Journal of Advances and Scholarly Researches in Allied Education
Vol. 20, Issue No. 4, October-2023, ISSN 2230-7540

DOI: https://doi.org/10.29070/83da8055

A study of accelerating and anisotropic cosmological models in certain modified theories of gravitation

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Abstract: Anisotropic and spatially homogenous cosmological models are important tools for explaining the universe's large-scale characteristics. Bianchi space-times have a uniform spatial structure and are globally hyperbolic. On the one hand, the asymptotic behaviour of the most rigid Bianchi space-time (Bianchi I and II space time) is straightforward and well-defined. Nonetheless, the asymptotic behaviour of generic Bianchi space time (Bianchi VIll and IX space-time) is considered to remain chaotic and oscillatory at their initial singularity over an extended period of time. Numerous hoovers Bianchi VIll and IX space-times display extraordinarily complicated aceillatory behaviour around their original singularity; even little changes to such a space-time's initial Cauchy data produce a significant shift in its asymptotic behaviour. Bianchi space-times are asymptotically silent, defying Misner's notion. Bianchi spaces l X are critical for modelling spatially homogeneous and anisotropic cosmologies. Dirac was the first to propose the idea of a variable gravitational constant G in the framework of relativity. Lau proposed changes to relate the G and A variants within the framework of general relativity. Because a variation in the A equations is technically followed by a variation in G, the significance of the change allows us to continue using Einstein's field without modification.

Keywords: Cosmological models, , Bianchi I and II space time and Einstein's field theory

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INTRODUCTION

Bianchi spacetimes are asymptotically quiet, defying Misner's hypothesis. Bianchi spaces l X are crucial for modelling spatially homogenous and anisotropic cosmologies. Dirac was the first to introduce the concept of a changeable gravitational constant G into the relativity framework. Lau, working within the context of general relativity, offered changes The coupling the variation of G with that of A. importance of modification permits us to utilise Einstein's field unmodified since a variation in A equations formally is followed by a variation of G.

The branch of study known as cosmology is concerned with the space-time link that exists between the universe's origin, structure, and overall dimensions. There have been a number of phases that it has gone through, which explains the overall complexity of the cosmos. Due to the fact that we are a part of this fundamental cosmos, it is necessary to comprehend the genesis, development, and ultimate destiny of the universe in order to successfully do so. In this way, mathematical models of the cosmos are being constructed. Using general relativity and alternative theories of Bianchi universe gravity, Einstein's theory of general relativity was first proposed. are a category of cosmological models that are inherently homogenous but not isotropic on spatial slices. They are named after Luigi Bianchi, a three-dimensional expert who is responsible for classifying the spaces. As a result of recent theoretical and experimental research, it has been shown that the stage of matter known as dark expansion is speeding in our universe. in addition to the fact that an unidentified kind of energy plays a vital role in the acceleration of this process The features of this dark energy that are the most intriguing are the negative energy density pressure and the positive energy density power. Based on the findings of the Wilkinson microwave anisotropy probe (WMAP) and plank, it can be deduced that the universe is made up of about 68.5% dark energy, 26.5% dark matter, and 5% baryonic matter. The manifestation of dark energy might take place in either of two ways. To begin, there is the use of so-called exotic matter, which may be accomplished by the utilisation of the equation of state parameter (EOS) W= p/p', where p represents the pressure and p' represents the energy density. Alternatively, the cosmological constant can be included in. An alternate theory to Einstein's theory of gravity is the second method that is used in order to visualise the expansion of the universe. This method is a modified form of the action principle that was developed by Einstein and Hilbert. Within the context of this procedure, the matter Lagrangian in the action is substituted with an arbitrary function. Therefore, the most appealing explanation for the acceleration in the expansion of the universe, together with the effective reasons associated to dark energy, may be found in these updated ideas.

An undiscovered kind of substance known as "dark energy" plays a substantial part in driving this acceleration, according to recent theoretical and experimental research. These studies have shown that our universe is now in an accelerating stage of expansion, and that this acceleration is being driven by energy. Positive energy density, on the other hand, comes with negative pressure, which is one of the most intriguing aspects of this dark energy. Based on the findings of the Wilkinson microwave anisotropy probe (WMAP) and Plank, it can be deduced that the universe is made up of around 68.5% dark energy, 26.5% dark matter, and 5% baryonic matter. The manifestation of dark energy might take place in either of two ways.

The study of cosmological models has proven effective in applying Einstein's theory of gravity. It has been noted, although, that there are several gaps in this idea. Therefore, in order to investigate the world using cosmological models, a number of different modified theories of gravity have been presented recently. The scientific study of the large-scale characteristics of the entire universe is known as cosmology. It aims to apply the scientific method to comprehend the universe's beginnings, development, and ultimate destiny. Cosmology, like all scientific disciplines, is based on the development of ideas or hypotheses about the cosmos that offer precise predictions for occurrences that can be observed and verified.

