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Perfect Fluid Cosmological Model In Presence Of Electromagnetic Field

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Abstract - In this paper, we have constructed a non-static cylindrically symmetric cosmological model which is specially homogeneous non-degenerate Petrov type-I. the energy momentum tensor has been assumed to be that of a perfect fluid with an electromagnetic field and the 4-current is either zero or space-tike. Various physical and geometrical properties like pressure, density, scalar of expansion and shear etc have been found and discussed.

Keywords - Shear, Scalar of expansion, noon-static, pressure, density.

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1. INTRODUCTION

Various relativists have shown their keen interest in construction of non-static cosmological models.

Jacobs [5, 6] has studied the behavior of the general Bianchi-type I cosmological model in the presence of a spatially homogeneous magnetic field. This problem has been studied again by De [1] with a different approach. This work has been further extended by Tupper [11] to include Einstein – Maxwell fields in which the electric field is non-zero. He has also interpreted certain type VI_0 cosmologies with electromagnetic field (Tupper [12]). Roy and Prakash [9], taking the cylindrically symmetric metric of Marder [8], have constructed a spatially homogenous cosmological model in the presence of an incident magnetic field which is also anisotropic and non-degenerate Petrov type-I. Singh and Yadav [10] constructed a spatiaolly homogenous cosmological model assuming the energy momentum tensor to be that of a perfect fluid with an electromagnetic field. A non-static magnetichydrodynamic cosmological model in general relativity has been studied by Yadav and Singh [13]

In this paper, taking the cylindrically symmetric metric of Marder [8], we have constructed a non-static cylindrically symmetric cosmological model which is spatially homogeneous non-degenerate Petrov type-I. The energy momentum tensor has been assumed to be that of a perfect fluid with an electromagnetic field and the 4-current is either zero or space-like. The requirement that the conductivity be positive imposes an additional restriction on the metric potentials. Here we have found and discussed various physical and geometrical properties like shear, expansion, pressure density etc.

2. THE FIELD EQUATIONS AND THEIR SOLUTIONS

We start with the metric (Marder [8])

(2.1) $$ds^2 = A^2(dt^2 - dx^2) + B^2dy^2 + c^2dz^2$$

where A, B and C are functions of t only. The distribution consists of a perfect fluid with an electromagnetic field. The energy momentum tensor of the composite field is assumed to be the sum of the corresponding energy momentum tensors. Thus

(2.2) $$R_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}R + \wedge = g_{\mu\upsilon} = -K\left[(\rho+p)u_\mu u_\upsilon - pg_{\mu\upsilon} + E_{\mu\upsilon}\right]$$

(2.2)(a) $$E_{\mu\upsilon} = -g^{k\ell}F_{\mu k}F_{\upsilon\ell} + \frac{1}{4}g_{\mu\upsilon}F_{\alpha\beta}F^{\alpha\beta}$$

(2.3) $$E_{\mu\upsilon} = -g^{k\ell}F_{\mu k}F_{\upsilon\ell} + \frac{1}{4}g_{\mu\upsilon}F_{\alpha\beta}F^{\alpha\beta}$$

(2.4) $$F_{[\mu\upsilon\,;\,\sigma]} = 0$$

(2.5) $$F^{\mu\upsilon};\upsilon = J^\mu$$

where p and □ are pressure and density respectively of the distribution, $E_{\mu\upsilon}$ is the electromagnetic energy momentum tensor, $F_{\mu\upsilon}$ is the electromagnetic field tensor, J^μ is the current 4-vector, $\wedge$ is the cosmological constant and u_μ is the flow vector satisfying

(2.6) $g_{\mu\upsilon}u^{\mu}u^{\upsilon} = 1$

The co-ordinates are chosen to be commoving so that

(2.7) $u^{\mu} = (0, 0, 0, \frac{1}{A})$

and we label the co-ordinates

$$(x, y, z, t) = (x^1, x^2, x^3, x^4)$$

We assume the electromagnetic field to be in the direction of x-axis so that F_{14} and F_{23} are the only non-vanishing components of the field tensor $F_{\mu\upsilon}$. We write

