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Some Fixed Point Results in Dislocated and Dislocated Quasi-Metric Spaces

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Abstract -In this articlewe will discuss the concepts of dislocated and dislocated quasi-metric spaces as well as established fixed point results in the aforementioned spaces. A few well-known discoveries that have been supported by the literature will be widened, clarified, and combined by our theorems.

Keywords - Fixed Point Theorem, Quasi Metric Space, Dislocated Quasi Metric Space

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PRELIMINARILY NOTES

To investigate the generality of Banach's Contraction principle and the concept of dislocated metrics, metric domains were studied in the framework of domain theory [3].Hitzler and Seda were the first mathematicians to examine the dislocated metric space in quasi-metric spaces in [6, 7]. In the disciplines of topology, electronics engineering, and logic programming, these metrics are very important. The rational type of contractive condition is used by D.S. Jaggi.

By generalizing the well-known Banach Contraction Principle in these spaces whenever the self distance for any point does not have to be equal to zero, the Hitzler and Seda concept of a dislocated metric space creates the sense of a dislocated metric space.

The term "dislocated quasi-metric space" was originally used by Zeyada et al. [14], and the Hitzler and Seda finding was generalised to such spaces. In recent years, Isufati [8], Aage and Salunke [2], and Rao-RangaSwamy [12] have studied dislocated and dislocated quasi-metric spaces.

In order to validate our conclusions, we will first provide a few definitions and theorems produced by other mathematicians**.**

Definition[14] : Let A be a non-empty and let $\delta : \mathrm{A} \times \mathrm{A} \rightarrow [0,\infty)$ be a function, called a distance function, satisfies:

$\delta 1$: $\delta(a,a)=0,$

$\delta 2$: $\delta(a,b)=\delta(b,a)=0$, then $a=b,$

$\delta 3$: $\delta(a,b)=\delta(b,a),$

$\delta 4$: $\delta(a,b)\le\delta(a,c)+d(c,b)$

for all $a,b,c\in \mathrm{A}$.

If the criteria δ1 through δ4 are met, then δ is referred to as a metric on A. It is referred to as a quasi metric space if it meets the requirements δ1, δ2, and δ4. It is referred to as a dislocated metric (or simply δ-metric) on A if conditions δ2, δ3, and δ4 are met, and as a dislocated quasi-metric (or simply δ q-metric) on A if just conditions δ2 and δ4 are met. A dislocated quasi-metric space is a set A that is not empty and has the δ q-metric, or (A, δ).

Definition [14]: A sequence $\{x_n\}$ in δ q-metric in *(A,* δ*)* is called Cauchy if for all $\in>0$, $\exists x_0\in N$, such that $\forall m,n\ge x_0$, $\delta(a_m,a_n)<\in$ or $\delta(a_n,a_m)<\in$.

If the afore mentioned issue is resolved by $\max\{\delta(a_m, a_n), \delta(a_n, a_m)\} < \in$, the sequence $\{x_n\}$ is δ q-metric space (*A*, δ) is called 'bi' Cauchy.

Definition [14]: A sequence $\{a_n\}$ δ q-converges to *A* if

$$\lim_{n\to\infty} \delta(a_n, a) = \lim_{n\to\infty} \delta(a, a_n) = 0$$

In this case *A* is called a δ_q-limit of a_n and we write $a_n \to a.$.

Proposition: In a δ_q-metric space, every convergent sequence is bi Cauchy.

Definition [14]: If every Cauchy sequence contained within the δ_q-metric space (*A*, δ) is δ_q-convergent, then it is said to be complete.

Lemma [14]: Every segment of the δ_q-convergent sequence to a point x_0 is δ_q-convergent to x_0.

Definition [14]: Assume that (*A*, δ) is a δ_q-metric space. If there is such a map $f : \mathrm{A} \to \mathrm{A}$, then it is said to be contracted if there exists $0 \le \lambda < 1$ such that $\delta[f(a), f(b)] \le \lambda\delta(a,b)$.

Lemma 1 [14]: Assume that (*A*, δ) is a δ_q-metric space. If $f : \mathrm{A} \to \mathrm{A}$ is a contraction function, then $f^n(a_0)$ is a Cauchy sequence for each $a_0 \in A$.