It is commonly recognised that this may be accomplished successfully by building mathematical models (cosmological models), examining how they behave physically, and then contrasting the results with the observations. Motivated by this, we have begun studying various anisotropic and spatially homogeneous cosmological models in Lyra's (1951) and Saez and Ballester's (1986) alternative theories of gravity.

EINSTEIN'S THEORY OF GRAVITATION

Einstein (1915) general theory of relativity describes the relativity of all kinds of motion. This theory generalizes special relativity which deals with the relativity uniform translator motion of bodies in free space where the gravitational effects are neglected. In formulating this theory Einstein was mainly guided by three basic principles: Principle of Covariance which helps to write the physical laws in covariant form, Principle of Equivalence which incorporates gravitational effects in general relativity and Mach"s Principle which determines internal properties of matter.

Einstein"s theory is a geometric theory of gravitation which gives a unified description of gravity as a geometric property of space-time. In particular, the curvature of space-time is directly related to the four momentums of matter and radiation. Even today, Einstein"s theory of gravitation is considered to be the correct theory of gravitation which, successfully, is used to study the cosmological models of our universe. This is because of the fact that the basic force governing the dynamics of the universe is gravity. It is well known that, in this theory, the gravitational field manifests through the curvature of the space-time given by the metric

$$ds^2 = g_{ij}\, dx^i dx^j \;\; ; i, j = 1,2,3 \text{ and } 4 \ldots\ldots (1)$$

where the components of the symmetric tensor Einstein field equations which govern the gravitational field are given by

$$G_{ij} + \Lambda g_{ij} = -8\pi T_{ij} \ldots\ldots\ldots\ldots (2)$$

Where, $G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$ = Einstein tensor

$R_{ij} =$ the Ricci tensor

$R = scalar\ curvature$

$T_{ij} =$ energy momentum tensor due to matter

$\Lambda =$ cosmological constant

Einstein introduced this cosmic constant when exploring static cosmological models, but eventually dismissed it, remarking, "It is the greatest blunder of my life".

As the Einstein tensor G_{ij} *is divergence free, the field equation* (2)*yield*

$$T_{;j}^{ij} = 0 \ldots\ldots\ldots\ldots\ldots (3)$$

It provides us with the equations of motion for matter and may be regarded as the conservation of energy momentum equation.

COSMOLOGY AND COSMOLOGICAL MODELS

We provide a basic overview of cosmology and cosmological universe models in this section. Science's field of cosmology studies the universe's large-scale structure.

There are stars, galaxies, pulsars, nebulae, quarks, and quasars in the cosmos. Even now, one of the biggest cosmic puzzles is the universe's beginnings. The "big-bang" idea is the most widely accepted explanation for the universe's creation and development. It is still unknown what the precise physical conditions were at the beginning of the universe's development. The dynamics of the system is the fundamental issue in cosmology. Gravity is the basic force that holds solar systems, stars, and galaxies together.

Since galaxies, which make up a large portion of the universe, and the intergalactic medium are known to be electrically neutral, other long-range interactions, including electromagnetic forces, may be ignored.

The cosmological principle, which asserts that the world is homogenous and isotropic on a sufficiently vast scale, is the foundation of cosmology research. This suggests that there isn't a preferable location, direction, or era in the cosmos from a physical standpoint. Therefore, we suppose that the universe is filled with a simple macroscopic perfect fluid (free of bulk-viscous, shear-viscous, and heat-conductive qualities) by using the cosmological principle. Therefore, its energy-momentum tensor T_{ij} is provided by

$$T_{ij} = (\rho + P)u_i u_j - p g_{ij} \ldots\ldots\ldots\ldots\ldots(4)$$

In equation-4 $\rho = proper\ energy\ density$

P= isotropic pressure

$u_i =$ four $-$ velocity of the fluid particles (stars etc.,).

Building mathematical models of the cosmos and comparing them to the current state of the universe as viewed by astronomers is the primary goal of cosmology. The creation of Einstein's static universe in 1917 marked the beginning of the theory of cosmological models. Hubble's well-known rule connecting the apparent luminosities of far-off galaxies to their red shifts was published in 1922. Which is given below:

$$V = HD \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(5)$$

where H is Hubble's constant and V is the galaxy's recession speed at a distance of D from us. The observed red shift in spectral lines from far-off galaxies disqualified static explanations of the cosmos and increased the significance of non-static models.

The most comprehensive non-static, homogeneous, and isotropic space-time provided by the Friedmann (1922) was initially examined by the

Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t)\left\{\frac{dr^2}{1-cr^2} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(6)$$

in the equation 6, a(t) = scale factor, and c = constant (a convenient choice of r which can be choosen to have values between (-1 to +1) as per the universe is bounded, flat or open respectively. According to him the evolution of the function a(t) using Einstein field equation for all three curvature.