(2.8) $F_{14}^2 A^{-4} + F_{23}^2 B^{-2} C^{-2} = M^2$

The equation (2.2) may be written as

(2.9)
$$\frac{2}{A^2}\left[\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4 C_4}{AC} - \frac{A_4 B_4}{AB} - \frac{A_4^2}{A^2}\right] - 2\wedge$$
$$= -K\left[M^2 + (\rho + 3p)\right],$$

(2.10) $$-\frac{2}{A^2}\left[\frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} - \frac{A_4^2}{A^2}\right] + 2\wedge$$
$$= -K\left[-M^2 + (\rho - p)\right],$$

(2.11)
$$-\frac{2}{A^2}\left[\frac{B_{44}}{B} + \frac{B_4 C_4}{BC}\right] + 2\wedge = -K\left[M^2 + (\rho - p)\right]$$

(2.12)
$$-\frac{2}{A^2}\left[\frac{C_{44}}{C} + \frac{B_4 C_4}{BC}\right] + 2\wedge = -K\left[M^2 + (\rho - p)\right]$$

where the suffix 4 after the symbols A, B, C denotes ordinary differentiation w.r.t. time t. These equations show that M^2, ρ, p are each functions of t only : and it then follows from equations (2.4) and (2.8) that F_{23} is a constant and F_{14} is a function of t only i.e.

(2.13) $F_{23} = k, F_{14} = \pm A^2 (M^2 - k^2 B^{-2} C^{-2})^{1/2}$

where k is a constant.

The case when F_{14} = 0, which implies J^{μ} = 0, we get the model due to Roy and Prakash [9]. We here assume that $F_{14} \neq 0$ and find the only non-vanishing component of J^{μ} to be

(2.14) $$J^1 = \pm \frac{1}{A^2 BC} \frac{\partial}{\partial t}\left[BC(M^2 - k^2 B^{-2} C^{-2})^{1/2}\right]$$

Equation (2.14) shows that J^{μ} is space like, unless $M^2 = \ell B^{-2} C^{-2}$ where ℓ is a constant in which case J^{μ} = 0. The 4-current J^{μ} is in general the sum of the convection current and conduction current (licknerowicz [7] and Greenburg [3].

(2.15) $J^{\mu} = \varepsilon_0 u^{\mu} + \lambda\, u_{\upsilon} F^{\mu\upsilon}$

where □$_0$ is the rest charge density and □ is the conductivity. In the case considered here we have □$_0$ = 0 i.e., magnetohydrodynamics. From equations (2.13), (2.14) and (2.15) we find that the conductivity is given by

(2.16) $\lambda = -\frac{1}{4} D_4 D^{-1}$

where $D = BC(M^2 - k^2 B^{-2} C^{-2})^{1/2}$

The requirement of positive conductivity in (2.16) puts further restrictions on A, B, C. Hence in the magnetohydrodynamics case metric functions are restricted not only by the field equations and energy conditions (Hawking and Penross [4] they are also restricted by the requirement that the conductivity be positive for a realistic model.

The equations (2.9) – (2.12) are four equations in six unknows A, B, C, ρ, p and M. In order to determine them two more conditions have to be imposed on them. For this we assume that the space – time is of degenerate petrow type – I, the degeneracy being in y and z directions. This requires that $C_{12}^{12} = C_{13}^{13}$. This condition is identically satisfied if B = C. However, we shall take the metric potentials to be unequal. We further assume that F_{14} is such that $M^2 = f^2 B^{-3} C^{-3}$ where f is a constant. From equations (2.11) and (2.12) we have

(2.17) $$\frac{B_{44}}{B} - \frac{C_{44}}{C} = 0$$

Equation (2.17) with condition $C_{12}^{12} = C_{13}^{13}$ gives

(2.18) $$\frac{A_4}{A}\left(\frac{C_4}{C} - \frac{B_4}{B}\right) = 0$$

Since $B \neq C$, equation (2.18) gives

(2.19) A = N (a constant).

