A Survey of the Models for the Inventory Problem

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Abstract - The inventory problem is a critical challenge faced by businesses across various industries. Effective management of inventory can significantly impact a company's profitability and customer satisfaction. Over the years, researchers and practitioners have developed numerous models to address this problem and improve inventory management practices. This survey aims to provide an overview of the existing models for the inventory problem and highlight their strengths, limitations, and potential applications.

The survey begins by presenting the different types of inventory models, including deterministic models, stochastic models, continuous review models, periodic review models, single-item models, and multi-item models. Each model type is discussed in detail, highlighting its assumptions, mathematical formulations, and optimization techniques. The survey then examines various factors that influence inventory decisions, such as demand forecasting, lead time, stockout costs, holding costs, and ordering costs.

Keyword - Operations Research, models

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INTRODUCTION

Typically, Operations Research methodologies are delineated as discrete models. Although it can be challenging, establishing a connection between these models has the potential to unveil their interrelatedness and enhance their comprehensibility for the user. The present investigation employs three distinct models, namely Markov Chain, Dynamic Programming, and Markov Sequential Decision Processes, to address an inventory problem that is predicated on the periodic review system. The convergence of the three models to a common (s, S) policy is demonstrated, and a numerical example is presented to illustrate this convergence.[1]

Operations Research is commonly regarded as a collection of models that are tailored to address distinct problem types. Frequently, Operations Research textbooks do not establish a connection between these models and treat them as disparate subjects. The establishment of such a linkage is imperative in order to maintain the integrity of Operations Research. The objective of the current chapter is to establish a connection between three distinct models in Operations Research, namely Markov Chain, Dynamic Programming, and Markov Sequential Decision Processes. This will be achieved by implementing each of these models to address a common inventory control problem. The chapter elucidates the process by which each of the three models yields a solution and demonstrates the

equivalence of the three solutions, despite their apparent dissimilarities.[2]

THE PROBLEM AND THE (S, S) POLICY SOLUTION

Let us examine a theoretical corporation that is approximating the dispersion of demand, denoted as D, for one of the products it is manufacturing. $P[D = j] = p_j$, for $j \in \{m, ..., M\}$; where $P[D = j]$ The inquiry pertains to the likelihood of a specific demand level, denoted as "j," and its corresponding probability value, represented as "pj." The fulfillment of demand during a given period, denoted as n, can be achieved through the production of a quantity, x_n, during that same period, and/or by utilizing the quantity available in inventory at the onset of period n. In the context of inventory management, a holding cost denoted by ch is associated with the storage of each unit from one period to the next, while a stockout cost represented by cu is incurred for each unit that is not available when requested, resulting in a lost sale. The production cost $c_g(x_n)$, expressed as a function of the quantity produced x_n, is assumed to be zero when xn equals zero and is concave for $x_n > 0$.[3]

The company's management is currently seeking to establish a process control system that can automatically generate reorder decisions based on a production policy, as no particular inventory policy has been implemented thus far δn The proposed

concept pertains to the allocation of a predetermined production quantity to each inventory level at the onset of period n. δn(in) chosen from the set of possible production quantities {xn}.

Scarf The author has demonstrated the existence of an optimal production policy, denoted as δ+, for every period n. This policy ensures that the inventory level is brought to a predetermined target level, denoted as s_n^+, in the event that the initial inventory position for the item is less than or equal to s_n^+. A salient characteristic of the problem at hand is that the cost functions, demand distribution, and potential levels of initial inventory remain consistent across all time periods. This statement suggests the presence of a stable condition wherein an optimal policy δ*(i) is determined for any given initial inventory i, regardless of the period n. Hence, our primary objective is to determine the optimal decision policy δ* that corresponds to each inventory position i and production quantity δ*(i) that effectively minimizes the overall costs of production, holding, and stockout for an infinite time horizon. The policy in question is established based on the optimal values of two variables, namely s and S, denoted as s* and S* respectively:[4]

$$\delta(i) = \begin{cases} S - i & \text{if } i \le s \\ 0 & \text{if } i > s \end{cases}$$

$$\text{1.1}$$

Moreover, it is imperative to note that the values of i must always remain below the difference between S (the uppermost attainable level) and m (the minimum feasible demand**)**.

