Theorem on Partial Symmetric Spaces

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Abstract - In this paper, we first introduce the class of partial symmetric spaces and then prove some fixed point theorems in such spaces.We introduce an analogue of the Hausdorff metric in the context of partial symmetric spaces and utilize the same to prove an analogue of the Nadler contraction principle in such spaces. Our results extend and improve many results in the existing literature. We also give some examples exhibiting the utility of our newly established results.

Keywords - partial symmetric; fixed point; contraction and weak contraction; Nadler's theorem,

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: INTRODUCTION AND PRELIMINARIES

The Banach contraction principle, which Banach [1] demonstrated to be a useful and potent tool in nonlinear analysis, was applied in the context of fixed point theory. Numerous approaches have been taken to expand and generalize this idea. In the study of fixed point theory, one may occasionally run into circumstances where none of the metric criteria are necessary. Numerous researchers obtained fixed point findings in semi-metric spaces [98] (also known as symmetric spaces) as a result of this feature. A mapping ∂_s on a non-empty set Ҳ is called symmetric if

∂_s (a, b) = ∂_s(b, a) and ∂_s (a, b) = 0 if and only if a = b, for all a, b ∈ Ҳ.

We refer to the pair (Ҳ,∂_s) as symmetric space. The uniqueness of the sequence's limit cannot be ensured since the function "∂_s" is not continuous in general due to the lack of triangular inequality. Wilson [12] proposed a number of related weaker criteria to get around the challenges that were previously mentioned. We have implemented weaker conditions in our setting, which will be explained shortly in the preliminary section. The study of fixed points for multi-valued contractions with the Hausdorff metric was started in 1969 by Nadler [13], who also extended the Banach fixed point theorem to set-valued contractive mappings.

As a generalization of partial metric spaces and symmetric spaces, the goal of this chapter is to present partial symmetric spaces and use them to demonstrate fixed point solutions for both single-valued and multivalued mappings.

We now introduce the partial symmetric space in order to expand the classes of partial metric spaces and symmetric spaces. This is done as follows:

Definition 1.1.: Let us assume that Ҳ be a nonempty set. A mapping $\wp: Ҳ \times Ҳ \to [0,\infty)$ is said to be a partial symmetric if (for all x, y, z∈ Ҳ):

($\wp$1) x = y if and only if $\wp$ (x, x) = $\wp$ (y, y) = $\wp$ (x, y);

($\wp$2) $\wp$ (x, x) ≤ $\wp$ (x, y);

($\wp$3) $\wp$ (x, y) = $\wp$ (y, x). Then the pair (Ҳ, $\wp$) is said to be a partial symmetric space.

A partial symmetric space (Ҳ, $\wp$) reduces to a symmetric space if $\wp$ (x, x) = 0, for all x ∈ Ҳ. Obviously, every symmetric space is a partial symmetric space but not conversely.

Example 1.1.: Let Ҳ = $\Re$ and define a mapping $\wp: Ҳ \times Ҳ \to [0,\infty)$

as follows , for all x, y ∈ Ҳ and α,β> 1): $\wp$ (x, y) = |x − y| α + |x − y| β . Then the pair (Ҳ, $\wp$) is a partial symmetric space.

Example 1.2.: Let Ҳ = [0, ∞) and define a mapping $\wp: Ҳ \times Ҳ \to [0,\infty)$ as follows for all x, y ∈ Ҳ and α,β> 1): $\wp$ (x, y) = (max {x , y} α + {x , y} β).

Then the pair (Ҳ, $\wp$) is a partial symmetric space.

Example 1.3. : Let Ҳ = [0, π) and define a mapping $\wp: Ҳ \times Ҳ \to [0,\infty)$ as follows, for all x, y ∈ Ҳ and a > 0): $\wp$ (x, y) = sin |x − y| + a. Then the pair (Ҳ, $\wp$) is a partial symmetric space.

Let (Ҳ, ℘) be a partial symmetric space. Then ℘-open ball with center x ∈ Ҳ and radius $\varepsilon > 0$ is defined by:

$\mathcal{B}_{\wp}(x,\varepsilon)$ ={y∈ Ҳ: ℘ (x, y)< ℘ (x, y) + ε}.

