



A Theoretical Analysis of Literature of Accelerating and Anisotropic Cosmological Models in Certain Modified Theories of Gravitation

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Abstract: Cosmology is the scientific study of the relationship between space and time, with an emphasis on the origin, structure, and nature of the universe. The universe has evolved through numerous stages, revealing its entire complexity. To fully fathom the nature of our world, we must first understand its origins, evolution, and ultimate fate. The process of creating mathematical models of the universe. Einstein's theory of general relativity explains the force of gravity, whilst other theories investigate gravitational consequences in a Bianchi universe. The class of cosmological models that are intrinsically homogeneous but not isotropic on spatial slices is named after Luigi Bianchi, a mathematician who categorised these spaces in three dimensions. After doing theoretical research, it is determined that infinitesimally tiny singularities do not entirely meet the energy density constraint, and instead, a rebound occurs in place of the universe's original singularity. The solutions have densities that exceed the maximum density and are effective for connecting anisotropic solutions, even in the absence of shear at the bounce. A cosmological model of the Bianchi type II that is both spatially homogeneous and totally anisotropic. In the context of general relativity, the model is filled with pressure-less matter and anisotropic modified Ricci dark energy, as well as an attracting massive scalar field. A solution to Einstein's field equations using Bianchi type II space-time, which provides a logical explanation for the inflationary state and isotropy in the absence of singularities or particle boundaries.

Keywords: Cosmology, General Relativity, anisotropy

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AN OVERVIEW

Examining the behaviour of the universe in terms of its acceleration and anisotropy using certain modified theories of gravity provides valuable understanding about its dynamics that goes beyond what General Relativity predicts. Here is a systematic methodology for doing such a study:

INTRODUCTION

Present an overview of the mainstream cosmological model, which is founded on the principles of General Relativity.

Discuss the need of revised gravitational theories to tackle problems like the rapid expansion of the universe and the characteristics of dark energy.

The work aims to investigate accelerating and anisotropic cosmological models under certain modified theories of gravity.

Theoretical Framework:

Explain the alternative theories of gravitation that are being considered, including $f(R)$ gravity, scalar-tensor theories, and braneworld models.

Provide the equations that describe the motion and behaviour of the cosmos in these updated theories.

Examine the ways in which these alterations impact the functioning of gravity in relation to General Relativity and its consequences for the study of cosmology.

Advancing Cosmological Models:

Analyse the theoretical processes included in the selected modified theories that cause the acceleration of expansion.

Provide explicit cosmological models resulting from these ideas that demonstrate rapid expansion.

Examine the observable indications of the rapid expansion of the universe, including the relationships between distance and redshift obtained from type I_a supernovae, the cosmic microwave background radiation, and baryon acoustic oscillations.

Anisotropic Cosmological Models: Investigate the emergence of anisotropy in the context of modified theories of gravity.

Examine certain cosmological models that include anisotropy and its theoretical forecasts.

Examine observational limitations on anisotropic models, particularly their consistency with tests of isotropy and observations of large-scale structure.

Empirical examinations and limitations:

Assess the compatibility of accelerating and anisotropic cosmological theories with empirical evidence.

Assess the forecasts of these models by examining several cosmological indicators, such as the cosmic microwave background, gravitational lensing, and galaxy surveys.

Examine any conflicts or inconsistencies between theoretical forecasts and observational limitations.

Cosmology is the scientific field that studies the link between space and time, focusing on the origin, structure, and nature of the cosmos. The cosmos has progressed through several phases, elucidating its overall complexity. In order to comprehend the nature of our universe, it is crucial to get a thorough understanding of its genesis, development, and ultimate destiny. The process of building mathematical models of the cosmos Einstein's theory of general relativity is a framework that describes the force of gravity, whereas other theories explore the gravitational effects in a Bianchi world. The class of cosmological models that are inherently homogeneous but not isotropic on spatial slices is named after Luigi Bianchi, a mathematician who classified these spaces in three dimensions. Recent theoretical and experimental research have shown that our universe is now in a period of accelerated expansion known as dark expansion. It has been discovered that an unidentified kind of energy is responsible for fueling this