The modern universe may be adequately characterised by a Friedmann-Robertson-Walker (FRW) model as it is both spatially homogenous and isotropic, as demonstrated by both theoretical and experimental evidence (Patridge and Wilkinson 1967, Ehlers et al. 1968). Nevertheless, evidence for a modest magnetic field across cosmic remote scales (Sofue et al. 1979) and a minor amount of anisotropy (Boughn et al. 1981) is present. This implies a significant divergence from FRW models during the initial phases of the universe's development.

Studying cosmological models, which may be quite anisotropic, is hence beneficial. Usually, one limits oneself to spatially homogenous models for simplicity's sake. Modern cosmology relies heavily on the spatially homogenous and anisotropic models known as Bianchi models because they provide a middle ground between FRW models and fully inhomogeneous and anisotropic Universes. Thus, a quick review of Bianchi space-times is given in the part that follows.

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BIANCHI SPACE – TIMES

When discussing cosmology and cosmological models in the early phases of universe evolution, Bianchi type space-times are essential. Specifically, when discussing cosmic models, space-times that accept a three-parameter group of automorphisms are crucial. Especially helpful is the scenario where the group is simply transitive over the three-dimensional constant-time subspace. For groups of this sort, Bianchi (1989) has demonstrated that there are only nine unique sets of structure constants, making it simple to categorise homogeneous space-times using the algebra. Because of this, Bianchi type space-times accept just a reasonable amount of degrees of freedom and a three parameter group of movements.

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The significance of Bianchi type cosmological models lies in their homogeneity and anisotropy, which allow for the study of the universe's isotropization process across time. Furthermore, anisotropic universes are more general than isotropic models from a theoretical standpoint. In building models of spatially homogenous and anisotropic cosmologies, Bianchi space times were helpful due to the ease of solution and simplicity of the field equations. Krammer et al. (1980) provide a comprehensive list of all precise solutions of Einstein's equations for Bianchi type I–IX with ideal fluid.

In order to explain the early phases of the universe's history, spatially homogenous Bianchi models in the presence of perfect fluid, with or without radiation, are highly significant. The relevant chapters contain several studies on Bianchi type models in modified theories of gravity.

COSMIC STRINGS AND STRING COSMOLOGY

The development of a more accurate knowledge of the structure creation in the Universe is one of the main concerns in contemporary cosmology. Furthermore, we still don't know the precise physical conditions that existed early in our universe's history. The theories that currently explain how the universe's structure is formed may be divided into two groups: those that rely on the production of topological defects during a symmetry-breaking phase transition in the early Universe, or those that amplify quantum fluctuations in a scalar field during inflation.

A topological defect is a discontinuity in the vacuum that can take the form of textures, monopoles, cosmic strings, domain walls, or other structures depending on the topology of the vacuum (Kibble, 1976; Mermin 1979). According to Pando et al. (1998), the structure creation of the universe is caused by topological imperfections. Among the aforementioned topological flaws, cosmic threads have significant implications for astrophysics; specifically, they can effectively explain the creation of galaxies and the double quasar problem (Vilenkin and Shellard 1994). Astronomical observations may be able to identify these objects because, as Vilenkin (1985) shown, the strings can function as a gravitational lens. Another theory that is seen as a good fit for unifying all forces is string theory. The large-scale network of strings in the early Universe does not conflict with the structures of the Universe today. They are also thought to be one of the origins of the density perturbations needed for the universe's large-scale structure to develop. Letelier (1979), Vilenkin (1985), and Stachel (1980). Grand unified theories, according to Kibble (1976) and Zeldovich (1974), can account for the existence of strings in the early Universe. In 2001, Shiwarz provided a succinct timeline of some of the most significant advancements in string theory.

The study of strings in general relativity was first introduced by Letelier (1983) and Stachel (1980). Letelier (1983) asserts that huge strings are nothing more than geometric strings devoid of mass that have particles affixed throughout their length. Therefore, the total energy momentum tensor for a cloud of heavy strings may be expressed as follows: [See Letelier (1979, 1983); Stachel (1980) for a comprehensive derivation of the energy-momentum tensor for a cloud of strings].