Similarly, the ℘-closed ball with center x ∈ Ҳ and radius $\varepsilon > 0$ is defined by:

$\mathcal{B}_{\wp}[x,\varepsilon]$ ={y∈ Ҳ: ℘ (x, y)≤ ℘ (x, y) + ε}.

The family of ℘-open balls, for all x ∈ Ҳ and $\varepsilon > 0$

$\mho_{\wp} = \{\mathcal{B}_{\wp}(x,\varepsilon)$: x ∈ Ҳ and $\varepsilon > 0\}$, form a basis of some topology τ℘ on Ҳ.

Lemma 1.4.: Let (Ҳ, τ℘) be a topological space and $\mathcal{F}$: Ҳ → Ҳ. If $\mathcal{F}$ is continuous then every convergent sequence $\{x_n\}$ ⊆ Ҳ such that x_n → x implies $\mathcal{F}x_n \to \mathcal{F}$x. The converse holds if Ҳ is metrizable.

Further down the road, we will need to define a few terms more thoroughly. These include full partial symmetric spaces, convergent sequences, and Cauchy sequences.

Definition 1.5. : A sequence $\{x_n\}$ in (Ҳ, ℘) is said to be ℘-convergent to x ∈ Ҳ, with respect to τ℘, if ℘ (x, x) = $\lim_{n\to\infty}\wp(x_n, x)$.

Definition 1.6.: A sequence $\{x_n\}$ in (Ҳ, ℘) is said to be ℘-Cauchy if and only if $\lim_{n\to\infty}\wp(x_n, x_m)$ exists and finite.

Definition 1.7. : A partial symmetric space (Ҳ, ℘) is said to be a ℘-complete if every ℘-Cauchy sequence $\{x_n\}$ in (Ҳ, ℘) is ℘-convergent with respect to τ℘ to a point x ∈ Ҳ such that ℘ (x, x) = $\lim_{n\to\infty}\wp(x_n, x)$ =$\lim_{n\to\infty}\wp(x_n, x_m)$.

We are now applying some definitions from the context of symmetric spaces to the context of partially symmetric spaces.

Definition 1.8.: Let (Ҳ, ℘) be a partial symmetric. For the sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ and x, y in Ҳ, we have

(Λ1) $\lim_{n\to\infty}\wp(x_n, x)$ = ℘ (x, x) and $\lim_{n\to\infty}\wp(x_n, y)$ = ℘ (x, y) imply x = y.

(Λ2) A partial symmetric ℘ is said to be 1-continuous if

$\lim_{n\to\infty}\wp(x_n, x)$ = ℘ (x, x) implies that $\lim_{n\to\infty}\wp(x_n, y)$ = ℘ (x, y).

(Λ3) A partial symmetric ℘ is said to be continuous if $\lim_{n\to\infty}\wp(x_n, y)$ = ℘ (x, y) and

$\lim_{n\to\infty}\wp(x_n, y)$ = ℘ (x, y).

Imply that

$\lim_{n\to\infty}\wp(x_n, y_n)$ = ℘ (x, y).

(Λ 4) $\lim_{n\to\infty}\wp(x_n, x)$ = ℘ (x, x) and $\lim_{n\to\infty}\wp(x_n, y_n)$ = ℘ (x, x) imply $\lim_{n\to\infty}\wp(y_n, x)$ = ℘ (x, x).

(Λ 5) $\lim_{n\to\infty}\wp(x_n, y_n)$ = ℘ (x, x) and $\lim_{n\to\infty}\wp(y_n, z_n)$ = ℘ (x, x) imply

$\lim_{n\to\infty}\wp(y_n, z_n)$ = ℘ (x, x) .

Remark 1.9. : It may be seen from Definition 1.8 that (Λ 3) ⇒ (Λ 2), (Λ4) ⇒ (Λ 1), and (Λ 2) ⇒ (Λ 1), but generally speaking, the opposite implications are not true.

