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A Study of Triple Derivations, Jordan U-Generalized of Semiprime Rings

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Abstract - Within the scope of this work, we prove, among other things, that if d is a Jordan derivation of a semiprime ring R, then R is a Jordan derivation of d, then d is a quarter derivation. Because of this, we are able to show that d can be derived three times. We further prove that if and only if R is a semiprime ring of character 2, and d acts as a homomorphism on R, then d is a central derivation. We prove that each semiprime ring R of character 2 with a u-generalized derivation G also has a corresponding u-*generalized derivation G. Similarly, we prove that G is a u-*generalized derivation if and only if R is an asemiprime ring of character 2. We also establish that R is commutative if and only if $G([x,y]) = 0$.

Keywords - Triple Derivations, Jordan U-Generalized, Semiprime Rings

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INTRODUCTION

Ring theory is a strong tool for studying issues of significant historical and scientific significance since it unifies numerous different areas of mathematics. Derivational rings are not the kind of topic that sees radical shifts in understanding. The connections between derivations and ring structure, however, have been the subject of much research over the last 50 years by a wide range of scholars. In algebra and analysis, it was a popular line of inquiry to consider whether or not a map could be comprehended in terms of simply its "local" aspects. Specifically, Herstein famously questioned whether or not a normal derivation yields a map that deviates the Composite Lie algebra over a subalgebra of Lie's using only prime rings. The pioneering finding in this topic may be found in Kaplansky's unpublished work on matrix algebras over a field. Martindale has investigated this for primitive rings when idempotent is present. The strong approach to functional identities was invented, and only then was Herstein's issue addressed in complete generality. In 1993, Bre sar found a solution to this issue for prime rings. Beidar and Chebotar solved the problem of constructing a Lie ideal of a prime ring. Whether or not a Lie derivation can be produced by a regular one was a debatable topic that required investigation; see, for example, Banning & Mathieu, Villena.

Algebraic number theory and the study of ideals had a significant role in the development of ring theory. Wilhelm Julius The foundations of ring theory were first established by the renowned German mathematician Richard Dedekind, however, Hilbert is credited with coining the term "ring." The natural numbers, algebraic number theory, and the definition of real numbers all owe a great deal to Dedekind's work in abstract algebra. The rudiments of ring theory were established in 1879 and 1894, respectively, thanks to the ideas of an ideal. When it comes to ring theory, an algebraic structure is crucial. The group ring, the division ring, In mathematics, rings are generalizations of the universal enveloping algebra and polynomial identities. These rings play an important role in the solution of many different issues in algebra and number theory. Topology and mathematical analysis are only two of the numerous branches of mathematics where rings appear often. In 1957, E. C. Posner introduced the concept of derivation to the field of ring theory. Improvements to the derivations in ring theory have led to the development of several different derivations, including the generalized derivation, the Jordan derivation, the symmetric bi-derivation, and the generalized Jordan derivation.

DERIVATIONS

Let A be a ring and B be a bimodule over A. A derivation d: A→B is an additive map that satisfies the Leibniz rule

$$d(xy) = xd(y) + d(x)y.$$

If B is an algebra over A and if we are given in addition a ring homomorphism θ:A→B, a twisted derivation concerning θ (or a θ-derivation) is an additive map d: A→B such that

$$d(xy) = \theta(x)d(y) + d(x)y.$$

When θ is the morphism defining the structure of A-algebra on B, a θ-derivation is nothing but a derivation. In general, if ı:A→B denotes the defining morphism shown above, verification is simple. θ−ı is a θ-derivation

You may use the information in this file to derive and twist derivations over commutative rings whose values are algebras. The collection of derivations (or -derivations) in this instance is a module over B.

TYPES OF RINGS

1. Associative ring
2. Nonassociative ring
3. Alternative ring
4. Lie Ring
5. Jordan ring
6. Associator
7. Commutator
8. Anti-commutator
9. Commutative Ring
10. Near Ring

TRIPLE DERIVATIONS OF SEMIPRIME RINGS

Here we show that each Jordan derivation of a semiprime ring R is also a quarter derivation of R. This allows us to demonstrate that d is triple derivable. We further show that d is a central derivation if and only if R is a semiprime ring of char. 2 and d operates as a homomorphism on R.