acceleration. The utmost. The intriguing attributes of this dark energy are its negative energy density and positive pressure. The findings from the Wilkinson Microwave Anisotropy Probe (WMAP) and the Planck satellite show that the cosmos consists of about 68.5% dark energy, 26.5% dark matter, and 5% baryonic matter. Dark energy may be represented in two alternative ways. One approach is to use exotic matter, which may be achieved either using the equation of state parameter (EOS) $W = p/p'$, where p represents pressure and p' represents energy density, or by considering the cosmological constant. An additional method used to depict the expansion of the universe involves a modified rendition of the Einstein-Hilbert action principle, which encompasses hypotheses that deviate from Einstein's theory of gravity. During this procedure, a function that is not predetermined is used to substitute the matter Lagrangian in the action. Therefore, the acceleration in the expansion of the cosmos, together with the associated effective causes,

Implications and future directions:

Examine the consequences of the study's discoveries on our understanding of gravity and the development of the cosmos.

Identify potential areas for further investigation, such as improving the accuracy of observational limitations, examining alternate hypotheses, or exploring new methods of observation.

Examine the wider consequences for the study of the universe and the basic laws of physics, such as the characteristics of dark energy and the applicability of General Relativity in the context of cosmology.

OBJECTIVE OF THE STUDY

1. This study focuses to explore literatures in the field of accelerating and anisotropic cosmological models in certain modified theories of gravitation
2. To assess the application areas of the accelerating and anisotropic cosmological models

METHODOLOGY

The research process for investigating accelerating and anisotropic cosmological models under certain modified theories of gravity often include a blend of theoretical modelling, numerical simulations, and observational investigation. Below is a suggested approach for doing research:

Literature Review methodology: Perform an extensive examination of current literature on altered theories of gravity, cosmological models that include acceleration, and cosmologies that exhibit anisotropy.

Enumerate the primary theoretical frameworks, observational limitations, and unanswered inquiries within the discipline.

The process of creating mathematical or computational models to represent and analyse theoretical concepts or systems.

Create theoretical models within the selected modified theories of gravity that integrate both accelerating expansion and anisotropic behaviour.

Compute the field equations that govern the motion of the cosmos, taking into account alterations to

General Relativity.

Apply mathematical frameworks to articulate the changes in cosmological parameters, such as the scale factor, energy density, and anisotropic stress.

Observational Analysis: Conduct a comparison between the theoretical predictions generated by simulated models and the observational data obtained from cosmic probes.

Conduct analysis on data obtained from observational surveys, which include measurements of the cosmic microwave background, supernova surveys, and galaxy clustering.

Utilise statistical techniques to evaluate the agreement between theoretical models and observational limitations.

LITERATURE REVIEW AND ANALYSIS OF IMPLICATIONS

Solanki et al. (2023) analyse the observed cosmic acceleration using a cosmology $f(R, L_m)$ model, where the major factor is bulk viscous matter in an anisotropic background. We derive the Friedmann equations that describe the gravitational interactions in $f(R, L_m)$ gravity, using the locally rotationally symmetric Bianchi type I metric. In addition, we analyse the functional form of the equations and derive the precise solutions for the field equations that correspond to our model dominated by viscous matter. In this model, we have a free parameter. The $H(z)$ data and the Pantheon data are merged using the χ^2 reduction approach and the Markov Chain Monte Carlo random sampling method to get the optimal values for the model's parameters. In addition, we analyse the characteristics of physical components such as density, the equation of state (EoS) and effective pressure parameters, the skewness parameter, and the statefinder diagnostic parameters. These factors help us understand the different stages of the Universe's history. The energy density demonstrates the expected upward trend, whereas the negative behaviour of the bulk viscous pressure causes the cosmos to expand. The effective equation of state (EoS) parameter favours the accelerating phase of the Universe's expansion. Moreover, the skewness parameter demonstrates the anisotropic nature of spacetime throughout the whole evolution of the Universe. Ultimately, we determined that our cosmological $f(R, L_m)$ model is situated in the quintessence region and demonstrates far-future behaviour similar to a de-Sitter universe when assessed using the statefinder diagnostic test.