2: FIXED POINT RESULTS IN PARTIAL SYMMETRIC SPACES

Let (Ҳ, ℘) be a partial symmetric space and $\mathcal{F}$: Ҳ → Ҳ. Then for every x ∈ Ҳ and for all i, j ∈ N, we define ℶ(℘,$\mathcal{F}$, x) = sup{℘ ($\mathcal{F}^i x, \mathcal{F}^j x$):i, j ∈ N }…. (2.1)

Definition 2.1. Let (Ҳ, ℘) be a partial symmetric space and $\mathcal{F}$: Ҳ → Ҳ be a mapping. Then mapping $\mathcal{F}$ is said to be ℘-contraction if

℘ ($\mathcal{F}$x, $\mathcal{F}$y) ≤ α℘ (x, y), ∀ x, y ∈ Ҳ, ……….. (2.2)

where α ∈ (0, 1).

We are currently demonstrating a counterpart of the Banach contraction principle within the context of partially symmetric spaces.

Theorem 2.2. Let (Ҳ, ℘) be a partial symmetric space and $\mathcal{F}$: Ҳ → Ҳ be a mapping. Let us assume that the following conditions are satisfied:

(i) $\mathcal{F}$ is ℘-contraction for some α ∈ [0, 1),
(ii) There exists x_0 ∈ Ҳ such that ℶ(℘,$\mathcal{F}$, x_0) < ∞,
(iii) Either
(a) $\mathcal{F}$ is continuous or
(b) (Ҳ, ℘) enjoys the (Λ1) property.

Then $\mathcal{F}$ has a unique fixed point x ∈ Ҳ such that ℘ (x, x) = 0.

Proof. Choose x_0 ∈ Ҳ and construct an iterative sequence $\{x_n\}$ by:
$x_1 = \mathcal{F}x_0,\ x_2 = \mathcal{F}x_1,\ x_3 = \mathcal{F}x_2, \cdots, x_n = \mathcal{F}x_{n-1}, \cdots$.
Now, from (5.2.2) (for all i, j ∈ N), we have
℘ ($\mathcal{F}^{n+i}x_0, \mathcal{F}^{n+j}x_0$)≤ α ℘ ($\mathcal{F}^{n-1+i}x_0, \mathcal{F}^{n-1+j}x_0$).
Since the above inequality holds for all i, j ∈ N, therefore by conditions (ii) and (2.2), we have
℘ ($\mathcal{F}^{n+i}x_0, \mathcal{F}^{n+j}x_0$)≤ α ℘ ($\mathcal{F}^{n-1+i}x_0, \mathcal{F}^{n-1+j}x_0$).
ℶ(℘,$\mathcal{F}$,$\mathcal{F}^n x_0$) ≤ αℶ(℘,$\mathcal{F}$,$\mathcal{F}^{n-1} x_0$)
Repeating this procedure, we have (for every n ∈ N)
ℶ(℘,$\mathcal{F}$,$\mathcal{F}^n x_0$) ≤ α^nℶ(℘,$\mathcal{F}$,x_0) ………….. (2.3)
Let n, m ∈ N such that m = n + p (for some p ∈ N). On using (2.3), we have
℘ ($\mathcal{F}^n x_0, \mathcal{F}^{n+p}x_0$)≤ ℶ(℘,$\mathcal{F}$,$\mathcal{F}^n x_0$) ≤ α^nℶ(℘,$\mathcal{F}$,x_0)
As ℶ(℘,$\mathcal{F}$,x_0) < ∞ and α ∈ (0, 1), we have

$\lim_{n,m\to\infty} \wp(x_n, x_m) = 0$, so that $\{x_n\}$ is a ℘-Cauchy sequence in Ӽ.

In view of the ℘-completeness of Ӽ, there exists x ∈ Ӽ such that $\{x_n\}$ ℘-converges to x. Now, we show that x ∈ Ӽ is a fixed point of $\mathcal{F}$.

Assume that $\mathcal{F}$ is continuous. Then x = $\lim_{n\to\infty} x_{n+1} = \mathcal{F}\left(\lim_{n\to\infty} x_n\right) = \mathcal{F}x$.Alternately , assume that (Ӽ, ℘) enjoys the (Λ1) property.