Lemma 2 [14]: In a δ_q-metric space, there are no other δ_q-limits.

Theorem 1 [14] : Let $f : \mathrm{A} \to \mathrm{A}$ be a continuous contraction function, and let (*A*, δ) be a complete δ_q-metric space. Consequently, f has a distinct fixed point.

The **Isufati [8**] proved the following conclusions in dislocated and dislocated quasi-metric spaces.

Theorem 2 [8]: Let $T : A \to A$ be a continuous mapping meeting the following requirement, with (*A*, δ) being the complete δ_q-metric space:

$$d(Tx,Ty) \le \alpha \frac{d(y,Ty)[1+d(x,Tx)]}{1+d(x,y)} + \beta d(x,y)$$

for all $x, y \in A$, $\alpha, \beta > 0$, $0 \le \beta < 1$. Then *T* has unique fixed point.

Theorem 3[8] : Let (*A*, δ) be a complete δ_q- metric space and let $T : \mathrm{A} \to \mathrm{A}$ be a continuous mapping satisfying the following conditions:

$$\delta(Tx,Ty) \le \alpha\delta(x,Ty) + \beta\delta(y,Tx) + \gamma\delta(x,y)$$

where α, β, γ are non negative, which may depends on both *x* and *y*, such that sup $\{2\alpha + 2\beta + \gamma : x, y \in X\}$<1. Then *T* has unique fixed point.

Theorem 4[8] : Let (*A*, δ) be a complete dislocated metric space. Let $f, g : A \to A$ be continuous mapping satisfying:

$$\delta(fx, gy) \le h \max\left\{\delta(x,y), \delta(x,fx), \delta(y,gy), \frac{\delta(x,gy)+\delta(y,fx)}{2}\right\}$$

for all $x, y \in A$ and $0 < h < 1$. Then *f* and *g* have common fixed point.

The findings from**Aage**and**Salunke [1, 2]** are as follows**:**

Theorem 5 [1] : Let (*A*, δ) be a complete δ_q-metric space. If $T : \mathrm{A} \to \mathrm{A}$ be a continuous mapping satisfying

$$\delta(Tx,Ty) \le \alpha\{\delta(x,Tx) + \delta(y,Ty)\}$$

for all $x, y \in X$ and $0 \le \alpha < 1/2$. Then *T* has a unique fixed point.

Theorem 6 [1] : Let (*A*, δ) be a complete δ_q-metric space. Let $T : A \to A$ be a continuous generalized contraction. Then *T* has a unique fixed point.

Theorem 7 [1] : Let (*A*, δ) be a complete dislocated metric space. Let $T : \mathrm{A} \to \mathrm{A}$ be continuous mapping satisfies;

$$\delta(Tx,Ty) \le \alpha\delta(x,y) + \beta\delta(x,Tx) + \gamma\delta(y,Ty)$$

$$+ \delta\frac{\delta(x,Tx)\delta(y,Ty)}{\delta(x,y)} + \mu\frac{\delta(x,Ty)\delta(y,Tx)}{\delta(x,y)},$$

for all $x, y \in A$ and $\alpha, \beta, \gamma, \delta, \mu \geq 0$ with $\alpha + \beta + \gamma + \delta + 4\mu < 1$. Then *T* has a unique fixed point.

Main Result:

We first present the following theorem in order to illustrate Theorem 3.1.20 in the context of dislocated quasi-metric spaces.

Theorem 8: Let Γ be a continuous self-mapping defined on a complete δ_q-metric space (*A*, δ). Further let Γ satisfy the contractive condition,

$$\delta(\Gamma x, \Gamma y) \leq \alpha \frac{\delta(x,\Gamma x).\delta(y,\Gamma y)}{\delta(x,y)} + \beta \frac{\delta(x,\Gamma y).\delta(y,\Gamma x)}{\delta(x,y)} + \gamma\delta(x,y)$$

. (1)

For all $x, y \in A$ and $\alpha, \beta, \gamma \geq 0$, with $\alpha + \beta + \gamma < 1$. Then Γ has a unique fixed point.