Maurya et al. (2023) have examined a cosmological model in $f(Q)$ gravity with string fluid in the LRS Bianchi type-I universe, which exhibits anisotropy. We have examined the function $f(Q) = Q + \alpha Q + 2\Lambda$, where Λ represents the cosmological constant and α is the model's free parameter. By using the Hubble function and the constant equation of state parameter ω , we have established a correlation between the matter energy density parameter Ω_m and the dark energy density parameter Ω_Λ . The most appropriate values have been included in the discussion and final result. We have identified a dark energy model during the transit phase and have reported our findings using cosmographic coefficients. Furthermore, by examining the Om diagnostic function for an anisotropic universe, we have determined that our model is a quintessence dark energy model.

Tarai and Kumar (2022) developed a viable exponential gravity model to explain the fast expansion of the cosmos in Bianchi VIh space-time. Specifically, the cosmological models are formulated and examined

while considering the estimated physical parameters. We observed that during the first phase of the phantom domain, the state parameter in both models increases to a more negative range. Subsequently, in the latter stages of growth, it transitions to the positive domain. Both theories have a positive effective cosmological constant, suggesting that the cosmos is undergoing accelerated expansion.

Agrawal and Sen, 2022. Currently, the available evidence indicates that the universe is sufficiently massive, homogenous, and isotropic. Nevertheless, it is still possible that some anisotropy may have emerged during the early stages of the universe's existence and then diminished throughout succeeding periods. The popularity of the homogeneous but anisotropic Bianchi models has grown as a result of this theory. Furthermore, modified gravity has been receiving significant attention as a result of the issues associated with the conventional cold dark matter (CDM) hypothesis within the framework of general relativity. The purpose of the present research is to investigate the Bianchi type-I cosmological model within the framework of $f(R, T)$ -modified gravity. Agrawal and Sen, 2022. Currently, the available evidence indicates that the cosmos is both sufficiently big in size and exhibits homogeneity and isotropy. Nevertheless, it is still possible that some anisotropy may have emerged during the early stages of the cosmos and then diminished over later periods. The popularity of the homogeneous but anisotropic Bianchi models has grown as a result of this theory. Furthermore, modified gravity is receiving significant attention as a result of the issues associated with the conventional cold dark matter (CDM) concept within the framework of general relativity. The present research investigates the Bianchi type-I cosmological model within the framework of $f(R, T)$ -modified gravity.

Pawar and Shahare (2020) provide two primary rationales for discussing the phenomenon of the expanding cosmos. The first aspect to consider is the entrance of the most unusual phenomenon in the mysterious universe: positive energy density and dark energy, which together impose a substantial negative pressure. This dark energy is responsible for the existence of the whole universe and also accelerates its growth. An further motive for discussing the evolution of the cosmos is the altered rendition of Einstein's field equations of general relativity (GR), which may be substantiated using the Einstein Hilbert action principle. This is corroborated by current astronomical observational data of Supernova Ia. This technique replaces an arbitrary function with the matter Lagrangian. Subsequently, these revised ideas emerge as the most convincing contender to see both the rapid expansion of the cosmos and the practical justifications for dark energy. Several revised theories of gravity have been proposed in an effort to understand the mechanisms behind the late-time acceleration, dark energy, and dark matter.

Within the framework of $f(R, T)$ gravity, Ram et al. (2013) documented the presence of a group of non-singular bouncing cosmological models belonging to a broad range of Bianchi models that are filled with a perfect fluid. Following the initial acceleration, the model undergoes deceleration for a certain duration. Furthermore, the model's physical behaviour is being analysed.

