5	0.4000	0.0800	1.414	3.000	0.707	0.2319	-0.0294
			10	0.2000	0.0200	1.414	3.000	0.707	-0.4659	-0.7255
			5	0.4750	0.0894	1.215	2.309	0.629	0.3181	0.0755
			10	0.2375	0.0223	1.215	2.309	0.629	-0.3762	-0.6137
TABED	2	0.5	5	0.5062	0.0925	1.394	3.141	0.601	0.3349	0.0781
		0	10	0.2531	0.0231	1.394	3.141	0.601	-0.3482	-0.6136
		-0.5	5	0.6000	0.1200	1.155	2.000	0.577	0.4583	0.2388
			10	0.3000	0.0300	1.155	2.000	0.577	-0.2163	-0.4499
			5	0.6938	0.1300	0.987	1.558	0.519	0.5389	0.3131
			10	0.3469	0.0325	0.987	1.558	0.519	-0.1624	-0.3816

# CONCLUSION

a study is conduct to Imperative for understanding the subject matter, comprises of general introduction about the concept of weighted distributions, their importance, literature review related to different statistical measures/techniques used and objectives of the study discusses the derivation of weighted version of Maxwell-Boltzmann distribution on considering the weight function  $w(x) = x \omega$ . The introduced version is named as weighted Maxwell-Boltzmann distribution and abbreviated as WMD. Expressions related to different statistical measures associated with WMD have been derived and studied in detail. Inverse sampling method is used to generate random numbers from WMD with the aid of "uniroot" function in R programming. Deals with the derivation of different statistical measures associated to four parameter extension of power function distribution. The extension of power function distribution has been introduced by using the concept of transmutation and weighted distribution Describes transmuted weighted exponential distribution.

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