Proof: Let x_0 be any random point in A. Set forth a sequence $\{x_n\}$ in *A* such that $x_1 = \Gamma(x_0)$, $x_2 = \Gamma(x_1), \ldots\ldots\ldots\ x_{n+1} = \Gamma(x_n), \ldots\ldots\ldots$

Replace x by x_{n-1} and *y* by x_n in (3.2.1), we have

$$\delta(x_n, x_{n+1}) = \delta(\Gamma x_{n-1}, \Gamma x_n)$$

$$\leq \alpha \frac{\delta(x_{n-1},\Gamma x_{n-1}).\delta(x_n,\Gamma x_n)}{\delta(x_{n-1},x_n)} + \beta \frac{\delta(x_{n-1},\Gamma x_n).\delta(x_n,\Gamma x_{n-1})}{\delta(x_{n-1},x_n)} + \gamma\delta(x_{n-1},x_n)$$

$$\leq \alpha \frac{\delta(x_{n-1},x_n).\delta(x_n,x_{n+1})}{\delta(x_{n-1},x_n)} + \beta \frac{\delta(x_{n-1},x_{n+1}).\delta(x_n,x_n)}{\delta(x_{n-1},x_n)} + \gamma\delta(x_{n-1},x_n)$$

$$\leq \alpha\delta(x_n, x_{n+1}) + \gamma\delta(x_{n-1}, x_n).$$

Therefore

$$\delta(x_n, x_{n+1}) \leq \frac{\gamma}{1-\alpha}\delta(x_{n-1}, x_n) = \lambda\delta(x_{n-1}, x_n) \cdot$$

Where $\lambda = \dfrac{\gamma}{1-\alpha} < 1$

Similar to it, we have

$$\delta(x_{n-1}, x_n) \leq \lambda\delta(x_{n-2}, x_{n-1})$$

As a result,

$$\delta(x_n, x_{n+1}) \leq \lambda^2\delta(x_{n-2}, x_{n-1})$$

In general, as we continue this process, $\delta(x_n, x_{n+1}) \leq \lambda^n\delta(x_1, x_0)$

Since $0 \leq \lambda < 1$ as $n \to \infty$, $\lambda^n \to 0$.

A has a -Cauchy sequence as a result. Therefore, in A, dislocated quasi converges to some u. Γ being a continuous mapping , we have

$$\Gamma(u) = \lim \Gamma(x_n) = \lim x_{n+1} = u.$$

Consequently, u is a fixed point on Γ.

Uniqueness: Consider u as a fixed point of Γ. Next by (1)

$$\delta(u,u) = \delta(\Gamma u, \Gamma u)$$

$$\leq \alpha \frac{\delta(u,\Gamma u).\delta(u,\Gamma u)}{\delta(u,u)} + \beta \frac{\delta(u,\Gamma u).\delta(u,\Gamma u)}{\delta(u,u)} + \gamma\delta(u,u)$$

$$\leq (\alpha + \beta + \gamma)\delta(u,u)$$

which only applies if $\delta(u,u) = 0$, since $0 \leq \alpha + \beta + \gamma < 1$ and $\delta(u,u) \geq 0$.

Thus $\delta(u,u) \geq 0$, if uis a fixed point of Γ. Assume that A has two fixed points, u and v, which are $\Gamma u = u$ and $\Gamma v = v$.

Then by (3.2.1) we have, $\delta(\Gamma u, \Gamma v) \leq \alpha \dfrac{\delta(u,\Gamma u).\delta(v,\Gamma v)}{\delta(u,v)} + \beta \dfrac{\delta(u,\Gamma v).\delta(v,\Gamma u)}{\delta(u,v)} + \gamma\delta(u,v)$

$$\delta(u,v) = \delta(\Gamma u, \Gamma v) \leq (\beta + \gamma)\delta(u,v),$$

That provides $\delta(u,v) = 0$, since $0 \leq (\beta + \gamma) < 1$ and $\delta(u,v) \geq 0$.

Similarly $\delta(v,u) = 0$ and hence *u* =*v*.

Thus fixed point of Γ is unique.

This completes the proof.

Theorem 9: Let *(A, d)* be a complete dislocated quasi-metric space. Let $\Gamma : A \to A$ be continuous mapping satisfies the condition;

for all $x, y \in X$ and $\alpha, \beta, \gamma \geq 0$, with $\alpha + \beta + 2\gamma + 2\delta < 1$. Then Γ has a unique fixed point.