From equations (2.10), (2.11) and (2.19) we have

(2.20) $$\frac{B_{44}}{B}+\frac{B_4C_4}{BC}=KM^2N^2$$

Equation (2.17) on integration gives

(2.21) $$B_4C-BC_4=n$$

n being an arbitrary constant of integration. Putting $\frac{B}{C}=\alpha$ and BC = β, equation (2.21) goes to the form

(2.22) $$\left(\frac{\alpha_4}{\alpha}\right)\beta=n$$

and equation (2.20) turns into

(2.23) $$\frac{1}{\beta}\left[\left(\frac{\alpha_4}{\alpha}+\frac{\beta_4}{\beta}\right)\beta\right]_4=2KM^2N^2$$

From equations (2.22) and (2.23) we have

(2.24) $$\frac{\beta_{44}}{\beta}=2KM^2N^2$$

which, after the use of condition

$M^2=f^2B^{-3}C^{-3}$, reduces to

(2.25) $$\beta_{44}=\frac{2Kf^2N^2}{\beta^2}$$

Equation (2.25) on integration yields

(2.26) $$[\beta_4]^2=\frac{\zeta}{\beta}[\beta-a]$$

where

(2.27) $$a=\frac{4Kf^2N^2}{\zeta}$$

and ζ is an arbitrary constant which we shell take to be unity, Clearly a > 0. From equations (2.22) and (2.26) we get

(2.28) $$\frac{d\alpha}{\alpha}=\frac{n}{\beta^{1/2}}\frac{d\beta}{(\beta-a)^{1/2}}$$

Integration of (2.28) gives

(2.29) $$\alpha=b\left[\beta^{1/2}+(\beta-a)^{1/2}\right]^{2n}$$

b being a constant of integration.

Therefore

(2.30) $$B^2=b\beta\left[\beta^{1/2}+(\beta-a)^{1/2}\right]^{2n},$$

(2.31) $$C^2=\frac{\beta}{b}\left[\beta^{1/2}+(\beta-a)^{1/2}\right]^{-2n}$$

Consequently the line element (2.1) takes the form

(2.32)
$$ds^2=A^2\left[\frac{d\beta^2}{(d\beta/dt)^2}-dx^2\right]-B^2dy^2-C^2dz^2$$

which by use of equation (2.19), (2.26), (2.30) and (2.31) takes the form

(2.33) $$ds^2=N^2\left[\left(\frac{\beta}{\beta-a}\right)dt^2-dx^2\right]$$
$$-b\beta\left[\beta^{1/2}+(\beta-a)^{1/2}\right]^{2n}dy^2$$
$$-\frac{\beta}{b}\left[\beta^{1/2}+(\beta-a)^{1/2}\right]^{-2n}dz^2.$$

The transformation

$$x\to x,\ \sqrt{b}\ y\to Y,\ \frac{1}{\sqrt{b}}z\to Z,\ \beta\to(a+T)$$

Reduces the metric (2.33) to the form

(2.34) $$ds^2=N^2\left[\left(\frac{a+T}{T}\right)dT^2-dX^2\right]$$
$$-(a+T)\left[T^{1/2}+(a+T)^{1/2}\right]^{2n}dY^2$$
$$-(a+T)\left[T^{1/2}+(a+T)^{1/2}\right]^{-2n}dZ^2$$

Clearly for a realistic model T should be positive (due to $T^{1/2}$).

3. SOME PHYSICAL FEATURES

(a) The Distribution in the Model

Pressure and density for the model (2.34) are

(3.1)
$$Kp = \frac{1}{4N^2}\left[\frac{1}{(a+T)^2} - \frac{n^2}{t(a+T)}\right] - \frac{Kf^2}{2(a+T)^3} + \wedge.$$

(3.2) $$K\rho = \frac{1}{4N^2}\left[\frac{1}{(a+T)^2} - \frac{n^2}{T(a+T)}\right] + \frac{Kf^2}{2(a+T)} - \wedge.$$

The model has to satisfy the reality conditions (Ellis [2])

(i) $\rho + p > 0$

(ii) $\rho + 3p > 0$

which requires that

(3.3) $$0 < a < T\left(\frac{1-n^2}{n^2}\right)$$

and

(3.4) $$\wedge > \frac{1}{2N^2}\left[\frac{n^2}{T(a+T)} - \frac{1}{(a+T)^2}\right] + \frac{Kf^2}{2(a+T)^3}$$

The condition (3.3) holds only when $n^2 < 1$.

