Nonlinear Fractio Al Partial Differential Equations: A new Analytical Technique for Solving

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Abstract - In many branches of science and engineering, nonlinear fractional partial differential equations (FPDEs) have shown to be effective instruments for simulating complicated processes. The intrinsic complexity of these equations and the nonlocal nature of fractional derivatives pose challenges to traditional techniques of solving them. In order to overcome these difficulties, this paper presents a brand-new analytical method that offers a practical and quick way to solve nonlinear FPDEs. The suggested approach combines [briefly outline the fundamental techniques, such as the Adomian decomposition and Laplace transform] to get precise results with little computing overhead. The technique's effectiveness and adaptability are shown by a number of examples, which also highlight its potential for broad use in fields including biological systems, fluid dynamics, and financial modelling.

Keywords: Nonlinear fractional partial differential equations, Analytical techniques, Fractional calculus

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INTRODUCTION

Fractional partial differential equations (FPDEs) have gained significant attention in recent years due to their ability to model various complex systems with memory and hereditary properties. Unlike integer-order differential equations, FPDEs incorporate fractional derivatives, which introduce nonlocality and can better describe phenomena in fields such as viscoelasticity, anomalous diffusion, and quantum mechanics. However, the complexity of these equations, particularly in their nonlinear forms, poses substantial challenges to traditional solution methods.

Several numerical and analytical approaches have been proposed to solve FPDEs, yet they often suffer from limitations such as high computational cost, instability, or restricted applicability. This paper presents a new analytical technique that combines [describe key methods or principles, e.g., the Laplace transform, series expansion, etc.] to effectively solve nonlinear FPDEs. The objective is to offer a reliable and efficient tool that can be applied across a wide range of disciplines, with a focus on demonstrating the practical utility of the method through a series of case studies.

NEW ANALYTICAL TECHNIQUE FOR SOLVING NON LINEAR FRACTIO AL PARTIAL DIFFERENTIAL EQUATIONS

In this paper, A NAT is shwo that can solve the following nonlinear first-order FPDEs:

$$\begin{cases} \mathcal{D}_t^q u(\bar{x},t) = f(\bar{x},t) + L(u(\bar{x},t)) + N(u(\bar{x},t)), \; m-1 < q < m \in \mathbb{N}, \\ \dfrac{\partial^r u(\bar{x},0)}{\partial t^r} = f_r(\bar{x}), \; r = 0,1,2,\ldots,m-1, \end{cases}$$

1.2.1

Where $L(u(\bar{x},t))$ and $N(u(\bar{x},t))$ perform their duties as linear and nonlinear operator on $u(\bar{x},t)$ withall partialderivatives, including fractional ones, and any others that could be involved, and $f(\bar{x},t), f_r(\bar{x})$ recognized as analytical operations, $\mathcal{D}_t^q$ where q is the magnitude of the caputo linear partial derivative with time and $\bar{x} = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$.

Through analytical piecewise solutions, the NAT can handle noonlinear first order partial differential equations (FPDEs). Before the ANT may be implemented, these outcomes must be demonstrated (Kormaz, E. 2013)

Theorem 1.2.1 Let $u(\bar{x},t)=\sum_{k=0}^{\infty}u_k(\bar{x},t),$ we define $u_\lambda(\bar{x},t)=\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t)$ where the parameter λ is an integer between zero and one. Afterwards, the unit operator $L(u_\lambda(\bar{x},t))$ fulfilss the condition listed below:

$$L(u_\lambda(\bar{x},t))=L(\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t))=\sum_{k=0}^{\infty}\lambda^k L(u_k(\bar{x},t)).$$

………………1.2.2

Theorem 1.2.2: Let $u(\bar{x},t)=\sum_{k=0}^{\infty}u_k(\bar{x},t),$ we define $u_\lambda(\bar{x},t)=\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t)$ where the parameter is an integer zero and one. Afterwards, the operator on nonlinear $N(u_\lambda(\bar{x},t))$ fulfills the condition listed below

$$N(u_\lambda(\bar{x},t))=N(\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t))=\sum_{n=0}^{\infty}\Big[\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}[N(\sum_{k=0}^{n}\lambda^k u_k(\bar{x},t))]_{\lambda=0}\Big]\lambda^n.$$