Proof : Let $\{x_n\}$ be a sequence in *A* defined as follows for an arbitary $x_0 \in X$, $\Gamma(x_0) = x_1$, $\Gamma(x_1) = x_2, \ldots\ldots \Gamma(x_n) = x_{n+1}, \ldots\ldots$,

Putting $x = x_{n-1}$ and $y = x_n$ in (2) we have,

$$\delta(\Gamma x_{n-1}, \Gamma x_n) = \delta(x_n, x_{n+1}) \le \alpha_1 \delta(x_{n-1}, x_n) + \alpha_2 \frac{\delta(x_{n-1}, \Gamma x_{n-1})\delta(x_n, \Gamma x_n)}{1+\delta(x_{n-1}, x_n)} + \alpha_3 \frac{\delta(x_{n-1}, \Gamma x_n)\delta(x_n, \Gamma x_{n-1})}{1+\delta(x_{n-1}, x_n)} + \alpha_4 \frac{\delta(x_{n-1}, \Gamma x_{n-1})\delta(x_{n-1}, \Gamma x_n)}{1+\delta(x_{n-1}, x_n)} + \alpha_5 \frac{\delta(x_n, \Gamma x_{n-1})\delta(x_n, \Gamma x_n)}{1+\delta(x_{n-1}, x_n)}$$

$$\le \alpha_1 \delta(x_{n-1}, x_n) + \alpha_2 \frac{\delta(x_{n-1}, x_n)\delta(x_n, x_{n+1})}{1+\delta(x_{n-1}, x_n)} + \alpha_3 \frac{\delta(x_{n-1}, x_{n+1})\delta(x_n, x_n)}{1+\delta(x_{n-1}, x_n)} + \alpha_4 \frac{\delta(x_{n-1}, x_n)\delta(x_{n-1}, x_{n+1})}{1+\delta(x_{n-1}, x_n)} + \alpha_5 \frac{\delta(x_n, x_n)\delta(x_n, x_{n+1})}{1+\delta(x_{n-1}, x_n)}$$

$$\le \alpha_1 \delta(x_{n-1}, x_n) + \alpha_2 \frac{\delta(x_{n-1}, x_n)\delta(x_n, x_{n+1})}{1+\delta(x_{n-1}, x_n)} + \alpha_3 \frac{\delta(x_{n-1}, x_{n+1})\delta(x_n, x_n)}{1+\delta(x_{n-1}, x_n)} + \alpha_4 \frac{\delta(x_{n-1}, x_n)\delta(x_{n-1}, x_{n+1})}{1+\delta(x_{n-1}, x_n)} + \alpha_5 \frac{\delta(x_n, x_n)\delta(x_n, x_{n+1})}{1+\delta(x_{n-1}, x_n)}$$

$$\le \alpha_1 \delta(x_{n-1}, x_n) + \alpha_2 \frac{\delta(x_{n-1}, x_n)\delta(x_n, x_{n+1})}{1+\delta(x_{n-1}, x_n)} + \alpha_3 \frac{\delta(x_{n-1}, x_{n+1})\delta(x_n, x_n)}{1+\delta(x_{n-1}, x_n)} + \alpha_4 \frac{\delta(x_{n-1}, x_n)[\delta(x_{n-1}, x_n) + \delta(x_n, x_{n+1})]}{1+\delta(x_{n-1}, x_n)} + \alpha_5 \frac{\delta(x_n, x_n)\delta(x_n, x_{n+1})}{1+\delta(x_{n-1}, x_n)}$$

$$\le (\alpha_1 + \alpha_4)\delta(x_{n-1}, x_n) + \alpha_2 \delta(x_n, x_{n+1}) + \alpha_3 .0 + \alpha_4 \delta(x_n, x_{n+1}) + \alpha_5 .0$$

$$(1 - \alpha_2 - \alpha_4)\delta(x_n, x_{n+1}) \le (\alpha_1 + \alpha_4)\delta(x_{n-1}, x_n)$$

$$\delta(x_n, x_{n+1}) \le \frac{(\alpha_1 + \alpha_4)}{(1 - \alpha_2 - \alpha_4)}\delta(x_{n-1}, x_n)$$

$$\delta(x_n, x_{n+1}) \le \lambda \delta(x_{n-1}, x_n)$$

$$0 \le \lambda = \frac{(\alpha_1 + \alpha_4)}{(1 - \alpha_2 - \alpha_4)} \prec 1$$