Now, we have ℘ $(\mathcal{F}x_n, \mathcal{F}x\ )$ ≤ ℘ $(x_n, x\ )$, which on taking n → ∞ implies that $\lim_{n\to\infty} \wp(x_{n+1}, \mathcal{F}x) = 0$.

Thus, from the (Λ1) property, $\mathcal{F}x$ = x. Therefore, x is a fixed point of $\mathcal{F}$. To prove the uniqueness of fixed point, let on contrary that x, y ∈ Ӽ such that $\mathcal{F}x$ = x and $\mathcal{F}y$ = y.

Then by the definition of ℘-contraction, we have ℘ (x, y) = ℘ $(\mathcal{F}x, \mathcal{F}y)$ ≤ α℘ (x, y), a contradiction. Hence, x = y, that is, x is a unique fixed point of $\mathcal{F}$.

Finally, we show that ℘ (x, x) = 0. Since $\mathcal{F}$ is ℘-contraction mapping, hence we have ℘ (x, x) = ℘ ($\mathcal{F}$x, $\mathcal{F}$x) ≤ α℘ (x, x) , this implies that ℘ (x, x) < 0 implying thereby ℘ (x, x) = 0. This completes the proof. Now, we recall the definition of Kannan-Ćirić contraction condition [14].

Definition 2.3.: Let (Ӽ, ℘) be a partial symmetric space. A mapping $\mathcal{F}$: Ӽ → Ӽ is said to be Kannan-Ćirić type ℘-contraction if

(for all x, y ∈ Ӽ)

℘ ($\mathcal{F}$x, $\mathcal{F}y$) ≤ α max{℘ (x, $\mathcal{F}$x), ℘ (y, $\mathcal{F}$y)}.....(2.4)

where α ∈ [0, 1).

Next, we prove a fixed point result via Kannan-Ćirić type ℘-contraction in the setting of partial symmetric spaces.

Theorem2.4.: Let (Ӽ, ℘) be a partial symmetric space and $\mathcal{F}$: Ӽ → Ӽ be a mapping. Assume that the following conditions are satisfied:

(i) $\mathcal{F}$ is a Kannan- Ćirić type ℘-contraction mapping,

(ii) $\mathcal{F}$is continuous.

Then $\mathcal{F}$ has a unique fixed point x ∈ Ӽ such that ℘ (x, x) = 0.

Proof. Take x_0∈ Ӽ and construct an iterative sequence $\{x_n\}$ by:

$x_1 = \mathcal{F}x_0$, $x_2 = \mathcal{F}^2x_0, x_3 = \mathcal{F}^3x_0, \cdots, x_n = \mathcal{F}^nx_0, \cdots$.

Now, we assert that $\lim_{n\to\infty} \wp(x_n, x_{n+1})$ =0. On setting x = x_n and y = x_{n+1} in (2.4), we get

$\lim_{n\to\infty} \wp(x_n, x_{n+1})$ =$\wp(\mathcal{F}x_{n-1}, \mathcal{F}x_n)$

≤ α max$\{\wp(x_{n-1}, \mathcal{F}x_{n-1}), \wp(x_n, \mathcal{F}x_n)\}$

≤ α max$\{\wp(x_{n-1}, x_n), \wp(x_n, x_{n+1})\}$ ....(2.5)

Assume that max $\{\wp(x_{n-1}, x_n), \wp(x_n, x_{n+1})\}$ =$\wp(x_n, x_{n+1})$.

Then, from (2.5), we have

$\wp(x_n, x_{n+1}))$ ≤ $\wp(x_n, x_{n+1})$, a contradiction (since α ∈ (0, 1)).

Thus, max $\{\wp(x_{n-1}, x_n), \wp(x_n, x_{n+1})\}$ =$\wp(x_{n-1}, x_n)$.

Therefore, (2.5) gives rise $\wp(x_n, x_{n+1})$ = α$\wp(x_{n-1}, x_n)$, for all n ∈ N.

Thus, inductively we have $\wp(x_n, x_{n+1}) = \alpha^n \wp(x_0, x_1)$ for all n ∈ N. On taking limit as n → ∞, we get $\lim_{n\to\infty} \wp(x_n, x_{n+1}) = 0$ ... . (2.6)

Now, we assert that $\{x_n\}$ is a ℘-Cauchy sequence.