……1.2.3

Proof. Applying the Maclaurin expansion allows one to $N(\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t))$ in reference to λ we have

$$\begin{aligned}N(u_\lambda(\bar{x},t))&=[N(\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t))]_{\lambda=0}+[\frac{\partial}{\partial\lambda}[N(\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t))]_{\lambda=0}]\lambda\\&+[\frac{1}{2!}\frac{\partial^2}{\partial\lambda^2}[N(\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t))]_{\lambda=0}]\lambda^2+\cdots\\&=\sum_{n=0}^{\infty}[\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}[N(\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t))]_{\lambda=0}]\lambda^n\\&=\sum_{n=0}^{\infty}\Big[\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}[N(\sum_{k=0}^{n}\lambda^k u_k(\bar{x},t)+\sum_{k=n+1}^{\infty}\lambda^k u_k(\bar{x},t))]_{\lambda=0}\Big]\lambda^n\\&=\sum_{n=0}^{\infty}\Big[\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}[N(\sum_{k=0}^{n}\lambda^k u_k(\bar{x},t)+\lambda^{n+1}u_{n+1}(\bar{x},t)+\lambda^{n+2}u_{n+2}(\bar{x},t)+\cdots)]_{\lambda=0}\Big]\lambda^n\\&=\sum_{n=0}^{\infty}\Big[\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}[N(\sum_{k=0}^{n}\lambda^k u_k(\bar{x},t))]_{\lambda=0}\Big]\lambda^n.\end{aligned}$$

Since $\frac{\partial\lambda^n}{\partial^n}\Big[N(\lambda^{n+1}u_{n+1}(\bar{x},t)+\lambda^{n+2}u_{n+2}(\bar{x},t)+\cdots)\Big]_{\lambda=0}=0.$

Defination 2.2.1: The function $E_n(u_0(\bar{x},t),u_1(\bar{x},t),\ldots,u_n(\bar{x},t))$ is constrained to

$$E_n(u_0(\bar{x},t),u_1(\bar{x},t),\ldots,u_n(\bar{x},t))=\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\Big[N(\sum_{k=0}^{n}\lambda^k u_k(\bar{x},t))\Big]_{\lambda=0}.$$

……….1.2.4

Remark 2.1. Let $E_n=E_n(u_0(\bar{x},t),u_1(\bar{x},t),\ldots,u_n(\bar{x},t))$ be defined in 1.2.1 next, in section 1.2.2 we apply the theory to get the nonlineast operator $N(u_\lambda(\bar{x},t))$ is synonymous with En when stated as: (Belic, M. 2016)

$$N(u_\lambda(\bar{x},t))=\sum_{n=0}^{\infty}\lambda^n E_n.$$

…………………..1.2.5

The analytical solution to the nonlinear partial differential equation (NAT) is given by the following theorem and is used to provide Equation (1.2.1) for the nonlinear fractional parametric differential.

Theorem 2.2.3: Let $m-1<q<m\in\mathbb{N}$ and $f(\bar{x},t),f_k(\bar{x})$ the analytical functions that are used consequently, a solution can be found for the equation (1.2.1) by

$$u(\bar{x},t)=f_t^{(-q)}(\bar{x},t)+\sum_{r=0}^{m-1}\frac{t^r}{r!}f_r(\bar{x})+\sum_{k=1}^{\infty}[L_t^{(-q)}(u_{(k-1)})+E_{(k-1)t}^{(-q)}(u_0,u_1,\ldots,u_{k-1})],$$

……1.2.6

Where $f_t^{(-q)}(\bar{x},t),L_t^{(-q)}(u_{(k-1)})$ and $E_{(k-1)t}^{(-q)}(u_0,u_1,\ldots,u_{k-1})$ serve as a symbol for the 1-order temporal fractional partial integral in $f(\bar{x},t)$, $L(u_{(k-1)})$ and $E_{(k-1)}(u_0,u_1,\ldots,u_{k-1})$ respectively proof. The analytic expansion that follow is predicated on the function $u(\bar{x},t)$ being a solution to equation (1.2.1)

$$u(\bar{x},t)=\sum_{k=0}^{\infty}u_k(\bar{x},t).$$

…………1.2.7

With these considerations in mind, we can find a solution to the nonlineat fractional partial differential equation (1.2.1)

$$\mathcal{D}_t^q u_\lambda(\bar{x},t)=\lambda[f(\bar{x},t)+L(u_\lambda(\bar{x},t))+N(u_\lambda(\bar{x},t))],\ \lambda\in[0,1],$$