Tiwari et al. (2021) have determined that the universe is both homogeneous and isotropic by their recent measurements made on a sufficiently enough scale. However, it is still possible that there was some anisotropy present in the early stages of the universe's development, but it was subsequently suppressed. The approach has led to increased interest in the homogeneous but anisotropic Bianchi models. Furthermore, the conventional Λ CDM model is also facing challenges related to general relativity, which has led to a significant increase in interest in modified gravity. This research focused on studying the

Bianchi type-I cosmological model within the framework of $f(R,T)$ -modified gravity. By adopting a specific formulation of the deceleration parameter based on cosmological principles, a model was developed that demonstrates a transition from initial slowdown to subsequent acceleration in the later stages. During later stages, the advanced model closely resembled isotropy. Expressions were constructed for the several parameters of the model, and the model's physical features were examined. Moreover, the inclusion of a variable cosmological-type parameter in the $f(R,T)$ gravity model first exhibited a large value but eventually stabilised at a constant value. This advancement may contribute to the resolution of the cosmological constant issue.

Tarai, Mishra, and Tripathy published a paper in 2018. Recent research has shown that the vacuum 5D BD field equations may be used to create a four-dimensional (4D) Brans-Dicke (BD) theory. This theory includes an effective matter field and a self-interacting potential. The theory is often referred to as a modified Brans-Dicke theory (MBDT). In this article, we investigate a specific sort of anisotropic cosmology known as the generalised Bianchi type I in the context of the 5D BD theory. We then calculate the matter that is present on a 4D hypersurface using the formalism obtained from the MBDT. They show that, in most cases, there will be inconsistencies in the field equations if the typical spatial scale factors vary with time, while the scale factor of the additional dimension remains constant, and the scalar field varies on both time and the fifth coordinate. Furthermore, we assume that the extra coordinate and the time are independent variables in the power-law equations, and that the scale factors and the scalar field depend on these variables. Hence, we categorise several categories of 5D spacetime solutions and analyse the specific one that results in a comprehensive Kasner link among the Kasner parameters.

The power law or logarithmic form of the induced scalar potential is derived. However, it vanishes when the scalar field is constant, even if it depends only on the fifth coordinate. The conservation rule is indeed satisfied by the induced energy-momentum tensor (EMT) in the MBDT approach. Our study focuses on many cosmic quantities, under the assumption that both the scalar field and the metric are time-dependent functions. Consequently, the model indicates that a fixed average Hubble value is not allowed and the energy-momentum tensor (EMT) satisfies the barotropic equation of state. Therefore, we make the assumption of a consistent deceleration parameter and establish the progression of the values in relation to the unchanging deceleration, the BD coupling, and the state parameters by considering the variation of the Hubble parameter. The decelerating expansion of the universe, which follows the same pattern as the flat FRW spacetime in general relativity, is facilitated by the weak energy condition via the presence of a diminishing extra dimension. The properties of the rigid fluid and radiation-dominated cosmos indicate an expanding universe that originated from a colossal explosion. Every fluid has a boundary, and with time, the rate of expansion diminishes. Once the permissible limits for the BD coupling parameters and deceleration are obtained, the model predicts that the cosmos becomes empty as time approaches infinity.

A recent study conducted by Galeev et al. (2021) has shown that, contrary to expectations, the irregularities in the uniform Bianchi I cosmology analysed within the context of a certain Horndeski theory are actually suppressed around the initial singularity. In this study, we extend the research of this phenomenon to include the whole Horndeski family. It is found that the K-essence and/or Kinetic Gravity Braiding theories do not display this characteristic, since the anisotropies grow as the distance from the singularity increases. Anisotropies are only suppressed during early periods in Horndeski models that include a Lagrangian including components that are quadratic and cubic in second derivatives of the scalar

field. Due to the anticipation that gravitational waves would travel at a variable velocity, these theories are often seen to be in conflict with the observed facts. The theories, however, align with the existing observations since the expected value of the speed at this time may closely approximate the speed of light with the required accuracy. The analysis focuses on two separate occurrences of these theories, each characterised by a postponed self-acceleration and an initial rapid expansion caused by the non-minimal coupling. During the early and late hours, the anisotropies tend to decrease and reach their lowest values, while they reach their highest values during middle intervals. The early inflationary stage's instability in reaction to inhomogeneous perturbations suggests that the universe may have had an inhomogeneous initial condition. It is probable that generic Horndeski models will exhibit stability.