Form (5.2.4), we have (for n, m ∈ N)

$$\wp(x_n, x_m) = \wp(\mathcal{F}x_{n-1}, \mathcal{F}x_{m-1})$$
$$\leq \alpha \max\{\wp(x_{n-1}, \mathcal{F}x_{n-1}), \wp(x_{m-1}, \mathcal{F}x_{m-1})\}$$
$$\leq \alpha \max\{\wp(x_{n-1}, x_n), \wp(x_{m-1}, x_m)\}$$

By taking limit as n, m → ∞ and using (2.6),

We have

$\lim_{n,m\to\infty} \wp(x_n, x_m) = 0$ .... (2.7)

Hence, $\{x_n\}$ is a ℘-Cauchy sequence.

Since (Ӽ, ℘) is ℘-complete, then there exists x ∈ Ӽ, such that

$\lim_{n\to\infty} \wp(x_n, x_m) = 0$.

Now, we have shown that x ∈ Ӽ is a fixed point of $\mathcal{F}$.

By the continuity of $\mathcal{F}$, we have

$$x = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} \mathcal{F}\, x_n = \mathcal{F}x.$$

Therefore, x is a fixed point of $\mathcal{F}$. For the uniqueness part, let on contrary that x, y ∈ Ӽ are such that $\mathcal{F}$x = x and $\mathcal{F}$y = y.

Then from (2.4), we have

℘ (x, y) = ℘ ($\mathcal{F}$x, $\mathcal{F}$y)

≤ α max{℘ (x, $\mathcal{F}$x), ℘ (y, $\mathcal{F}$y)}

≤ α max{℘ (x, x), ℘ (y, y)}.

So, either ℘ (x, y) ≤ α℘ (x, x) or ℘ (x, y) ≤ α℘ (y, y), which is a contradiction. Therefore, x is a unique fixed point of$\mathcal{F}$.

Finally, to show that P(x, x) = 0. Then from (2.4), we have

℘ (x, x) = ℘ ($\mathcal{F}$x, $\mathcal{F}$x)

≤ α max{℘ (x, $\mathcal{F}$x), ℘ (x, $\mathcal{F}$x)}

≤ α max{℘ (x, x), ℘ (x, x)}.

Which implies that ℘ (x, x) < 0 implying thereby ℘(x, x) = 0.

This completes the proof.

Now, we present some fixed point results for Ćirić quasi contraction in the setting of partial symmetric spaces. We start with the following definition.

Definition 2.5.: Let (Ӽ, ℘) be a partial symmetric space and $\mathcal{F}$: Ӽ → Ӽ be a mapping. Then $\mathcal{F}$ is said to be ℘-weak contraction if for all x, y ∈ Ӽ and α ∈ (0,

1) ℘ ($\mathcal{F}$x, $\mathcal{F}$y) ≤ α max {℘ (x, y), ℘ (x, $\mathcal{F}$x), ℘ (y, $\mathcal{F}$y), ℘ (x, $\mathcal{F}$y), ℘(y, $\mathcal{F}$x)}

…. (2.8)

Proposition 2.6. Let $\mathcal{F}$ be a ℘-weak contraction for any α ∈ (0, 1). If x is a fixed point of $\mathcal{F}$, then ℘ (x, x) = 0.

Proof. Suppose x ∈ Ӽ is a fixed point of $\mathcal{F}$. Since $\mathcal{F}$ is a ℘-weak contraction, so that

℘ (x, x) = ℘ ($\mathcal{F}$x, $\mathcal{F}$x)

≤ α max { ℘ (x, x), ℘ (x, $\mathcal{F}$x), ℘ (x, $\mathcal{F}$x), ℘ (x, $\mathcal{F}$x, ℘ (x, $\mathcal{F}$x) }

= α max {℘ (x, x), ℘ (x, x), ℘ (x, x),℘ (x, x),℘ (x, x)}

= α ℘ (x, x)

This is implies that ℘ (x, x) < 0 implying thereby ℘ (x, x) = 0.