1.2.8

Under the starting condition provided by (Hammad, D.2012)

$$\frac{\partial^r u_\lambda(\bar{x},0)}{\partial t^r}=g_r(\bar{x}),\quad r=0,1,2,\ldots,m-1.$$

…….1.2.9

Since this is a nonlinear fractional partial differential equation, we may use the answer to solve (2.2.8)

$$u_\lambda(\bar{x},t)=\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t).$$

………………..1.2.10

We derive it by applying theorem 1.2.1 to both sides of the starting value issue (2.2.8) and taking the Riemann-Liouville time fractioanl partial integral of order q.

$$u_\lambda(\bar{x},t)=\sum_{r=0}^{m-1}\frac{t^r}{r!}\frac{\partial^r u_\lambda(\bar{x},0)}{\partial t^r}+\lambda\mathcal{I}_t^q[f(\bar{x},t)+L(u_\lambda)+N(u_\lambda)].$$
…………1.2.11

The starting condition is supplied by (2.2.1) which allows us to rewrite equation (2.2.11) as:

$$u_\lambda(\bar{x},t)=\sum_{r=0}^{m-1}\frac{t^r}{r!}g_r(\bar{x})+\lambda[f_t^{(-q)}(\bar{x},t)+\mathcal{I}_t^q[L(u_\lambda(\bar{x},t))]+\mathcal{I}_t^q[N(u_\lambda(\bar{x},t))]].$$
……..1.2.12

Additionally we get that we want by changing (1.2.10) into (1.2.12)

$$\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t)=\sum_{r=0}^{m-1}\frac{t^r}{r!}g_r(\bar{x})+\lambda[f_t^{(-q)}(\bar{x},t)+\mathcal{I}_t^q[L(\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t))]+\mathcal{I}_t^q[N(\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t))]].$$
……1.2.13

Equation (12.13) is transformed by using Theorems 1.2.1 and 1.2.2

$$\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t)=\sum_{r=0}^{m-1}\frac{t^r}{r!}g_r(\bar{x})+\lambda f_t^{(-q)}+\mathcal{I}_t^q\lambda\sum_{k=0}^{\infty}\lambda^k[L(u_k(\bar{x},t))]+\mathcal{I}_t^q\lambda\sum_{n=0}^{\infty}\left[\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}[N(\sum_{k=0}^{n}\lambda^k u_k(\bar{x},t))]_{\lambda=0}\right]\lambda^n.$$
…….1.2.14

Equation (1.2.14) is then solved using definition 1.2.1 and remark 1.1

$$\sum_{k=0}^{\infty}\lambda^k u_k(\bar{x},t)=\sum_{r=0}^{m-1}\frac{t^r}{r!}g_r(\bar{x})+\lambda f_t^{(-q)}(\bar{x},t)+\mathcal{I}_t^q\lambda\sum_{k=0}^{\infty}[\lambda^k L(u_k(\bar{x},t))]+\mathcal{I}_t^q\lambda\sum_{n=0}^{\infty}E_n\lambda^n.$$
……1.2.15

The commponents in equation (1.2.15) may be expressed as identical powers of λ which allows us Derive (Erdelyi, 1965)

$$\begin{cases}u_0(\bar{x},t)=\sum_{r=0}^{m-1}\frac{t^r}{r!}g_r(\bar{x}),\ u_1(\bar{x},t)=f_t^{(-q)}(\bar{x},t)+L_t^{(-q)}u_0(\bar{x},t)+E_{0t}^{(-q)},\\ u_k(\bar{x},t)=L_t^{(-q)}u_{(k-1)}(\bar{x},t)+E_{(k-1)t}^{(-q)},\ k=2,3,\ldots.\end{cases}$$
……..1.2.16