Similarly ,

$$\delta(x_{n-1}, x_n) \le \lambda \delta(x_{n-2}, x_{n-1})$$

Therefore, we get

$$\delta(x_n, x_{n+1}) \le \lambda^2 \delta(x_{n-2}, x_{n-1})$$

Continuing in this way, we have

$$\delta(x_n, x_{n+1}) \le \lambda^n \delta(x_0, x_1).$$

Since $0 \le \lambda < 1$, so for $n \to \infty$, we have $\delta(x_n, x_{n+1}) \to 0$. Similarly we show that $\delta(x_{n+1}, x_n) \to 0$. Hence $\{x_n\}$ is a Cauchy sequence in complete dislocated quasi-metric space *A*. So there is a point $u \in X$ and since Γ is continuous, therefore $\Gamma(u) = \Gamma(\lim x_n) = \lim \Gamma(x_n) = \lim x_{n+1} = u$. Thus *u* is a fixed point of Γ.

Uniqueness: Let *u* be a fixed point of Γ i.e. $\Gamma u = u$. Then by condition (3.2.2), we have,

$$\delta(\Gamma u, \Gamma u,) \le \alpha_1 \delta(u, u) + \alpha_2 \frac{\delta(u, \Gamma u)\delta(u, \Gamma u)}{1+\delta(u, u)} + \alpha_3 \frac{\delta(u, \Gamma u)\delta(u, \Gamma u)}{1+\delta(u, u)} + \alpha_4 \frac{\delta(u, \Gamma u)\delta(x, \Gamma y)}{1+\delta(u, u)} + \alpha_5 \frac{\delta(u, \Gamma u)\delta(u, \Gamma u)}{1+\delta(u, u)}$$

$$\delta(\Gamma u, \Gamma u,) \le (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)\delta(u, u)$$

Which is true only if $\delta(u, u) = 0$ $(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)$<1.

Thus, $\delta(u, u) = 0$, for a fixed point u of Γ. Similarly $\delta(v, v) = 0$

Let *u*, *v* be fixed points of Γ, i.e. $\Gamma u = u$ and $\Gamma v = v$.

$$\delta(\Gamma u, \Gamma v,) \le \alpha_1 \delta(u, v) + \alpha_2 \frac{\delta(u, \Gamma u)\delta(v, \Gamma v)}{1+\delta(u, v)} + \alpha_3 \frac{\delta(u, \Gamma v)\delta(v, \Gamma u)}{1+\delta(u, v)} + \alpha_4 \frac{\delta(u, \Gamma u)\delta(u, \Gamma v)}{1+\delta(u, v)} + \alpha_5 \frac{\delta(v, \Gamma u)\delta(v, \Gamma v)}{1+\delta(u, v)}$$

$$\delta(u, v) = \delta(\Gamma u, \Gamma v,) \le \alpha_1 \delta(u, v) + +\alpha_3 \delta(u, v)$$

$$\delta(u, v) \le (\alpha_1 + \alpha_3)\delta(u, v)$$

This implies that $\delta(u, v) = 0 = \delta(v, u)$, since $(\alpha_1 + \alpha_3) \prec 1$.

Further, $\delta(u, v) = 0 = \delta(v, u)$ gives u=v.

Hence Γ has unique fixed point.

This completes the proof.

Theorem 10: Let *(A, d)* be a complete dislocated metric space. Let $\Sigma, \Gamma : A \to A$ be continuous mapping satisfying:

$$\delta(\Sigma x,\Gamma y)\le \alpha \max\begin{Bmatrix}\delta(x,\Sigma x)+\delta(y,\Gamma y),\\ \delta(y,\Gamma y)+\delta(x,y),\\ \delta(x,\Sigma x)+\delta(x,y)\\ \delta(\Sigma x,\Gamma y)+\delta(x,y)\end{Bmatrix}$$
(2)

for all $x, y \in A$ and $\alpha \in [0,1/2)$. Then $\Sigma \,\&\, \Gamma$ have common fixed point.