Pacif and Mishra (2015) provide an explanation for an isotropic dark energy model within the framework of General Relativity. This model is constructed by constraining the Hubble parameter in the backbone of Bianchi-I space-time. The energy density is seen to be initially negative as a result of anisotropy, but it turns positive when anisotropy is no longer present. Our model's behaviour is shown to be analogous to that of the CDM model via the use of a state finder diagnostic.

Sahoo et al. (2017) conducted a study on a spatially homogeneous anisotropic Bianchi type-I universe that included matter with bulk viscosity. This study incorporates a time-varying deceleration parameter (DP) to accurately solve the field equations, resulting in an accelerating cosmos. The physical and kinematical aspects of both ideas are extensively analysed in consideration of the probable future development of the cosmos. We have examined the properties of energy density, WEC (Weighted Energy Consumption), DEC (Direct Energy Consumption), and SEC (System Energy Consumption) in both cases. Our findings indicate that the universe is undergoing accelerated expansion in both models that have a matter component with bulk viscosity. Furthermore, we have shown that the three sets of kinematical data align with the cosmic jerk parameter.

Balakin & Zimdahl published a nonminimal extension of the Einstein-Maxwell equations in 2005. This expansion was motivated by the influence of one-loop vacuum polarisation effects in curved space-time. This approach is used for modelling magnetic fields in Bianchi I models. Several exact solutions for the nonminimal system are derived, some of which elucidate an isotropization phenomenon. We provide evidence that inflationary solutions exist, in which the cosmological constant is determined by the nonminimal coupling parameters. In addition, we have found an isotropic de Sitter solution in which the nonminimal coupling leads to a "screening" effect on the magnetic field.

The four-dimensional anisotropic Bianchi models are analysed by Chaubey (2014) using the Einstein-Maxwell dilaton field equations. After deriving the generic solutions, their characteristics were analysed. The exact solutions for the field equations of two separate physically plausible cosmologies have been determined. An in-depth analysis of the cosmological parameters has been provided, coupled with data indicating that the solutions gradually approach the isotropic Friedmann-Robertson-Walker cosmology model.

Reddy et al. (2019) investigated a cosmological model of anisotropic dark energy in the framework of the general theory of relativity. The model was studied in Kantowski-Sachs space-time and included an anisotropic dark energy fluid connected to a mass-less scalar field. A physically plausible dark energy

cosmological model is offered by using a hybrid expansion law proposed by Akarsu et al. (JCAP. 01, 022: 2014) and a relationship involving the metric potential. The dark energy density, skewness parameter, and equation of state (EoS) parameter of the model are determined, along with their scientific importance. The validity of modern cosmology observations is supported by the seamless shift of the cosmos from initial deceleration to subsequent acceleration, as seen by the model's jerk and deceleration parameters.

The field equations in a modified theory of gravity, as proposed by Harko et al. (Phys Rev D 84: 024020, 8), are determined using a spatially homogeneous and anisotropic LRS Bianchi type-I metric, as described by Santhi Kumar and Satyannarayana (2017). One may formulate cosmological models that accurately correspond to a universe consisting of dust and a fake vacuum. A discourse is conducted on certain kinematic and physical attributes of each model. These theories may have physical importance when considering the early stages of the universe's development.