We know that an additive mapping $d: R \to R$ is called a Jordan derivation if $d(x^2) = d(x)x + xd(x)$ for all x in R. An additive map d from a ring R to R derivation is a triple derivation if $d(xyx) = d(x)yx + xd(y)x + xyd(x)$ hold for all $x,y \in R$ and d is a quarter derivation if $d(xyzx) = d(x)yzx + xd(y)zx + xyd(z)x + xyzd(x)$ hold for all $x,y,z \in R$. Throughout this section, R will denote a semiprime ring and Z its center.

First we prove the following Lemmas:

Lemma 1: Let R be any ring and let $T(a) = \{r \in R / r(ax - xa) = 0$ for all $x \in R\}$ for all $a \in R$. Then T(a) is a two-sided ideal of R.

Proof: Clearly $T(a)$ is a left ideal of R. It remains to show that if $u \in T(a)$, $x \in R$, then $ux \in T(a)$. But then, for all $r \in R$, $u(axr - xra) = 0$. Thus $u\{(ax - xa)r + x(ar - ra)\} = 0$. Since $u \in T(a)$, $u(ax - xa) = 0$, and so we have that $ux\,(ar - ra) = 0$ for all $r \in R$. Then $ux \in T(a)$. hence the lemma is proved.

Lemma 2; If R is a prime ring and if a ∊ R is not in Z, the center of R, then 7(a) = (0).

Proof: Since $a \notin Z$, for some $b \in R$, $ab - ba \neq 0$. If $T(a) \neq (0)$,), then $T(a)\ \ (ab - ba) = (0)$. So the right ideal 7(a) is annihilated by a nonzero element. By the definition of a prime ring, either $T(a) = (0)$ or $ab - ba = 0$. Since $a \notin Z$, $ab - ba \neq 0$. So $T(a) = (0)$

Now we prove the following results:

Theorem 1; If d is a Jordan derivation of a semiprime ring R, then dis a quarter derivation, that is, d(abca) = d(a)bea + ad(b)ca + abd(c)a + abcd(a) for all a,b,c ∊ R.

Proof:
We have $d(a^2) = ad(a) + d(a)a$. We replace a by $a{+}bc$. Then

$$d(abc + bca) = d(a)bc + ad(b)c + abd(c) + d(b)ca + bd(c)a + bcd(a), \quad 4.1.1$$

for all $a,b,c \in R$.

Consider $W = d(a(abc + bca) + (abc + bca)a)$.

$= d(a)\ (abc + bca) + a\ d(abc + bca) + d(abc + bca)a + (abc + bca)\ d(a)$

$= d(a)\ abc + d(a)\ bca + a(d(a)bc + ad(b)c + abd(c) + d(b)ca + bd(c)a + bcd(a)) + (d(a)bc + ad(b)c + abd(c) + d(b)ca + bd(c)a + bcd(a))a + abcd(a) + bca\ d(a))$.

$W = d(a)abc + 2d(a)bca + ad(a)bc + a^2d(b)c + a^2bd(c) + 2ad(b)ca + 2abd(c)a + 2abcd(a) + d(b)ca^2 + bd(c)\ a^2 + bcd(a)a + bcad(a)$.

On the other hand,

$W = d(a(abc + bca) + (abc + bca)a) = d(a^2bc + bc\,a^2 + 2abca)$.

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$W = d(a(abc + bca) + (abc + bca)a) = d(a^2bc + bc\,a^2 + 2abca)$.

$= d(a^2)bc + a^2d(b)c + a^2bd(c) + d(b)c\,a^2 + bd(c)\,a^2 + bcd(a^2) + 2d(abca)$.

$= ad(a)bc + d(a)abc + a^2bd(c) + d(b)ca^2 + bd(c)a^2 + bcad(a) + bcd(a)a + 2d(abca)$.