Proof: Let $x_0 \in A$ be arbitrary. Define the sequence $\{x_n\}$ by, $x_1=\Sigma(x_0)$, $x_2=\Gamma(x_1)$, ... $x_{(2n)}=\Gamma(x_{2n-1})$, $x_{2n+1}=\Sigma(x_{2n})$.........,

Replace x by x_{2n} and *y* by x_{2n+1} in (3) we have,

$$\delta(x_{2n+1},x_{2n+2})=\delta(\Sigma x_{2n},\Gamma x_{2n+1})$$

$$\le \alpha \max\begin{Bmatrix}\delta(x_{2n},\Sigma x_{2n})+\delta(x_{2n+1},\Gamma x_{2n+1}),\\ \delta(x_{2n+1},\Gamma x_{2n+1})+\delta(x_{2n},x_{2n+1}),\\ \delta(x_{2n},\Sigma x_{2n})+\delta(x_{2n},x_{2n+1})\\ \delta(\Sigma x_{2n},\Gamma x_{2n+1})+\delta(x_{2n},x_{2n+1})\end{Bmatrix}$$

$$\le \alpha \max\begin{Bmatrix}\delta(x_{2n},x_{2n+1})+\delta(x_{2n+1},x_{2n+2}),\\ \delta(x_{2n+1},x_{2n+2})+\delta(x_{2n},x_{2n+1}),\\ \delta(x_{2n},x_{2n+1})+\delta(x_{2n},x_{2n+1})\\ \delta(x_{2n+1},x_{2n+2})+\delta(x_{2n},x_{2n+1})\end{Bmatrix}$$

$$\le \alpha[\delta(x_{2n},x_{2n+1})+\delta(x_{2n+1},x_{2n+2})]$$

Therefore,

$$\delta(x_{2n+1},x_{2n+2})\le \frac{\alpha}{1-\alpha}\delta(x_{2n},x_{2n+1})$$

and $\delta(x_{2n+1},x_{2n+2})\le \lambda\delta(x_{2n},x_{2n+1})$

where $\lambda=\dfrac{\alpha}{1-\alpha}$, $\quad 0\le\lambda<1.$

Similarly

$$\delta(x_{2n},x_{2n+1})\le \lambda\delta(x_{2n-1},x_{2n})$$

and so $\delta(x_{2n+1},x_{2n+2})\le \lambda^2\delta(x_{2n-1},x_{2n})$

Continue in this manner, we have,

$$\delta(x_{2n+1},x_{2n+2})\le \lambda^n\delta(x_0,x_1)$$

Since $0\le\lambda<1$, as $n\to\infty$, $\lambda^n\to 0$. Thus $\{x_n\}$ is a Cauchy sequence in a complete dislocated metric space *A*. Therefore there exists a point $u\in A$ such that $x_n\to u$. Therefore the subsequences $\{\Sigma x_{2n}\}\to u$ and $\{\Gamma x_{2n}\}\to u$, Since $\Sigma \,\&\, \Gamma$ are continuous, so we must have $\Sigma u=u$ and $\Gamma u=u$. Thus *u* is a common fixed point of $\Sigma \,\&\, \Gamma$.

Uniqueness: Let *u*, *v* be common fixed point of $\Sigma \,\&\, \Gamma$. Then by the condition (3.2.3),

$$\delta(u,v)=\delta(\Sigma u,\Gamma v)$$

$$\le \alpha \max\begin{Bmatrix}\delta(u,u)+\delta(v,v),\\ \delta(v,v)+\delta(u,v),\\ \delta(u,v)+\delta(u,v)\\ \delta(u,v)+\delta(u,v)\end{Bmatrix}$$

Replacing *v* by *u*, we get,

$\delta(u,u)\le 2\alpha\delta(u,u)$. Since $2\alpha<1$, we have $\delta(u,u)=0$. Similarly we have $\delta(v,v)=0$. In this way, we have $\delta(u,v)\le\alpha\delta(u,v)$. Since $0\le\alpha<1/2$, we have $\delta(u,v)=0$. Similarly we have $\delta(v,u)=0$ and so $u=v$.

Hence $\Sigma \,\&\, \Gamma$ have a Common Unique fixed Point.

The proof is completed.

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