A Study on M/M/3 Queuing Model on Waiting time Reduction in a Hospital

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Abstract - This paper contains the analysis of queuing system in hospital management is the mathematical approach to the analysis of waiting lines in any setting where arrival. The goal of this paper explains the problem in their urgency of medical cases with respect to allocation problem of the patients and utilization the waiting line. We assume first service times first come first served queue discipline.

Keywords - M/M/3 Queuing Model, Waiting time, Hospital

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INTRODUCTION

Queuing theory is a mathematical approach to the study of waiting lines. Long waiting time in a health care system indicates the lack in management of the system. This project is an attempt to analyze the use of queuing theory in a local health care clinic. Also considers the clinic as a multi server queuing system following Poisson arrival and Exponential service based system. A comparison of the results using MATLAB pertaining to single server and multiple server queuing system is also provided.

QUEUING THEORY CONCEPT

Queuing is a challenge for all healthcare systems. In the present world, considerable research project has been done on how to improve queuing systems in various hospital settings. This, unfortunately, has not been the case in developing countries. This paper seeks to contribute to this subject by analyzing the queuing situation in government hospitals and also to bring its practical value to how decision making can be enhanced in hospitals. Queuing theory is a potent mathematical approach to the analysis of waiting lines performance parameters in healthcare delivery systems (Oscan, 2006). It has increasingly become a common management tool for decision making in the developed world. Hospital Services operations particularly, outpatient department plays a crucial role in providing quality health care for multi-speciality hospitals (Carman,1990). The outpatient department (OPD) often acts as profit centre in hospital operations not only for investing in new technology but also to curb losses on inpatient services (Green, 2006). However, despite the importance of OPD, hospitals fail to address complaints regarding long waiting time caused majorly due to observable queues which results in patient dissatisfaction (Kim 2t.al, 2009). Thus, hospital service operations should have a smooth flow to satisfy the patient's expectations by redesigning their systems and adapting to the best practices and tools with improved processes (Natrajan, 2006) which has a huge scope in developing economies such as India (Natchair et al., 1994). The OPD of a hospital acts as a bridge between hospital and community, hence it is very important to plan the OPD with the idea of maximizing the utilisation and quick turnover. (McQuarrie, 1983). It is imperative to have effective co-operation between the medical services and the support line services catering to the OPD requirements (Kritchanchai, 2012). The interpersonal skills of the medical personnel, availability of medicine, hospital infrastructure and medical information plays an important role in managing OPD and create a positive influence on patient satisfaction (Natarajan, 2006). Hence, it is necessary to focus on optimizing waiting time in hospital operations for the benefit and wellbeing of patients The OPD of a hospital acts as a bridge between hospital and community, hence it is very important to plan the OPD with the idea of maximizing the utilisation and quick turnover. (McQuarrie, 1983). It is imperative to have effective co-operation between the medical services and the support line services catering to the OPD requirements (Kritchanchai, 2012). The interpersonal skills of the medical personnel, availability of medicine, hospital infrastructure and medical information plays an important role in managing OPD and create a positive influence on patient satisfaction (Natarajan, 2006). Hence, it is necessary to focus on optimizing waiting time in hospital operations for the benefit and wellbeing of patients. The OPD of a hospital acts as a bridge between hospital and community, hence it is very important to plan the OPD with the idea of maximizing the utilisation and quick turnover. (McQuarrie, 1983). It is imperative to have effective co-operation between the medical services and the

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The OPD of a hospital acts as a bridge between hospital and community, hence it is very important to plan the OPD with the idea of maximizing the utilization and quick turnover. Planning is the most important aspect of establishing a hospital. If the ready plan is good all may go well as we want. If the plan is not thought out carefully the work may never be completed. Planning a new hospital starts with setting goals for the platform, without which the organization cannot have a definite direction or focus. This is followed by the study of the external environment of the organization, and the internal and external resources with which the goals set are to be achieved. This exercise facilitates selection of the means by which to achieve goals within a reasonable cost. Queuing theory was developed by A.K. Eland in 1904 to help determine the capacity requirements of the Danish telephone system (see Brock Meyer et al. 1948). It has since been applied to a big e range of service industries including banks, airlines, and telephone call centers (e.g. Brewton1989, Stern and Harsh 1980, Halloran and Byrne 1986, Briscoe et al 1995, and Brigandine et al 1994) as well as emergency systems such as police patrol, fire and ambulances (e.g. Larson 1972, Koalas et al 1975, Chest and Barrack 1981, Green and Koalas 1984, Taylor and Huxley 1989). It has also been applied in various healthcare system to settings as we will discuss later in this chapter. This models can be useful in identifying different levels of equipment, and beds as well as in making decisions about resource allocation and the design of new services.

QUEUING SYSTEM

A basic queuing system is a service system where "patient" arrive to a hospital of "servers". The "patient" may be the images waiting to read. Similarly, a "server" is the person or thing that provides the service. So when analyzing time delays for patients in the emergency Department (ED) awaiting admission to the hospital, the relevant servers Would be inpatient beds.