By comparing the above two expressions for W, we obtain

$2d(abca) = 2[d(a)bca + ad(b)ca + abd(c)a + abcd(a)]$.

Hence $d(abca) = d(a)bca + ad(b)ca + abd(c)a + abcd(a)$.

Theorem 2: When R is a semiprime ring, then d is a triple derivation if and only if d is a Jordan derivation of R, that is, d(aba) = d(a)ba + ad(b)a + abd(a) for all a,b∊R.

Proof:
We have $d(a^2) = ad(a) + d(a)a$. We replace a by $a{+}b$. Then we

obtain

$$d(ab + ba) = d(a)b + ad(b) + d(b)a + bd(a),$$

for all $a,b \in R$.

Consider $W = d(a(ab + ba) + (ab + ba)a)$.

$= d(a)\ (ab + ba) + a\ d(ab + ba) + d(ab + ba)a + (ab + ba)d(a)$.

$= d(a)ab + d(a)ba + a\ (d(a)b + ad(b) + d(b)a + bd(a)) + (d(a)b + ad(b) + d(b)a + bd(a))a + abd(a) + bad(a)$.

$= d(a)ab + 2d(a)\ ba + ad(a)b + a^2d(b) + 2abd(a) + 2ad(b)a + bad(a) + d(b)a^2 + bd(a)a$.

$W = d((a^2b + ba^2) + 2aba) = d(a^2)b + a^2d(b) + d(b)\ a^2 + bd(a^2) + 2d(aba)$.

$$= d(a)ab + ad(a)b + a^2d(b) + d(b)\ a^2 + bad(a) + bd(a)a + 2d(aba).$$

By comparing the above two expressions for W, we obtain

$2d(aba) = 2(d(a)ba + ad(b)a + abd(a))$.

$2d(aba) = 2(d(a)ba + ad(b)a + abd(a))$.

Hence $d(aba) = d(a)ba + ad(b)a + abd(a)$.

We linearize the result of Theorem 4.1.2 by replacing a by $a+c$, we arrive at

Corollary 1: If a,b $\in$ R and if ab = 0, then d(ba) = d(b)a + bd(a).

Proof: By Theorem 2.1.7, d(a)b + ad(b) = 0. But d(ba) = d(ab + ba) =d(a)b + ad(b) + d(b)a + bd(a) = d(b)a + bd(a).

Corollary 2: If a,b $\in$ R and if ab = 0, then for all c € R, d((ba)c) = d(ba)c + bad(c).

Proof: By Theorem 2.1.3, d(cab + bac) = d(c)ab + d(b)ac + cd(a)b +bd(a)c + cad(b) bad(c).

Since ab = 0, then d((ba)c) = d(b)ac + bd(a)c + bad(c) = c(d(a)b+ ad(b))

= d(b)ac + bd(a)c + bad(c) by Theorem 2.1.7.

= d(ba)c + bad(c) by the Corollary 2.1.1.

Let V = {a $\in$ R/d(ax) = d(a)x + ad(x) for all x $\in$ R}.

By Corollary 2.1.2, if ab =0, then ba $\in$V.

For the sake of convenience of writing we introduce the symbol, for a,b$\in$R, of a' to mean a' = d(ab) — d(a)b — ad(b).

We note

$$a^{b+c} = a^b + a^c \qquad 2.1.6$$

and $a^b = -b^a$.

2.1.7

Equation is a of the fact that d is a Jordan derivation of R.

for all a,b $\in$ R.

JORDAN U-GENERALIZED DERIVATIONS OF SEMIPRIME RINGS

Bresar showed that if we have a semiprime ring R, then D is uniquely defined by a function G, and that G must be a derivation if and only if G is a function from R to R and an additive mapping such that. G(x,y) = G(x)y + xD(y) for any x,y in R. Ashraf and Rehman provided evidence that R is really a set of interconnected characters. When R has a commutator that is not a zero divisor, then any Jordan generalized derivation on R is a generalization of R.