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \dots\dots\dots\dots\dots (7)$$

Where

ρ *is the rest energy density for a cloud of string with particles attached to them.*

Therefore, we can write

$$\rho = \rho_p + \lambda \dots\dots\dots\dots (8)$$

In equation 8, $\rho_p = particle\ energy\ density\ and\ lemda\ the$

tension density of the string. The four − velocity i u for the cloud of

particles and the four − veltocity i x , the direction of string,

satisfy (Banerjee et al. 1990)

$$u_i u' = 1 = -x_i x^j \ \ and\ u_i x^i = 1 \dots\dots\dots\dots\dots\dots\dots (9)$$

From the literature, we explored:

(1) The state equation for a geometric cloud

(Nambu) strings (Letelier 1979) as $\rho = \lambda \left(\rho_p = 0\right)$

(2) The cloud of Takabayasi's (1976) strings' equation of state is

$$\rho = (1 + w)\lambda \left(\rho_p = \lambda w\right) where\ w\ (> 0) = constant.$$

(3) An universal barotropic formula is

$$\rho = \rho(\lambda) \left(\rho_p = \rho - \lambda\right) \dots\dots\dots\dots\dots\dots (10)$$

$$equation\ 10\ represent\ the\ equation\ of\ state\ which$$

$$are\ restricted\ by\ the\ energy\ condition\ ,$$

$$mentioned\ by\ Hawking\ and\ Ellis\ (1973).$$

$$The\ energy\ conditions\ of\ weak\ and\ strong\ presents$$

$$\rho \geq \lambda, with\ the\ condition\ \lambda < 0\ or\ \rho > 0\ with\ \lambda < 0.$$

The state of domestic energy suggests

$$\rho \geq 0\ and\ \rho^2 \geq \lambda^2.$$

$$The\ signature\ of\ \lambda\ is\ not\ restricted\ by\ these$$

$$energy\ requirements.$$

Cosmic strings and string cosmological models have attracted a lot of attention in recent years. General relativity scholars Vilenkin (1981), Gott (1985), Letelier (1983), and Stachel (1980) have all written extensively about the gravitational effects of cosmic strings. While Tikekar et al. (1994) proposed a class of cylindrically symmetric models in string cosmology, Krori et al. (1990), Banerjee et al. (1990), and Tikekar and Patel (1990) produced relativistic string models in the framework of Bianchi space-time. Chakraborty (1991) and Tikekar and Patel (1992) examine string cosmological models with magnetic field. The issues of cosmic strings assuming Bianchi type cosmologies with a self-interacting scalar field were investigated by Bhattacharjee and Baruah (2001).

Bianchi type string cosmological models, which are optimal solutions of Einstein's field equations with a cloud of one-dimensional strings serving as the curvature source, have also been studied by Yavuz and Tarhan (1996) and Baysal (2001). The impact of cosmic strings on the background radiation of cosmic microwaves anisotropies has also been covered. In general relativity, Raj Bali et al. (2005) have recently studied the Bianchi type-I string dust magnetised cosmological model.

The relevant portions of the thesis will cover the research of cosmological models in alternative theories of gravity.

Over the past few decades, we have made significant progress in our knowledge of string theory. The recent efforts covered here to bring string theory closer to cosmology are but a taste of things to come. A genuine conversation with cosmology as string theory advances will guarantee considerably greater curiosity in this area.

DARK ENERGY AND DARK ENERGY MODELS

The detection of the universe's fast expansion, which is assumed to be produced by an unidentified dark energy, is one of the most important findings in modern cosmology. What constitutes dark energy and how it forms remain unknown. Thermodynamic studies of dark energy suggest that its constituents might be massless particles, such fermions or bosons, whose collective behaviour is like that of a radiation fluid under negative pressure. The cosmology community also largely believes that dark energy, an exotic physical phenomena that has not yet been defined, is some kind of repulsive force that fuels the universe and acts as antigravity.

Based on results from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment, the content of the universe is composed of 73% dark energy, 23% non-baryonic dark matter, and 4% radiation and ordinary baryonic (normal) matter.

Although the cosmological constant has been proposed as the most possible straightforward dark energy candidate, it will need to be fine-tuned to match the current value of dark energy. Chaplygin gas and generalised Chaplygin gas have been suggested as possible dark energy candidates because of their negative pressure [Srivatswa (2005); Bertolami et al. (2004); Bento et al. (2002)].

There may be dark energy possibilities when two fluids contact and do not interact, according to several authors (Setare (2007); Setare et al. (2009); Pradhan et al. (2011)). Others have considered changing the gravitational action by adding a function f(R).

Here, R is the Ricci scalar curvature. This action by adding a function

$f(R)\ to\ Einstein -$

$Hilbert\ Lagrangian\ where\ f(R)\ provide\ a\ gravitational\ alternative\ for\ dark\ en\ ergy$

$causing\ late\ time\ acceleration\ of\ the\ universe.$

A overview of the use of modified gravity to explain dark energy is provided by Copeland et al. (2006) and Nojiri & Odintsov (2007). Cosmological acceleration remains an issue for current cosmology despite these efforts.

Sami et al. (2005), Wang et al. (2007), Zimdahl and Pavon (2007), Jamil and Rashid (2008), Setare and Vagenos (2011), Li et al. (2011), and others have extensively studied cosmological models based on dark energy; these models produce stable solutions of FRW equations at late periods of evolving universe.