In 2020, Alam, Ullah, and Chowdhury conducted a study on diagnosing a cosmological model known as the spatially homogeneous and anisotropic Bianchi type-I model in the framework of general theory of relativity. The model includes a perfect fluid with a quadratic equation of state. We have used a connection between metric potentials to get a conclusive and predictable answer. The expansion and deceleration of the universe are shown in the accurate solution of Einstein's field equations. The study also investigated the physical and kinematic characteristics of the model, taking into account specific limits within the parameters of the quadratic equation of state.

As per the research conducted by Banerjee, A., Banerjee, N., and Santos, N. O. In the framework of Nordtvedt's scalar-tensor theory of gravity, anisotropic cosmological models from 1985 are being considered. The focus is placed on Bianchi type I models. The equation of state for perfect fluid models is given by $p = \epsilon p$. In the context of empty space, the value of ϵ is equal to 1, and the solutions may be obtained using Dicke's conformally transformed units. This is done by assuming that ω and ϕ have two separate and independent functional relationships. Furthermore, a comparison is drawn between their characteristics and the models presented by Brans-Dicke theory.

Kofman, L., Sakhni, V., and Starobinskii, A. A. In 1983, it was proposed that in the absence of classical matter, the vacuum of quantum fields may be influenced by a self-consistent gravitational field. This can result in the creation of a uniform, uneven space-time metric with a symmetry group consisting of six parameters. The metric is used to determine the average values of the energy-momentum tensors for a massive scalar field and massless conformally covariant fields. A Green function for a scalar field with mass is formulated.

Adhav et al. (2007) investigated Bianchi type-III cosmological models that include a bulk viscous fluid. The bulk viscosity coefficient is postulated to be a power function of the mass density. Recent supernovae measurements have shown that the cosmological constant $\dot{Y}$ is positive and exhibits a decreasing trend over time. The models also include coverage of certain geometrical and physical features. The study of cosmological models of the Bianchi type is of heightened interest due to the presence of isotropic special instances and the ability to have varying amounts of anisotropy at particular points in cosmic periods. Consequently, they are regarded as a commendable representation of our cosmos due to this characteristic. Based on our analysis, we have found that most of the cosmological models we have examined accurately

depict the matter in the universe as a "perfect fluid" or "dust," which refers to a distribution without any pressure.

Saha (2011) observed that the dispersion of matter has been considered in the framework of a Bianchi type-II (BII) cosmological model. The non-zero off-diagonal component of the Einstein tensor imposes strict constraints on the possible choices for matter distribution. Specifically, it has been shown that if the energy-momentum tensor of the related matter field only includes non-zero diagonal components, then the matter distribution must be completely isotropic for a locally rotationally symmetric Bianchi type-II (LRS BII) space-time. The behaviour of the Universe on a large scale is mostly characterised by cosmological models that are both spatiotemporally homogeneous and anisotropic. These theories have been thoroughly studied in the framework of General Relativity in order to get a precise understanding of the early Universe. In this note, we focus our analysis only on a Bianchi type-II (BII) space-time.

Corichi & Montoya (2012) conducted a quantitative investigation of the effective equations of the Bianchi II model using the dynamics of "improved" loop quantum cosmology. A scalar field with zero mass acts as the origin of matter. We systematically analyse the solution space, focusing on the properties of several geometric observables. We provide evidence that infinitesimally small singularities do not fully satisfy the limitation on energy density, and instead, a rebound occurs in lieu of the initial singularity of the universe. The shear displays a maximum of four local maxima and may reach zero at the point of rebound. The scale factors may have a maximum of three directional bounces, whereas the expansion has one global bounce. In the context of the isotropic and Bianchi I situations, this allows for solutions that exhibit densities beyond the maximum density. The solutions are successful in connecting anisotropic solutions, even in cases when there is no shear at the bounce. Additionally, the asymptotic behaviour of these solutions is shown to be similar to that of a Bianchi I model. All the known information about Bianchi I are given. Solutions are present in the "vacuum limit" when almost all dynamics is driven by anisotropies. Our results provide a plausible explanation for the behaviour of universal spacelike singularities in the presence of loop-geometric corrections, considering the importance of Bianchi II in the setting of the Bianchi IX model and the Belinskii, Khalatnikov, Lifshitz hypothesis.