We than get the solution of the second equation, which is (1.2.8) by plugging (1.2.16) into the first equation (1.2.10) We may now derive from equations (1.2.7) and (1.2.10)

$$u(\bar{x},t)=\lim_{\lambda\to 1}u_\lambda(\bar{x},t)=u_0(\bar{x},t)+u_1(\bar{x},t)+\sum_{k=2}^{\infty}u_k(\bar{x},t).$$
.1.2.17

By applying the starting conditions and looking at (1.2.17) we may observe that

$$\frac{\partial^k u(\bar{x},0)}{\partial t^k}=\lim_{\lambda\to 1}\frac{\partial^k u_\lambda(\bar{x},0)}{\partial t^k}.$$
…..1.2.18

Which implies that $g(\bar{x})=f(\bar{x})$. using equation (1.2.16) into equation (1.2.17) the proof is completed. The analytical solution to the nonlinear fractional partial differential equation (1.2.1) is provided by (1.2.6) and we prove its convergence and maximum absolute error in the following theorems (Singh, M.2011)

A convergene theorem is tarted in theorem 1.2.4. A Banach space is denoted by B. Afterwards, if there is an exist, the solution series of equation (1.2.16) will converge to $S\in B$. $\gamma,\ 0\le\gamma<1$ such that $\|u_n\|\le\gamma\|u_{(n-1)}\|\ for\ \forall n\in\mathbb{N}$.

Proof. The following partial sums are used to define the sequence Sn:

$$\begin{cases}S_0=u_0(\bar{x},t),\ S_1=u_0(\bar{x},t)+u_1(\bar{x},t),\\ S_2=u_0(\bar{x},t)+u_1(\bar{x},t)+u_2(\bar{x},t),\\ \vdots\\ S_n=u_0(\bar{x},t)+u_1(\bar{x},t)+u_2(\bar{x},t)+\cdots+u_n(\bar{x},t).\end{cases}$$
……1.2.19

{Sn} must be proven to be a Cauchy sequence in the Banach space B. Towards this end, we take into account

$$\|S_{n+1}-S_n\|=\|u_{n+1}(\bar{x},t)\|\le\gamma\|u_n(\bar{x},t)\|\le\gamma^2\|u_{n-1}(\bar{x},t)\|\le\cdots\gamma^{n+1}\|u_0(\bar{x},t)\|.$$
…….1.2.20

For every $n,n'\in\mathbb{N},\ n\ge n'$, equation (1.2.20) and the triangle inequality were applied in a sequential manner resulting in

$$\begin{aligned}\|S_n-S_{n'}\|&=\|S_{n'+1}-S_{n'}+S_{n'+2}-S_{n'+1}+\cdots+S_n-S_{n-1}\|\\&\le\|S_{n'+1}-S_{n'}\|+\|S_{n'+2}-S_{n'+1}\|+\cdots+\|S_n-S_{n-1}\|\\&\le\gamma^{n'+1}\|u_0(\bar{x},t)\|+\gamma^{n'+2}\|u_0(\bar{x},t)\|+\cdots+\gamma^n\|u_0(\bar{x},t)\|\\&=\gamma^{n'+1}(1+\gamma+\cdots+\gamma^{n-n'-1})\|u_0(\bar{x},t)\|\\&\le\gamma^{n'+1}\left(\frac{1-\gamma^{n-n'}}{1-\gamma}\right)\|u_0(\bar{x},t)\|.\end{aligned}$$
……………1.2.21

Since $0<\gamma<1$, so $1-\gamma^{n-n'}\le 1$. Then

$$\|S_n-S_{n'}\|\le\frac{\gamma^{n'+1}}{1-\gamma}\|u_0(\bar{x},t)\|.$$
…………1.2.22

Since $u_0(\bar{x},t)$ is bounded, then

$$\lim_{n,n'\to\infty} \|S_n - S_{n'}\| = 0.$$

……………………………..1.2.23

Hence in a Banach space B,{Sn} is a Cauchy sequence and the series solution with respect to equation (1.2.17) converged. The evidence is now complete. (He,J.H.1999)

Theorem 2.2.5: for nonlinear FPDEs (1.2.1) the greatest absolute truncation error for the solution series (1.2.7) is approximated to be

$$\sup_{(\bar{x},t)\in\Omega} \left|u(\bar{x},t) - \sum_{k=0}^{n'} u(\bar{x},t)\right| \le \frac{\gamma^{n'+1}}{1-\gamma} \sup_{(\bar{x},t)\in\Omega} |u_0(\bar{x},t)|,$$

….1.2.24

Where the region $\Omega \subset \mathbb{R}^{n+1}$.