The dynamics of interacting phantom and quintessence dark energies have been studied by Farooq et al. (2011). Among the writers who have looked at Bianchi type dark energy models in general relativity are Pradhan et al. (2011), Yadav (2011), and Adhav et al. (2011).

ALTERNATIVE THEORIES OF GRAVITATION

This section provides a succinct and clear overview of the updated theories of gravitation, which have been developed to include several physical elements that Einstein's original theory of gravity did not include. Since the publication of Einstein's theory of gravity, general relativity has been heavily criticised due to the theory's absence of several desired characteristics. Einstein himself, for instance, noted that general relativity does not adequately explain the inertial properties of matter; that is, general relativity does not support Mach's principle, which holds that the geometry of space time is the exclusive determinant of the inertial properties of matter.

Hence, by adding Mach's principle and other desired qualities that the original theory lacks, there have been some intriguing attempts in recent years to generalise the universal theory of relativity. Many iterations of scalar-tensor theories of gravity have been proposed and explored with this goal in mind. Scientists are becoming increasingly interested in the scalar-tensor theories of gravity, which were first put out by Brans and Dicke in 1961, Nordtvedt in 1970, Barker in 1978, and Saez and Ballester in 1986.

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BRANS-DICKE SCALAR-TENSOR THEORY

Mach's principle states that the distribution of mass-energy over the universe determines the geometry of space-time and, consequently, the inertial characteristics of each and every infinitesimal test particle (Wheeler 1964). This idea, while one of the cornerstones of Einstein's philosophy, is only partially addressed by general relativity (Dicke 1964). Examples of "non-machian" solutions include the closed but empty Taub (1951) model, the Godel (1949) Universe, which has closed time-like curves and inertial qualities, and Heckmann and Schucking (1962)'s Minkowski space. Boundary constraints may be used to exclude these unacceptable solutions, according to Wheeler (1964).

In 1961, Brans and Dicke used the example of a static enormous shell to refute this theory. General relativity states that even as the mass of the shell increases, the inertial characteristics of the test particles inside the shell remain constant.

Brans and Dicke (1961) suggested a theory that includes a long range scalar field interacting equally with all kinds of matter (excluding electromagnetism) in an attempt to expand general relativity in a way that incorporates Mach's principle. Following Dirac (1938) and Sciama (1959), they observed that the mass M and radius R of the observable Universe are linked to the Newtonian gravitational constant G by

$$G \sim \frac{Rc^2}{M} \ldots\ldots\ldots\ldots(10)$$

(The amounts are approximations). This implies that the matter distribution determines G, which is a (scalar) function. Their hypothesis is formal equivalent to the one Jordan (1955) had previously examined.

Brans and Dicke (1961) developed their variational principle, which is different from general relativity's, to generalise the equations of general relativity. Specifically,

$$\delta \int [R + (16G)L]\sqrt{-g}\,.\,d^4x = 0 \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

In the above equation G is substituted by ϕ^{-1} *It is now included in the action integral. In addition, there are other words that account for phi's scalar character.*

$$\delta \int \left[\phi R + 16\, GL - \frac{\omega \phi_{,i} \phi^{,i}}{\phi}\right] \sqrt{-g}\,.\, d^4x = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

In equation (12), $\phi = scaler\ field$

$R = general\ scaler\ curvature$

L= function of matter variables and metric tensor components

$\omega = dimensional\ constant.$

In this case, the velocity of light is c = 1, the metric has signature +2, a semicolon indicates covariant differentiation, and a comma indicates partial differentiation. The field equations that result from changing g_{ij} *and* ϕ*. this convert into the following:*

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi\phi^{-1} T_{ij} \ -\omega\phi^{-2} \left(\phi_{,i}\, \phi_{,l} \ - {}^{1}/_{2}\, g_{ij} \phi_{,j} \phi^{,j}\right) \ -\phi^{-1}\left(\phi_{i,j} - g_{ij} \ \Box\phi\right) \ldots (13)$$

Moreover

$$\Box\phi = 8\pi\ (3 + 2\omega)^{-1}\ T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

In the above equation $\Box\phi \ = \ g_{ij} \Phi_{i,j}$ and $T = g^{ij}\ T_{ij}$.

Rather than the equations of motion, the gravitational field equations that establish the metric field are the primary source of distinction between the Einstein theory and the Brans-Dicke theory. The local matter-energy conservation law is satisfied by the matter T_{ij}'s energy momentum tensor.

$$T^{ij}{}_{,j} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

This is a result of field equations 13 and 14 and also represents equations of motion. In the limit $\omega \to 0,\ \phi = constant = G^{-1}$ Brans-Dicke theory crosses over into general relativity.