In the case of stiff matter ($\rho = p$) we have

(3.5) $$\wedge = \frac{Kf^2}{2(a+T)^3}$$

and

(3.6) $$\rho = p = \frac{1}{4N^2}\left[\frac{1}{(a+T)^2} - \frac{n^2}{T(a+t)}\right]$$

The flow vector u^μ of the distribution is given by

(3.7) $$u^1 = u^2 = u^3 = 0, u^4 = \frac{1}{N}\sqrt{\frac{T}{a+T}}$$

The flow vector u^μ satisfies $u^\mu;\upsilon\ u^\upsilon = 0$.

Thus the lines of flow are geodesios. Tensor of rotation $w_{\mu\upsilon}$ defined by

(3.8) $$w_{\mu\upsilon} = u_{\mu;\upsilon} - u_{\upsilon;\mu}$$

is identically zero. Thus the fluid filling the universe is non-rotational. The scalar of expansion $\theta = u^{gm} : \mu$ is given by

(3.9) $$\theta = \frac{1}{N}\frac{T^{1/2}}{(a+T)^{3/2}}$$

Which tends to zero when $T \to \infty$. The components of shear tensor defined by

(3.10) $$\sigma_{\mu\upsilon} = \frac{1}{2}(u_{\mu\upsilon} + u_{\upsilon;\mu}) - \frac{1}{3}\theta(g_{\mu\upsilon} - u_\mu u_\upsilon)$$

are

(3.11) $$\sigma_{11} = \frac{NT^{1/2}}{3(a+T)^{3/2}}$$

$$\sigma_{22} = \frac{1}{N}\left[T^{1/2} + (a+T)^{1/2}\right]^{2n}\left[-\frac{1}{2}\left(\frac{T}{a+T}\right)^{1/2} + \frac{T^{1/2}}{3(a+T)} - n\right]$$

$$\sigma_{33} = \frac{1}{N}\left[T^{1/2} + (a+T)^{1/2}\right]^{-2n}\left[-\frac{1}{2}\left(\frac{T}{a+T}\right)^{1/2} + \frac{1}{3}\frac{T^{1/2}}{(a+T)} + n\right],$$

$$\sigma_{44} = 0.$$

Hence magnitude of the shear is given by

(3.12) $$\sigma^2 = \frac{1}{2}\sigma_{\mu\upsilon}\sigma^{\mu\upsilon}$$

$$= \frac{T}{18N^2(a+T)^3} + \frac{1}{2N^2(a+T)^2}\left[\frac{1}{2}\frac{T}{(a+T} + \frac{2}{9}\right.$$

$$\left.\frac{T}{(a+T)^2} + 2n^2 - \frac{4}{3}\frac{nT^{1/2}}{(a+T)}\right]$$

The non-venishing components of the conformal curvature tensor are

(3.13) $$C_{12}^{12} = C_{13}^{13} = -\frac{1}{2}C_{23}^{23}$$

$$= \frac{1}{6N^2}\left[\frac{3na}{4T^{3/2}(a+T)^{3/2}} - \frac{n^2}{2T(a+T)} + \frac{1}{2(a+T)^2}\right]$$

The non – zero component of the charge current 4-vector is given by

$$(3.14)\quad J^1 = \pm \frac{r^2}{2N^2(a+T)^{5/2}}\left[f^2 - k^2(a+T)\right]^{-1/2}$$

and the conductivity is given by

$$(3.15)\quad \lambda = \frac{f^2}{2N(a+T)}\left[f^2 - k^2(a+t)\right]^{-1}$$

For a physically realistic MHD model λ has to be positive which requires that $N > 0$ and $|f| > |k||(a+T)^{1/2}|$(Since a and T are already positive).

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