Naidu (2019) investigated a cosmological model of the Bianchi type-II, which is both spatially homogenous and completely anisotropic. The model is filled with pressure-less matter and anisotropic modified Ricci dark energy, which is accompanied by an attractive massive scalar field in the framework of general relativity. The linearly variable deceleration parameter is used to create a deterministic model of the cosmos. The models' physical and kinematical parameters, which are pertinent to the ongoing cosmological discourse, provide a scientific examination of the dark energy notion. The hypothesis aligns with the existing evidence from contemporary cosmology.

Bali (2022) presented a solution to Einstein's field equations within the framework of Bianchi type II space-time. This solution allows for the presence of dust and a massless scalar field with negative energy as sources, together with a cosmological component. The models adhere to the conservation equation, and the result is determined by Hoyle and Narlikar as the creation field expands over time. Their model's deceleration parameter undergoes a transition from a decelerating to an accelerating phase upon formation. Comparing various cosmological models shows a transition from a slowing down to a speeding up phase,

regardless of whether there is a creation event or not. The present model provides a natural explanation for the inflationary state and isotropization, without any singularities or particle horizons. Theoretical superiority of creation field cosmological models over Big Bang theories stems from these qualities. The principle of least action is used to construct the creation field and Einstein field equation. Additionally, as Moffat said, the Lagrangian formulation of the variable cosmological component is obtained. Furthermore, there is ongoing discourse on the current pace of creation and its tangible consequences. Visual representations are used to provide instructive examples of how cosmic parameters evolve over time.

Clarkson, C., and Shafieloo, A. The year 2010. The focus on the dark energy issue has led to speculation that the Friedmann-Lemaître-Robertson-Walker models themselves may not provide the appropriate set of background models, along with the potential that Λ CDM is inaccurate. We investigate the potential formulation of tests for the dominant framework in cosmology by using direct observations of $H(z)$. These indicators alone can be used to detect deviations from a flat Λ CDM model. When Hubble rate data is combined with supernovae distances, it allows for an investigation of the Copernican principle and the assumption of homogeneity in the standard model. This is independent of dark energy or a gravitational metric theory. This analysis presents an alternative approach that also provides a model-independent measure of flatness. This measure effectively separates the computation of curvature obtained from dark energy. We analyse these experiments by use both a calculated Hubble rate derived from recent observations of the baryon acoustic oscillations, as well as Hubble rate measurements obtained from age data. These trials provide intriguing prospects for the future, despite the inadequate data available to make definitive judgements.

FINDING AND CONCLUSION

Solanki et al. (2023) analyze the observed cosmic acceleration using a cosmology $f(R, L_m)$ model, focusing on bulk viscous matter in an anisotropic background. They derive Friedmann equations and analyze the functional form of the equations, resulting in precise solutions for field equations. The study also examines physical components such as density, equation of state (EoS), effective pressure parameters, skewness parameter, and statefinder diagnostic parameters. Authors establish a correlation between matter energy density and dark energy density parameters, identifying a dark energy model during the transit phase and determining it is a quintessence dark energy model.

A viable exponential gravity model is being developed to explain the fast expansion of the cosmos in Bianchi VIh space-time. They observed that the state parameter in both models increases to a more negative range during the first phase of the phantom domain, then transitions to the positive domain in the latter stages of growth. Agrawal and Sen (2022) discuss the phenomenon of the expanding cosmos, citing positive energy density and dark energy as the main factors. Ram et al. (2013) documented the presence of non-singular bouncing cosmological models within the framework of $f(R, T)$ gravity, which undergo deceleration after initial acceleration and are analyzed for physical behavior.