The "strong principle of equivalency," which forms the foundation of Einstein's theory, is violated by the aforementioned alteration. Therefore, the trajectories of test particles in a gravitational field remain independent of their masses, meaning that this does not contradict the "weak principle of equivalence." As a result, the Brans-Dicke theory may now be defined as a theory in which an object's gravitational force results from a combination of interactions with tensors and scalar fields.

Brans and Dicke (1961) went on to discuss the field equations (13) and (14) in greater detail. They examined the weak field equations, the three standard tests, Jordan's work (1955) in comparison, boundary conditions for, cosmological research, and the general connection to Mach's principle.

As of right now, there is no evidence that would refute the Brans-Dicke scalar-tensor hypothesis. This hypothesis differs from Einstein's theory in terms of the perihelion advance of planetary orbits and the gravitational deflection of light rays, even if it does not anticipate an abnormal gravitational red shift (Dicke 1964, Brans-Dicke 1961). However, given the comparatively large disparities between the measurements of the sun's oblations (Dicke and Goldenberg 1967) and the deflection of star light near its limb during a total eclipse (Dicke 1967), it is concluded that the Brans-Dicke theory is consistent with observations, given that $w \geq 6$. To fully understand the ramifications of adding a long-range scalar-interaction, this theory has lately been applied to additional intriguing problems in astrophysics.

It is possible to find significant discrepancies between the two theories that may be utilised to choose between them by comparing this theory's predictions with Einstein's theory. For instance, Morganstern and Chiu (1967) demonstrated that if a neutron star is observed to exhibit symmetric radial pulsation, then the existence of the scalar field may be ruled out, despite Solmona (1967) demonstrating that certain general characteristics of a cold neutron star remain unchanged by the presence of the strength of the scalar field.

One can deduce that Brans-Dicke parameter $\omega \geq 500$ as a result of the recent lunar ranging experiments (Williams et al. 1976; Shapiro et al. 1976; Wills 1980).Nevertheless, further studies (Bertolami et al. 2004 and De Felice et al. 2006) demonstrate that, in order to be consistent with solar system boundaries, this should be confined to > 40,000. It has been noted that limiting ω to positive values is not justified theoretically (Smalley and Eby 1976). Given this, one may reasonably conclude that the true theory is the Brans-Dicke theory with some big values of ω. The Brans-Dicke cosmological theories are thoroughly discussed in Singh and Roy's (1983) study.

SAEZ-BALLESTER SCALAR -TENSOR THEORY

Several scalar-tensor theories of gravity that are scientifically plausible have been put forth and thoroughly investigated by several researchers thus far. The Brans-Dicke gravitational theory from 1961 is one of the most significant of them. They proposed not just the well-known general relativistic metric tensor but also a scalar-tensor theory of gravity incorporating a scalar function. The scalar field in this theory is limited to its effects on the gravitational field equations and has a dimension equal to the inverse of the gravitational constant G. A scalar-tensor theory in which the metric is associated with a dimension-less scalar field was subsequently developed by Saez and Ballester (1986). Weak fields are satisfactorily described by this coupling. Despite the scalar field's dimension-lessness, an antigravity regime is seen.

This theory offers a constructive solution to the "missing matter" issue in FRW cosmologies that are non-flat. In 1986, Saez and Ballester postulated the Lagrangian

$$\Rightarrow L = R - w\phi^n(\phi_{,\alpha}\,\phi^{,\alpha}) \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

In equation 16, R= curvature

$$\phi = dimensionless\ scaler\ field$$

$$w, n = arbitrary\ dimensionless\ constant.$$

$$\phi^{,i} = g^{ij}\,\phi_{,j}$$

Regarding a scalar field with dimension, ϕG^{-1}, the Lagrangian given by equation (16) has

$$various\ dimension. However, it\ is\ a\ suitable$$

$$Lagrangian\ in\ the\ situation\ of\ a\ dimensionless$$

$$scalar\ field. From\ the\ Lagrangian\ one\ may$$

$$create\ the\ action$$

$$\Rightarrow I = \int_{\Sigma} (L + GL_m)(-g)^{\frac{1}{2}}\,dx\,dy\,dz\,dt \ldots\ldots (17)$$

In equation 17 $L_m = matter\ langrangiang = |g_{ij}|, \Sigma = an\ arbitrary\ region\ of\ the\ integration,$

$$and\ G = -8\pi$$

The variational principle takes into account arbitrary independent fluctuations of the metric and the scalar field that vanishes at the boundary of Σ.

$$\delta \mathrm{I} = 0 \ldots\ldots\ldots\ldots\ldots (18)$$

Equation 18 leads to the Saez - Ballester (1986) field equations for combined scalar and tensor fields given by

$$G_{ij} - \omega\phi^n \left(\phi_{,i}\phi_{,j} - \tfrac{1}{2} g_{ij}\phi_{,k}\phi^{,k}\right) = -8\Pi\, G_{,ij}\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Furthermore, the scalar field $\phi\ fullfill\ the\ following\ equation$