Theorem 2.7.: Let (Ӽ, ℘) be a partial symmetric space and $\mathcal{F}$: Ӽ → Ӽ be a mapping. Suppose that the following conditions hold:

(i) $\mathcal{F}$ is a ℘-weak contraction for some α ∈ [0, 1),
(ii) there exists x_0∈ Ӽ such that ℶ (℘,$\mathcal{F}$, x) < ∞,
(iii) $\mathcal{F}$is continuous.

Then $\mathcal{F}$ has a unique fixed point.

Proof. Assume x_0∈ Ӽ and construct an iterative sequence $\{x_n\}$ by:

$x_1 = \mathcal{F}x_0$, $x_2 = \mathcal{F}^2x_0, x_3 = \mathcal{F}^3x_0, \cdots, x_n = \mathcal{F}^nx_0, \cdots$.

Let n be an arbitrary positive integer. Since $\mathcal{F}$ is a ℘-weak contraction, for all i, j ∈ N, we have

℘ $(\mathcal{F}^{n+i}x_0, \mathcal{F}^{n+j}x_0)$≤ α max { ℘ $(\mathcal{F}^{n-1+i}x_0, \mathcal{F}^{n-1+j}x_0)$,

℘ $(\mathcal{F}^{n-1+i}x_0, \mathcal{F}^{n+i}x_0)$, ℘ $(\mathcal{F}^{n-1+j}x_0, \mathcal{F}^{n+j}x_0)$,

℘ $(\mathcal{F}^{n-1+i}x_0, \mathcal{F}^{n+j}x_0)$, ℘ $(\mathcal{F}^{n-1+j}x_0, \mathcal{F}^{n+i}x_0)$}

Since above inequality is true for all i, j ∈ N, therefore by the conditions (ii) and (5.3.1), we have

ℶ(℘, $\mathcal{F}$, $\mathcal{F}^nx_0$) ≤ α ℶ(℘, $\mathcal{F}$, $\mathcal{F}^{n-1}x_0$)

Continuing this process indefinitely, we have (for all n ≥ 1)

ℶ(℘, $\mathcal{F}$, $\mathcal{F}^nx_0$) ≤ α^nℶ(℘, $\mathcal{F}$, x_0) ……….(2.9)

Now, for each n, m ∈ N such that m = n + p (for some p ∈ N), we have (due to (2.9))

℘$(\mathcal{F}^nx_0, \mathcal{F}^{n+p}x_0)$≤ ℶ(℘, $\mathcal{F}$, $\mathcal{F}^nx_0$)

≤ α^nℶ(℘, $\mathcal{F}$, x_0)

Since ℶ(℘, $\mathcal{F}$, x_0) < ∞ and α ∈ (0, 1), we have

$\lim_{n,m\to\infty}$ ℘$(x_n, x_m) = 0$, so that $\{x_n\}$ is a ℘-Cauchy sequence in Ӽ.

In view of the ℘-completeness of Ӽ, there exists x ∈ Ӽ such that $\{x_n\}$℘-converges to x.

Now, we show that x is a fixed point of $\mathcal{F}$. By the continuity of $\mathcal{F}$, we have

$$x = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} \mathcal{F}x_{n} = \mathcal{F}x.$$

Therefore, x is a fixed point of $\mathcal{F}$.

For the uniqueness part, let on contrary that x, y ∈ Ӽ are such that $\mathcal{F}x$ = x and

$\mathcal{F}y$ = y. Thus, by using the condition (2.8), we have

℘ (x, y) = ℘ ($\mathcal{F}x$, $\mathcal{F}y$)

≤ α max ℘ (x, y), ℘ (x, $\mathcal{F}x$), ℘ (y, $\mathcal{F}y$), ℘ (x, $\mathcal{F}y$), ℘ (y, $\mathcal{F}x$)

= α max { ℘ (x, y), ℘ (x, x), ℘ (y, y), ℘ (x, y), ℘ (y, x) }.

By using the property (2℘), we have ℘ (x, y) ≤ α℘ (x, y), a contradiction so that ℘ (x, y) = 0 which implies that x = y. Thus, $\mathcal{F}$ has a unique fixed point.

This completes the proof.

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