Proof Therorem 1.2.4 stated that

$$\|S_n - S_{n'}\| \le \frac{\gamma^{n'+1}}{1-\gamma} \sup_{(\bar{x},t)\in\Omega} |u_0(\bar{x},t)|.$$

…………1.2.25

But we assume that $S_n = \Sigma_{k=0}^{n} u(\bar{x},t)$ and since $n \to \infty$, we obtain $S_n \to u(\bar{x},t)$, next we can rewrite the equation (1.2.25) as

$$\|u(\bar{x},t) - S_n\| = \left\|u(\bar{x},t) - \sum_{k=0}^{n'} u(\bar{x},t)\right\| \le \frac{\gamma^{n'+1}}{1-\gamma} \sup_{(\bar{x},t)\in\Omega} |u_0(\bar{x},t)|.$$

………1.2.26

That being said the most extreme case of a mistake is

$$\sup_{(\bar{x},t)\in\Omega} \left|u(\bar{x},t) - \sum_{k=0}^{n'} u(\bar{x},t)\right| \le \frac{\gamma^{n'+1}}{1-\gamma} \sup_{(\bar{x},t)\in\Omega} |u_0(\bar{x},t)|,$$

………………1.2.27

And with that the proof is finished.

1.2.2 Discuttion and numerical results

Table 1.1 provides that numerical value of the solution to example 1.2.11 for a number x and t value for q=0.5, 0.75 and 1 when q ranges from 0.5 to 1.75 we may compare the solution numerical values for various x and t value to example 1.2.1.2 in table 1.2 for q=0.50 and q=0.75, the approximate solution to Example 1.2.1.1 is shown in Figure I.I, plotted against a number of x-t values. Figure 1.2 shows the graphs of the approximate and exact solutions for Exam-pie 1.2.1.1 for various values of x and t when q =I. (Tajadodi, H. 2010)

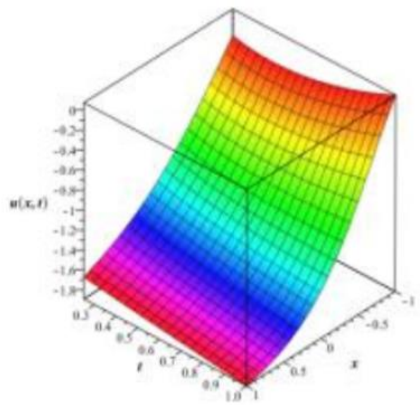
Table 1.1: umerical values of the precise and approximate solutions for Example 12.1.1

for q = 0.5,0.75, 1 among various choices ofx,t

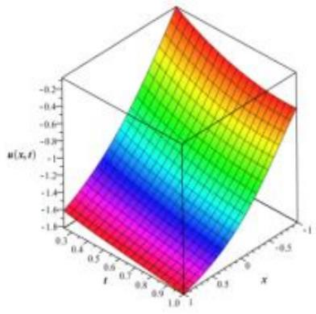
		=0.5	q = 0.75	q=I		Absolute Error at q=1
X	t	u(x, t)	u(x, t)	u(x, t)	UEX(X, t)	uEX(X, t) - u(x, t)I
	0.25	-0.73475	-0.6 I 587	-0.48622	-0.48650	b.000233
-0.5	0.50	-0.82575	-0.82914	-0.736403	-0.74008	b.003674
	0.75	-0.81728	-0.94952	-0.94678	-0.9649 I	b.018129
	0.25	-1.1720 I	-1.06435	-0.96475	-0.96490	b.000159
D	0.50	-1.29271	-1.23203	-1.15638	-1.15876	P.002377
	0.75	-1.36508	-1.34165	-1.31108	-1.32229	P.011207
	0.25	-1.47394	-1.39198	-1.32222	-1.32229	P.000062
D.5	0.50	-1.58180	-1.51238	-1.45700	-1.45788	b.000882
	0.75	-1.66942	-1.60008	-1.56485	-1.56880	b.003946

Table 1.2:Numerical values of the approximate and precise solution among several variables ofx,t for Example 2.2.1.2 with q = 0.5,0.75,1 and *a*= 2.