Here we show that if R is a semiprime ring of character 2, then G is a u-generalized derivation of R in the Jordan sense. We prove a similar result for the case when G is a Jordan u-*generalized derivation of a semiprime ring R of char. If the answer to #2 is "yes," then G is a u-*generalization. If G([x,y]) = 0, then we demonstrate that R is commutative.

We know that an additive mapping $G: R \to R$ is a Jordan generalized derivation if there exists a derivation D from R to R such that $G(x^2) = G(x)x + xD(x)$ for all x in R. An additive mapping D from R to itself is a u-derivation if D(xy) = D(x)u(y) + xD(y) hold where u is a homomorphism of R, for all x,y in R. An additive mapping $G: R \to R$ is a u-generalized derivation if there exists a derivation D from R to R such that G(xy) = G(x)

Theorem 3: If G is a Jordan u-*generalized derivation of a semiprime ring R of char. 2, then G is an u-*generalized derivation.

Proof: Since $G(x^2) = u(x)\ G(x) + D(x)x.$ If we replace x by x+y, then

$$G(xy + yx) = u(y)G(x) + u(x)G(y) + D(x)y + D(y)x,$$

,

for all x,y $\in$ R.

Consider $W = G(x(xy + yx) + (xy + yx)x)$.

$$= u(xy + yx)G(x) + D(xy + yx)x + u(x)\ G(xy + yx) + D(x)\ (xy + yx).$$

$$= u(y)u(x)\ G(x) + 2u(x)u(y)\ G(x) + u(y)\ D(x)x + D(y)x^2 + 2u(x)D(y)x + 2D(x)yx + u(x)u(x)\ G(y) + u(x)D(x)y + D(x)xy.$$

On the other hand,

$$W = G(x^2y + yx^2 + 2xyx) = G(x^2y) + 2G(xyx) + G(yx^2).$$

$$= u(y)u(x)\, G(x) + u(y)D(x)x + D(y)x^2 + 2G(xyx) + u(x)u(x)G(y)$$

$$+ u(x)\, D(x)y + D(x)xy.$$

By comparing the two expressions for W, we obtain

$$2G(xyx) = 2\,[u(x)u(y)\, G(x) + u(x)D(y)x + D(x)yx].$$

$$G(xyx) = u(x)u(y)\, G(x) + u(x)\, D(y)x + D(x)yx.$$

$$= u(yx)\, G(x) + D(yx)x.$$

We put z = yx, then $G(xz) = u(z)G(x) + D(z)x$ which implies that G is a

u-*generalized derivation.

Theorem 4: Assume that R is a char. 2 semiprime ring and that G is a non-zero Jordan u-generalization of R. It is known that R is commutative if and only if G([xy]]) = 0.

Proof: We substitute xy for x in G([x,y]) = 0. Then we obtain

$$0 = G([xy,y]) = G([x,y])\, u(y) + [x,y]\, D(y),$$ and so

$$[x,y]\, D(y) = 0, \text{ for all } x,y \in R.$$

We replace x by rx, then [rx,y] D(y) = 0 for all x,y,r ∈ R.

$$\Rightarrow [r,y]\, xD(y) = 0.$$

$$\Rightarrow [r,y]\, RD(y),\ \forall\, x \in R.$$

Now we choose a family $P = \{p_\alpha/\alpha\in\wedge\}$ of prime rings R such that $\cap P_\alpha=\{0\}$ and let p denote a fixed one of the pg. From the above equation it follows that $[r,y] \in p$ or $D(y) \in p$ for all y $y \in R$. Let $A = \{y\in R/y\in Z\}$ and $B = \{y\in R/D(y) = 0\}$. Then A and B are two additive subgroups of (R,+) such that $R = A \cup B$. However a group cannot be the union of proper subgroups. Hence either R = A or R = B. If R = A, then R ⊂ Z and so R is commutative. If R = B, then R = 0 which contradicts the hypothesis. So, we must have r ∈ Z, for all r ∈ R. Hence R is commutative.

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