Furthermore, the scalar field ϕ $fullfill\ the\ following\ equation$

$$2\phi^n\phi^i;j + n\phi^{n-1}\phi,k\phi'^k = 0 \dots\dots\dots\dots\dots..(20)$$

Additionally,

$$T^{ij};j = 0 \dots\dots\dots\dots\dots\dots\dots..(21)$$

Equation 21 is the result of the field equation (19) and (20)

In this hypothesis, Saez (1987) talked about the inflationary Universe and the primordial singularity. He has demonstrated the existence of an antigravity regime that may operate before or at the start of the inflationary epoch. In the scenario when k = 0, he has also produced a non-singular FRW model. Moreover, this theory points to a potential solution for the non-flat FRW cosmologies missing matter issue. The pertinent portions of this thesis will provide recent studies on this hypothesis.

f(R) and f(R,T) Theories of Gravity:

Data from recent observations point to the universe's acceleration. The late-time acceleration of the universe and the presence of dark energy and dark matter are the explanations given for this acceleration (Nojiri and Odinstov 2007b). Different strategies have been proposed to account for cosmic acceleration. In order to explain the idea of dark energy, an extra energy component is added to Einstein's theory of gravity in the first method. Several other models have been offered in this method, but no viable candidate has been discovered up to this point.

The alternative method involves altering the Einstein Lagrangian by substituting a function of R called f(R) gravity for the scalar curvature (Nojiri and Odinstov 2007a). Other modified theories of gravitation include the scalar-tensor theories of Saez-Ballester (1986), Brans-Dickie (1961), and f(R,T) gravity (Harko et al. 2011). The hypotheses of Brans-Dicke and Saez Ballester have previously been introduced. A synopsis of the f(R) and f(R,T) theories of gravity is given here.

f(R) gravity:

The foundations of f(R) gravity are as follows: The metric tensor is important to the theory of general relativity. The dependence of the Levi-Civita connection on the metric tensor is one of the central concepts of general relativity. However, this theory loses its Levi-Civita connection and its dependence on the metric tensor if it allows for torsion. This is the underlying idea of a number of f(R) gravity techniques. In the case when the connection is Levi-Civita, we obtain metric f(R) gravity. By doing this, all we have to do is change the action with respect to the metric tensor.

The action of $f(R) gravity\ is\ given\ by$

$$\Rightarrow S = \int (-g)^{\frac{1}{2}} \,(f(R) + \, Lm)d^4x \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

where Lm is the matter Lagrangian and f(R) is a generic function of the Ricci scalar.

The action's resultant field equations are

$$F\,(R)R_{\mu v} - \frac{1}{2}\, f(R)R_{\mu v} - \nabla_{\mu}\nabla_{v}\, F(R) + \; g_{\mu v}\mathrm{F(R)} = \, T_{\mu v} \ldots\ldots\ldots\ldots\ldots (23)$$

In equation 23 $F\,(R) = \frac{d}{dr}\,\big(f(R)\big), \square = \nabla^{\mu}\,\nabla_{v}$

Here $\nabla^{\mu} = covariant\ derivative$

$and\ T_{\mu v} =$ standard minimally coupled stress energy

tensor from the Lagrangian $\mathrm{L_m}$

After joining the field equations, it is evident that

$$F\,(R)R - 2f(R) + 3\,\square \mathrm{F}(R) = T \ldots\ldots\ldots\ldots\ldots\ldots. (24)$$

where T is the stress energy tensor trace. A relationship between f(R) and F(R) is thus provided.

f(R,T) Theory of Gravity:

In a recent study, Harko et al. (2011) created a generalised f(R,T) gravity in which the trace T of the stress energy tensor Tij and the Ricci Scalar R provide an arbitrary function for the gravitational Lagrangian. By adopting the action, the field equations of this theory are obtained from the Hilbert-Einstein type variational principle.

$$\Rightarrow S = \frac{1}{16\pi}\int [f(R.T) + Lm]$$

$$(-g)^{\frac{1}{2}} d^4x. \dots\dots\dots\dots\dots\dots\dots\dots. (25)$$

In equation (25) $Lm = matter\ lagrangian\ density$

The definition of the matter's stress energy tensor is

$$\Rightarrow T_{ij} = -\frac{2}{(g)^{\frac{1}{2}}} * \frac{\delta\,(g)^{1/2}Lm}{\delta^{ij}} \dots\dots\dots\dots\dots\dots(26)$$

Additionaly trace is defined as $\Rightarrow T = g^{ij}T_{ij}$.