COLLECTION OF DATA

Arrival and Departure of Queuing In Hospital (OPD) Table

Time	10:30–10:45		10:45-11:00		11:00-11:15		11:15-11:30		11:30-11:45		Average Arrival	Average Departure
Days	Arrivals	Departure	Arrivals	Departure	Arrivals	Departure	Arrivals	Departure	Arrivals	Departure		
Monday	72	67	72	72	72	70	78	76	74	73	74	72
Tuesday	80	77	81	75	70	70	81	75	81	73	79	74
Wednesday	77	70	69	77	69	73	78	79	74	79	73	76
Thursday	71	74	80	70	69	76	80	77	74	79	75	75
Friday	65	67	78	78	73	70	79	75	78	74	75	73
Saturday	62	63	75	78	78	68	79	73	76	71	74	71
Total	427	418	455	450	431	427	475	455	457	449	90	88

EVALUATION OF THE CHARACTERIZATION

From the above table

1) Average number of arrivals to the system per 15 min $(\lambda) = 90$ per 15 min

2) Average number of departure from the system $(\mu) = 88$ per 15 min

3) Since $c = 3, \mu = 5.86$ per 15 min

To find the crowd intensity $\rho = \frac{\lambda}{c\mu}$

Here $\lambda = 90$ $\mu = 5.86$ and $c = 3, \rho = 0.340$

4) To find $p_0 = \left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!} \left(\frac{\lambda}{\mu}\right)^c \frac{c\mu}{c\mu - \lambda} \right]^{-1}$

$= [1 + 15.314 + 117.256 + 143.650]^{-1} = P_0 = 0.00360$

The results of characteristics of our system

$P(n > c)$ =probability that an arrival has to wait

$$P(n > c) = \frac{c\mu \left(\frac{\lambda}{\mu}\right)^{cp_0}}{c! \, (c\mu - \lambda)} = 0.2980$$

Probability that an arrival enters the service without wait is

$1 - p(n > c) = 1 - 0.2980 = 0.702$

Average queue length L_q

$$L_Q = \frac{c\mu \left(\frac{\lambda}{\mu}\right)^{cp_0}}{(c - 1)! \, (c\mu - \lambda)^2} = 60.4823$$

Average number of customers in the system $[L_s]$

$L_s = L_q + \frac{\lambda}{\mu} = 75.796$

Average waiting time of an arrival $[W_q]$

$Wq = \frac{L_q}{\lambda} = 0.674$

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Average waiting time an arrival spends in the system [Ws]

$$W_s = W_q + \frac{1}{\lambda} = 0.685$$

Verification of the letter's formulae

The relationships between the characteristics are given by means of the following formulae known as little formulae there are

$$L_s = \lambda W_s$$

Our findings also satisfy the little's formula.

To verify $L_s = \lambda W_s$ from equation

In this section, we have obtained that $L_s = 75.796$

From equation we have found that

$$\lambda W_s = 90 \times 0.685 = 75.796 = L_s \text{ (appr.)}$$

To prove $L_q = \lambda W_q$ From the equation we see that

$$L_q = 60.4823$$

Also form equation,

$$\lambda W_q = 90 \times 0.674 = 60.47 = L_q.\text{(appr.)}$$

OBSERVATIONS

In our study we have made the following observations the following results.

a) Traffic intensity, $\rho = 0.340$

b) Probability that there are no customers in he system, $P_0 = 0.00360$

c) Probability that an arrival has to wait, $P(n > c) = 0.2968$

d) Probability that an arrival enters the service without wait $1 - P(n > c) = 0.702$

e) Average queue length at any time $L_s = 75.796$ customers

f) Average number of customer in the system at any time $L_s = 75.796$ customers

g) Average waiting time of an arrival in the queue, $W_q = 0.674$

h) Average waiting time of arrival spend in the system W_s =0.685

i) Average number of arriving rate, λ =90 per 15min

j) Average number of service rate, μ= 88 per 15 min

SUMMARY

a) An average of 90 patient are arriving to the hospital in 15 min

b) Average of 88 patient are being served in the hospital in 15 min

c) The crowd intensity $\rho = \frac{\lambda}{c\mu} = 0.340 < 1$ and therefore the queuing system followed in the hospital

d) At any time an average of 76 patients are found waiting in the queue.

e) At any time an

CONCLUSION

In our study of this system does not only serve the people requesting he service in hospital but also utilize their time to do other activities. The advantage of using this open source it could benefit to society as a whole. Not only one hospital can have benefit with the current system design and setting, but multiple hospitals can be served at the same time. An individual hospital can follow its own queues with a given power user as a queue administration. With this design, cost sharing arrangement is possible among hospitals without having any budget to spend for the extra development. Multi-server queuing can be used to estimate the average waiting time, queue length, number of servers and service rates.

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