		=0.5	q = 0.75	q=I		Absolute Error at q=1
X	T	u(x, t)	u(x, t)	u(x, t)	UEX(X, t)	luEx(x, t) - u(x, t)I
	0.25	-0.73475	-0.61587	-0.48622	-0.48650	0.000233
-0.5	0.50	-0.82575	-0.82914	-0.736403	-0.74008	0.003674
	0.75	-0.81728	-0.94952	-0.94678	-0.96491	0.018129
	0.25	-1.17201	-1.06435	-0.96475	-0.96490	0.000159
0	0.50	-1.29271	-1.23203	-1.15638	-1.15876	0.002377
	0.75	-1.36508	-1.34165	-1.3 I 108	-1.32229	0.011207
	0.25	-1.47394	-1.39198	-1.32222	-1.32229	0.000062
0.5	0.50	-1.58180	-1.51238	-1.45700	-1.45788	0.000882
	0.75	-1.66942	-1.60008	-1.56485	-1.56880	0.003946

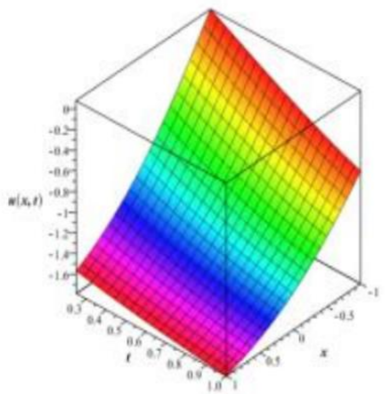


(a) The graph of the approximate solution u(x,t) for Example 2.2.1.1 for various values of x,t when q = 0.50.

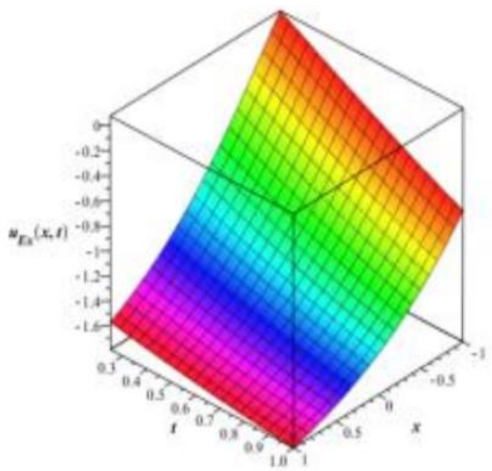


(b) The graph of the approximate solution u(x,t) for Example 2.2.1.1 for various values of x,t when q = 0.75.

Figure 1.1:The graphs show thee approximate solution u(x,t) for Example 22.1.1 for various values of x,t for q = 0.50 and q = 0.75.



(a) The graph of the approximate solution u(x,t) for Example 2.2.1.1 for various values of x,t when q = l .(Daftardar-Gejji, V. 2006)



(b) The graph for Example 2.2.1.l's precise answer, uEX (x,t), among various x,t values.

Figure 1.2:The graphs show Example 12.1.1's approximate and exact answers for various values of x,t when q = 1.

CONCLUSION

Enhancing the modelling capabilities of these potent mathematical instruments requires the creation of new analytical methods for solving nonlinear fractional partial differential equations. This study presents a strategy that overcomes many of the drawbacks of conventional procedures, offering a strong and effective approach. This approach provides a considerable increase in accuracy and processing efficiency by merging [reiterate core techniques]. The method's successful application to a number of cases demonstrates how widely applicable it might be in scientific and engineering applications. In order to further improve the analytical toolkit accessible to academics working in this field, future study will investigate additional expansions of this approach and its application to more complicated and high-dimensional FPDEs.

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