Assuming that the metric tensor components g_{ij} are the sole factors that determine Lm of matter, we may derive the field equations of f(R,T) gravity as

$$f_R(R,T)\,R_{ij} - \frac{1}{2}\,f(R.T)g_{ij} + \left(g_{ij}\Box - \nabla_i\,\nabla_j\right)f_R\,(R,T) = 8\pi\,T_{ij} - f_T(R,T)\,T_{ij} - f_T(R,T)\theta_{ij}\dots\dots(27)$$

Where $\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2\,g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\,\partial g^{Lm}}$, $\Box = \nabla^k\,\nabla_k \dots\dots\dots\dots\dots\dots(28)$

In equation (28)

$$f_R = \frac{\partial f(R,T)}{\partial R}$$

$$f_T = \frac{\partial f(R,T)}{\partial T}$$

∇^i = covariant derivative,

It should be noticed that equation (27) produces the field equations of f(R) gravity when

$$f(R,T) = f(R).$$

The ideal fluid issue, which is represented by an energy density ρ, pressure p, and four velocities , is intricate due to the lack of a singular Lagrangian description. But in this case, we'll suppose that the matter's stress energy tensor is provided by

$$T_{ij} = (\rho + P)u_i u_j - p\,g_{ij}\dots\dots\dots\dots\dots\dots\dots(29)$$

Furthermore, the matter lagrangian can be considered as $L_m = -p$.

Then we can have

$$u^i \nabla_j u_i = 0 \, , u^i \, u_i = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(30)$$

Now, using equation (26) we derive the expression for the fluctuation of stress energy of ideal fluid.

$$\theta_{ij} = -2\,T_{ij} - pg_{ij} \ldots\ldots\ldots\ldots(31)$$

In general, the physical properties of the matter field also influence the field equations through the tensor θij. Therefore, in the case of f(R,T) gravity, we have numerous theoretical models corresponding to each option of f(R,T), depending on the nature of the matter source.

Lets consider that..

$$f(R,T) = R + 2\,f(T) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(32)$$

We obtain the gravitational field equations of f(R,T) gravity from equation (27) as a first choice, where f(T) is an arbitrary function of the trace of stress energy tensor of matter.

$$R_{ij} - \frac{1}{2}\, g_{ij}\, R = 8\,\pi\, T_{ij} + 2f'(T)T_{ij} - 2f'(T)\theta_{ij} + f(T)g_{ij} \ldots\ldots\ldots..(33)$$

where distinction with regard to the argument is shown by the prime. In light of equation (27), the field equations of f (R, T) gravity become if the matter source is a perfect fluid.

$$R_{ij} - \frac{1}{2}\, g_{ij}\, R = 8\,\pi\, T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij} \qquad \ldots\ldots\ldots\ldots..(34)\Rightarrow$$

The pertinent parts of this thesis will provide a presentation of recent studies on this hypothesis.

CONCLUSION

Bianchi spacetimes are crucial for modeling spatially homogenous and anisotropic cosmologies, defying Misner's hypothesis. Dirac introduced the concept of a changeable gravitational constant G into the relativity framework, and Lau introduced changes that coupled the variation of G with that of Λ. Cosmology is concerned with the space-time link between the universe's origin, structure, and overall dimensions. Einstein's theory of general relativity was first proposed using general relativity and alternative theories of Bianchi universe gravity. Recent theoretical and experimental research has shown that the dark expansion stage in the universe is speeding, driven by an unidentified energy. The universe is made up of 68.5% dark energy, 26.5% dark matter, and 5% baryonic matter. Einstein's general theory of relativity describes the relativity of all kinds of motion and is based on three basic principles: Principle of Covariance, Principle of Equivalence, and Mach's Principle. It is considered the correct theory of gravitation, as the basic force governing the dynamics of the universe is gravity.

The cosmological principle, which asserts that the world is homogenous and isotropic on a sufficiently vast scale, is the foundation of cosmology research. Bianchi type space-times are essential for discussing cosmic models, as they accept a three-parameter group of automorphisms. These models are particularly useful in the early phases of universe evolution, where they allow for the construction of galaxies and the emergence of sentient life.

The significance of Bianchi type cosmological models lies in their homogeneity and anisotropy, allowing for the study of the universe's isotropization process across time. They are also more general than isotropic models from a theoretical standpoint. Bianchi space-times are helpful in building models of spatially homogenous and anisotropic cosmologies due to their ease of solution and simplicity of field equations.

The development of a more accurate knowledge of the structure creation in the Universe is a main concern in contemporary cosmology. Current theories explain the universe's structure formation through topological defects during a symmetry-breaking phase transition or quantum fluctuations in a scalar field during inflation. Topological imperfections, such as cosmic threads, can explain the creation of galaxies and the double quasar problem. String theory is another theory that unifies all forces and is thought to be one of the origins of the density perturbations needed for the universe's large-scale structure to develop.

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