Moreover, it is found the universe to be both homogeneous and isotropic, with some anisotropy present in the early stages of development. This led to increased interest in homogeneous but anisotropic Bianchi models and modified gravity. This research focused on studying the Bianchi type-I cosmological model within the framework of $f(R, T)$ -modified gravity. The study adopted a specific formulation of the

deceleration parameter based on cosmological principles, demonstrating a transition from initial slowdown to subsequent acceleration in later stages. The inclusion of a variable cosmological-type parameter in the $f(R,T)$ gravity model initially exhibited a large value but eventually stabilised at a constant value, potentially contributing to the resolution of the cosmological constant issue.

The vacuum 5D BD field equations can be used to create a four-dimensional (4D) Brans-Dicke (BD) theory, also known as a modified Brans-Dicke theory (MBDT). They investigated the generalised Bianchi type I in the context of the 5D BD theory and found that the conservation rule is satisfied by the induced energy-momentum tensor (EMT) in the MBDT approach. The study suggests that the universe may have had an inhomogeneous initial condition, and generic Horndeski models may exhibit stability. Pacif and Mishra (2015) provided an explanation for an isotropic dark energy model within the framework of General Relativity, which is constructed by constraining the Hubble parameter in the backbone of Bianchi-I space-time.

Sahoo et al. (2017) studied a spatially homogeneous anisotropic Bianchi type-I universe with matter with bulk viscosity, incorporating a time-varying deceleration parameter to solve field equations and resulting in an accelerating cosmos. They found that the universe is undergoing accelerated expansion in both models, and the three sets of kinematical data align with the cosmic jerk parameter.

Balakin & Zimdahl published a nonminimal extension of the Einstein-Maxwell equations in 2005, which is used for modeling magnetic fields in Bianchi I models. Chaubey (2014) analysed four-dimensional anisotropic Bianchi models using the Einstein-Maxwell dilaton field equations, determining the exact solutions for two separate physically plausible cosmologies. Reddy et al. (2019) investigated a cosmological model of anisotropic dark energy in the general theory of relativity, using a hybrid expansion law and a relationship involving the metric potential.

Harko et al. proposed a modified theory of gravity using a spatially homogeneous and anisotropic LRS Bianchi type-I metric, which may correspond to a universe consisting of dust and a fake vacuum. Alam, Ullah, and Chowdhury conducted a study on diagnosing the spatially homogeneous and anisotropic Bianchi type-I model in 2020, focusing on the expansion and deceleration of the universe.

Kofman, Sakhni, and Starobinskii (1983) proposed that in the absence of classical matter, the vacuum of quantum fields may be influenced by a self-consistent gravitational field, creating a uniform, uneven space-time metric with a symmetry group consisting of six parameters. Adhav et al. (2007) investigated Bianchi type-III cosmological models, finding that most accurately depict the matter in the universe as a "perfect fluid" or "dust," which refers to a distribution without any pressure.

Saha (2011) studied the dispersion of matter in a Bianchi type-II (BII) cosmological model, focusing on the non-zero off-diagonal component of the Einstein tensor. The study found that if the energy-momentum tensor of the related matter field only includes non-zero diagonal components, then the matter distribution must be completely isotropic for a locally rotationally symmetric Bianchi type-II (LRS BII) space-time.

They found that infinitesimally small singularities do not fully satisfy the limitation on energy density, and instead, a rebound occurs in lieu of the initial singularity of the universe. The solutions exhibit densities beyond the maximum density and are successful in connecting anisotropic solutions, even in cases without

shear at the bounce. A cosmological model of the Bianchi type-II, which is both spatially homogenous and completely anisotropic. The model is filled with pressure-less matter and anisotropic modified Ricci dark energy, accompanied by an attractive massive scalar field in the framework of general relativity.

A solution to Einstein's field equations within the framework of Bianchi type II space-time, providing a natural explanation for the inflationary state and isotropization without singularities or particle horizons.

The potential formulation of tests for the dominant framework in cosmology using direct observations of $H(z)$ and Hubble rate data. These trials provide intriguing prospects for the future, despite the inadequate data available to make definitive judgments.

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