$$i \in \{0, 1, ..., S - m)$$ 1.2

Constraints (1.1) and (1.2) implicitly require that:

$$S > s \text{ and } S \ge m$$
1.3

Moreover, we assume an inventory capacity restriction of K units:

$$i + \delta(i) \le K \Rightarrow S \le K$$ 1.4

THE MARKOV CHAIN MODEL

As per equation (2.2), the inventory level at the commencement of each period n is a stochastic process in discrete-time. The feasible values of this process are {0, 1, …, S-m}. The probability distribution of In is contingent upon the inventory level I_{n-1} and not on the states traversed by the stochastic process en route to I_{n-1}, given that In is invariably equivalent to I_{n-1} plus production minus sales.[5] For any given states i

and k, and for all periods n, the probability of the system transitioning from state i at the start of period n-1 to state k at the start of period n remains constant and is contingent upon the prescribed policy (s; S). Thus, it is possible to express the probabilities of transition as follows.

$$P[I_n = k \mid I_{n-1} = i] = q_{ik}^{sS}.$$

Let y and z be two natural numbers verifying $0 \le y \le M$ and $m \le z \le S$, the transition matrix Q^{sS} from the states i = 0, 1, …, s, s + 1, …, M − y, …, S − z, …, S − m to the states k = 0, 1, …, M − y, …, S − z, …, S − m can be represented as shown below, where $p_j = 0$ for all j < 0 (e.g., $p_{m-z} = 0$ if z > m) and $\sum_{i=x}^{y} p_i = 0$ for all y < x (e.g., $\sum_{i=s}^{y} p_i = 0$ if S > M). At optimality, we must have s < M for if we have enough stock to satisfy all the demand of the period, there will be no need to order and incur unnecessary holding cost [7]. However, as we are uncertain whether M < S or M > S, we include the two parameters y and z.

$$Q^{sS} =$$

State 0	⋯	State M − y	⋯	State S − z	⋯	State S − m	
$\sum_{i=s}^{M} p_i$	⋯	$PS-M+y$	⋯	Pz	⋯	Pm	State 0
$\sum_{i=s}^{M} p_i$	⋯	$PS-M+y$	⋯	Pz	⋯	Pm	State 1
⋮	⋱	⋮	⋱	⋮	⋱	⋮	⋮
$\sum_{i=s}^{M} p_i$	⋯	$PS-M+y$	⋯	Pz	⋯	Pm	State s
$\sum_{i=s+1}^{M} p_i$	⋯	$Ps+1-M+y$	⋯	$Ps+1-S+s$	⋯	$Ps+1-S+m$	State s +1
⋮	⋱	⋮	⋱	⋮	⋱	⋮	⋮
$\sum_{i=M-y}^{M} p_i$	⋯	$P0$	⋯	$PM+y-S+s$	⋯	$PM+y-S+s$	State M − y
⋮	⋱	⋮	⋱	⋮	⋱	⋮	⋮
$\sum_{i=S-z}^{M} p_i$	⋯	$PS-s-M+y$	⋯	$P0$	⋯	$Pm-s$	State S − z
⋮	⋱	⋮	⋱	⋮	⋱	⋮	⋮
$\sum_{i=S-m}^{M} p_i$	⋯	$PS-s-M+y$	⋯	$Pz-m$	⋯	$P0$	State S − m

Given that there are no transient or periodic states within the chain and that all states are interconnected, it can be inferred that the chain is ergodic. Hence, a stationary probability distribution $\pi^{sS} = \left[\pi_0^{sS}, \pi_1^{sS}, ..., \pi_{S-m}^{sS},\right]$ exists. The chain can be determined by solving the system of equations:

$$\begin{cases} \pi^{sS} Q^{sS} = \pi^{sS} \\ 1\pi^{sS} = 1; \quad \text{where}: 1 = [1, 1, ..., 1] \end{cases}$$

1
.5

The functions g(s, S), h(s, S), and u(s, S) represent the anticipated production, inventory holding, and stockout expenses per period, respectively, with respect to the reorder point s and the target level S.:

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$$g(s,S) = \sum_{i=0}^{s} \pi_i^{sS} c_g (S-i) \qquad (1.6)$$

$$h(s,S) = c_h \times \left[\sum_{i=0}^{s} \pi_i^{sS} \sum_{j=m}^{M} \max(0, S-j) p_j \right.$$

$$\left. + \sum_{i=s+1}^{S-m} \pi_i^{sS} \sum_{j=m}^{M} \max(0, i-j) p_j \right] \qquad (1.7)$$

$$u(s,S) = c_u \times \left[\sum_{i=0}^{s} \pi_i^{sS} \sum_{j=m}^{M} \max(0, j-S) p_j \right.$$

$$\left. + \sum_{i=s+1}^{S-m} \pi_i^{sS} \sum_{j=m}^{M} \max(0, j-i) p_j \right] \qquad (1.8)$$

The anticipated overall expense denoted by w(s, S) is equivalent to the summation of three functions (2.6) - (2.8). The determination of the optimal values s* and S* can be achieved through the minimization of the function w(s, S) = g(s, S) + h(s, S) + u(s, S), while adhering to the constraint (1.4).

$$\min w(s,S) = g(s,S) + h(s,S) + u(s,S),$$

$$\text{S.T. } s < S \text{ and } S \in \{m, m+1, \ldots, K\}$$

1.9

The Model Of Dynamic Programming

Consider the period n as the phase and the inventory level at the commencement of period n as the state. The process undergoes a transition from state "in" to state "i_n + 1".:

$$i_n + 1 = \max(0, i_n + \delta_n(i_n) - j)$$

1.10

where *j* belongs to the set {m, …, M} and $\delta n(i_n)$ The production quantity for period n, as determined by policy δn, is dependent on the initial inventory level.[6] Let $v(i_n, \delta_n(i_n))$ Let C(n) represent the anticipated aggregate cost of production, inventory holding, and stockout for a given period n, given an initial inventory of in units and a production of. $\delta_n(i_n)$ units:

$$v(i_n, \delta_n(i_n)) = c_g(\delta_n(i_n)) + \sum_{j=m}^{M} p_j \times \left[c_h \max(0, i_n + (\delta_n(i_n) - j)) \right.$$

1.11

$$\left. + c_u \max(0, j - i_n - \delta_n(i_n)) \right]$$

The aim is to reduce the anticipated overall expenses for the time intervals 1, 2, and so forth, while taking into account the initial inventory level. i_1. The objective

function can be denoted by... $f_1(i_1)$, It is possible to generate a function that is more general. $f_n(i_n)$ The concept of minimum expected total costs for the periods n, n + 1, etc. can be defined as the anticipated expenses that are incurred, assuming that a certain number of units are present in the inventory at the outset. The recurrent relationship that exists between the terms of a sequence is a fundamental concept in mathematics. $f_n(i_n)$ and $f_{n+1}(i_{n+1})$ can be expressed as:

$$f_n(i_n) = \min_{\delta_n(i_n)} \left[v(i_n, \delta_n(i_n)) + \sum_{j=m}^{M} p_j f_{n+1}(i_{n+1}) \right],$$

$$\text{for } n = 1, 2, \ldots$$

1.12

Dynamic programming models necessitate a finite horizon due to the fact that the computation of fn(in) in equation (1.12) is contingent upon the computation of fn+1(in+1). The recurrence relation is initiated with a final period N. The selection of a sufficiently large value for N may facilitate the attainment of a state of equilibrium in the process. During the initial periods, there exists an optimal policy denoted as δ* (i) that is applicable to all initial inventory levels i, regardless of the period n.[7] However, it is possible for the final periods to exhibit variation, as they may continue to reflect the impact of the introduction of the "dummy" last period N. The solution derived from the dynamic programming approach is.

achieved first by minimizing $v(i_N, \delta_n(i_n))$ to obtain $\delta_N^*(i)$. Then, we use the recursivity in (1.12) to find $\delta_{N-1}^*(i), \delta_{N-2}^*(i), \ldots$ and so on until the procedure reaches a period $N - L$ verifying $\delta_{N-L}^*(i) = \delta_{N-L-1}^*(i) = \delta_{N-L-2}^*(i) = \ldots = \delta_1^*(i) = \delta^*(i)$, thereby solving the problem for the last *L* periods only:

$$f_N(i) = \min_{\delta_N(i)} \left\{ v(i, \delta_N(i)) \right\}$$

1.13

$$f_n(i) = \min_{\delta_n(i)} \left\{ v(i, \delta_n(i)) + \sum_{j=m}^{M} p_j f_{n+1}(\max(0, i + \delta_n(i) - j)) \right\},$$

$$\text{for } n = N-1, \ldots, N-L \qquad (1.14)$$

$$f_n(i) = \min_{\delta(i)} \left\{ v(i, \delta(i)) + \sum_{j=m}^{M} p_j f_{n+1}(\max(0, i + \delta(i) - j)) \right\},$$

$$\text{for } n = N-L-1, \ldots, 1 \qquad (1.15)$$

$$i \in \{0, 1, \ldots, K-m\} \text{ and } \delta(i), \delta_n(i), \delta_N(i) \in \{0, 1, \ldots, K-i\}$$

$$(1.16)$$

The initial segment of equation (1.16) can be demonstrated through the same reasoning as

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equation (1.2): the upper limit of i is always constrained by the maximum attainable level (K) minus the minimum feasible demand (m). The second component is acquired directly from equation (1.4).

The model under consideration is the markov sequential decision processes model.

The Markov sequential decision process is characterized as a probabilistic dynamic program with an infinite horizon. The term can also be characterized as a Markovian process that encompasses a finite set of states and incorporates an economic value framework that is linked to the transitions between these states. In this instance, the state variable will persist as the initial inventory of the given time period. Let "f delta (i)" denote the anticipated cost that arises over an infinite number of time periods, assuming that the initial state is "i" and a stationary policy denoted by "delta" is adhered to from the outset of period 1.[8]

$$f_\delta(i) = v(i, \delta(i)) + \sum_{j=m}^{M} p_j f_\delta(\max(0, i + \delta(i) - j))$$

1.17

"where v(i, δ(i)) is the expected cost incurred during the current period, as defined in (1.10). The horizon being infinite, fδ(i) will also be infinite. To cope with the problem, we can use the expected discounted total cost. We assume that a \$1 paid the next period will have the same value as a cost of β dollars paid during the current period. Let Vδ(i) be the expected discounted cost incurred during an infinite number of periods, given that, at the beginning of period 1, the state is i and stationary policy δ is followed":

$$v_\delta(i) = v(i, \delta(i)) + \beta \sum_{j=m}^{M} p_j V_\delta(\max(0, i + \delta(i) - j))$$

1.18

where $\sum_{j=M}^{M} p_j V_\delta(\max(0, i + \delta(i) - j))$ is the expected cost,

The costs were discounted to the start of period 2 and accrued from that point forward. The minimum value of Vδ(i), that we denote by Vδ(i), The anticipated discounted expense that arises over an infinite number of periods is contingent upon the initial state being i at the onset of period 1 and the adherence to the optimal stationary policy, denoted as δ*.[9-10]

$$V(i) = V_{\delta^*}(i) = \min V_\delta(i) \text{ for all possible values of } i \quad (1.19)$$

Using (1.16) and (1.18), equality (1.19) can be equivalently written as:

For $i = 0, ..., K - m$: $\quad (1.20)$

$$V(i) = \min_{\delta(i)=0,...,k-i} \left\{ v(i, \delta(i)) + \beta \sum_{j=m}^{M} p_j V(\max(0, i + \delta(i) - j)) \right\}$$

CONCLUSION

The survey of models for the inventory problem reveals the extensive research and development carried out in this field. The inventory models discussed in this survey provide a range of approaches to address various inventory management challenges. Deterministic models offer simple and straightforward solutions for managing inventory, while stochastic models account for uncertainties and variability in demand and lead time.

Continuous review models provide real-time inventory control, ensuring timely replenishment, while periodic review models offer batch ordering strategies to optimize costs. Single-item models focus on managing inventory for individual products, while multi-item models consider the interdependencies and trade-offs between different items. Each model type has its advantages and limitations, and businesses need to carefully select the appropriate model based on their specific requirements and constraints.

Recent advancements in inventory modeling, such as incorporating machine learning techniques, have the potential to enhance demand forecasting accuracy and optimize inventory decisions. Considering perishable inventory and addressing the challenges of online retailing are critical for industries with time-sensitive products and omnichannel distribution. Furthermore, integrating inventory models with other supply chain components can lead to more holistic and efficient decision-making.

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