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Applications of Equation of Continuity

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Abstract - Equation of continuity is interesting equation arising in fluid mechanics. The equation being computationally attractive and is highly beneficial. In this paper we will solve real life problem with help of equation of continuity. This equation also finds its applications in flows where compressibility is not significant.

Keywords - Continuity, Volume, Density, Incompressible, Potential kind, law of conservation of mass

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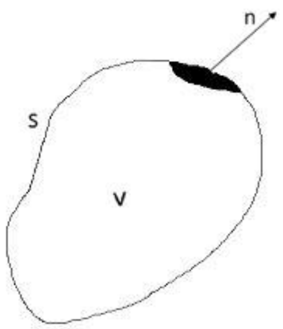
INTRODUCTION

In this paper we will throw light on equation of continuity and its interesting applications. Let us consider fluid flowing through some region of space from external source .Equation of continuity is derived using law of conservation of mass which states "The total mass of fluid within a region in absence of source or sink remain conserved". Let ρ be density of fluid at a point P. For calculation of density we need to know mass of small volume V of fluid containing point P.

Consider arbitrary surface S bounding volume V drawn in a region where fluid flows.



Total mass of fluid within volume V $=\int \rho dV$

Rate of increase of this mass in volume V $=\frac{\partial}{\partial t}\int \rho dV$

Mass of fluid leaving the surface S per unit time = $\int \rho\, \vec{q}\,\hat{n} dS$

By gauss divergence theorem, mass of fluid leaving the surface S per unit time = $\int div(\rho\vec{q})dV$

By law of conservation of mass, Rate of increase of mass in volume V = Mass of fluid entering S per unit time

$$\frac{\partial}{\partial t}\int \rho dV = -\int div(\rho\vec{q})dV$$

$$\frac{\partial}{\partial t}\int \rho dV + \int div(\rho\vec{q})dV = 0$$

$$\int \left(\frac{\partial \rho}{\partial t} + div(\rho\vec{q})\right)dV = 0$$

This is true for arbitrary dV

Therefore, $\frac{\partial \rho}{\partial t} + div(\rho\vec{q}) = 0$ … (1)

Equation (1) is known as equation of continuity.

SOME SPECIAL CASES

1). If the fluid flow is steady (i.e. flow properties are independent of time) .Then equation (1) becomes

$$div(\rho\vec{q}) = 0$$

2). If flow is considered incompressible (i.e. volume and density of fluid does not change with applied pressure).Then Equation (1) becomes $\nabla\vec{q} = 0$

3). If velocity is of potential kind then $\vec{q} = -\nabla\emptyset$; Equation (1) will takes form $\frac{d\rho}{dt} - \rho\nabla^2\emptyset = 0$

Example: A spherical bubble of gas is formed as result of explosion under water that was at rest

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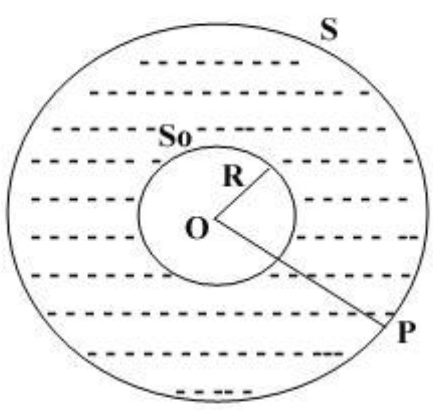
initially. Gas obeys adiabatic law of pressure (p V^{ω} = constant) during expansion . (Here p is pressure, V is volume, ω is constant) .It is required to find radius R of gas bubble at any time t after explosion, taking radii R_0 at time t =0.

Solution :



Radius of gas bubble can easily be found using simple adiabatic law of pressure and equation of continuity.

Consider concentric spherical surface in liquid region and velocity of fluid at any point in radial direction is $\vec{q} = q\hat{r}$

Since $\vec{q}=\frac{dr}{dt}\hat{r}$

We have $\vec{q} = q\hat{r} = \frac{dr}{dt}\hat{r} = -\frac{\partial \emptyset}{\partial r}\hat{r}$

At r = R, $\vec{q} = \dot{R}$

By equation of continuity, volume of fluid entering V at S_0 per unit time = volume of fluid leaving V at S per unit time

$$4\pi R^2\dot{R} = 4\pi r^2\dot{r}$$

$$\dot{r} = \frac{R^2\dot{R}}{r^2} = -\frac{\partial \emptyset}{\partial r}$$

$$\emptyset = \frac{R^2\dot{R}}{r}$$

If pressure at infinity is zero, using Bernoulli equation of motion

We have pressure p at r =R given by p = $\rho(\frac{3}{2}\dot{R}^2 + R\ddot{R})$

By adiabatic law of pressure pV^{ω} = constant

We have $p(\frac{4}{3}\pi R^3)^{\omega} = p_0(\frac{4}{3}\pi R_0{}^3)^{\omega}$

$$\frac{p}{p_0} = \left(\frac{R_0}{R}\right)^{3\omega}$$

$$\rho(\frac{3}{2}\dot{R}^2 + R\ddot{R}) = p_0\left(\frac{R_0}{R}\right)^{3\omega} \quad \text{… (2)}$$

Clearly this is second order differential equation in R (t) which can be solved easily.

Substitute $\dot{R}^2$= s, equation (2) will take the form as

$$\dot{R}^2 = s = \frac{2p_0 R_0{}^{3\omega} R^{3-3\omega}}{\rho R^3(3-3\omega)} + \frac{c}{R_0{}^3}$$

Since $\dot{R}$ =0 when R=R_0

We have c = $\frac{2p_0 R_0{}^3}{3\rho(\omega-1)}$

$$\dot{R} = \left(\frac{2p_0}{\rho}\left(\frac{R_0{}^3}{R^3} - \frac{R_0{}^4}{R^4}\right)\right)^{1/2}$$

$$\frac{dR}{dt} = \dot{R} = \left(\frac{2p_0}{\rho}\left(\frac{R_0{}^3}{R^3} - \frac{R_0{}^4}{R^4}\right)\right)^{1/2} \quad \text{…(3)}$$

Integrating equation (3) we get radius of gas bubble at any time t.

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