Mathematical Model for the MHD Flow through Porous Medium with Heat Transfer

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Abstract

This paper analyzed the effect of magnetic field on 3-D flow of an incompressible viscous fluid passes through a porous plate with the following assumption; The upper plate is subjected to a constant suction while the lower plate to a transverse sinusoidal injection velocity distribution. The plate temperature is constant. A magnetic field is applied in the direction normal to the plate. Three dimensional MHD flow. The governing nonlinear partial differential equations are the solved numerically using the finite difference method and the expression for the velocity and the temperature field are obtained. The effect of Hartman number M and suction parameter R are observed on the velocity and temperature distribution.

Keywords: Hartmann number, Prandtl number, permeability parameter, suction parameter, Grashof number

1. Introduction:

The steady flow of an electrically conducting fluid has many applications in biological and engineering problems such as magneto hydrodynamics (MHD) generators, plasma studies, nuclear reactors, geothermal energy extraction, the boundary layer control in the field of aerodynamics, blood flow problems, petroleum technology to steady the movement of natural gas, and in oil and water industries through the oil channel etc.

Gersten and Gross (1974) studied on the three dimensional convective flow and heat transfer through a porous medium, while Gulab and Mishra (1977) expressed an idea through the equation of motion for MHD flow. Raptis (1983) worked on the free convective flow through a porous medium bonded by the infinite vertical plate with oscillating plate temperature and constant suction, and again Raptis and Perdikis (1985) further worked on the free convective flow through a highly porous medium bounded by the infinite vertical porous plate with constant suction. Although in above studies the investigators have restricted themselves to two-dimensional flows, but there may arise situations, where the flow field may be essentially three-dimensional. Therefore Singh (1991) worked on three dimensional MHD flow past a porous plate, and again Singh (1993) also worked in the same direction and studied the problem of three dimensional viscous flow and heat transfer along a porous plate.

Ahmed and Sharma (1997) discussed about the three-dimensional free convective flow of an incompressible viscous fluid through a porous medium with uniform free steam velocity, while Singh (1999) again studied about a three-dimensional Couette flow with transpiration cooling by applying transverse sinusoidal injection velocity at the stationary plate velocity. Kim (2000) discussed the unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, and Kamel (2001) discussed about the unsteady MHD convection through porous medium with combined heat and mass transfer with heat source/sink. Kumar et al (2004) discussed about the Hall current on MHD free- convection flow through porous media past a semi-infinite vertical plate with mass transfer, while Muhammad et al (2005) discussed the effects of Hall current and heat transfer on the flow due to a pull of eccentric rotating. Attia (2006) observed the unsteady MHD Couette flow and heat transfer of dusty fluid with variable physical properties. Hakan (2007) worked on the natural convection in porous triangular enclosures with a solid adiabatic fin attached to the horizontal wall international communications in heat and mass transfer. Tzer-Ming Jeng (2008) worked on a porous model for the square pin-fin heat sink situated in a rectangular channel with laminar side-by-pass flow.

The main purpose of this work is to analyze the effects of electrically conducting three dimensional, viscous incompressible fluid through a porous plate with the observation of the velocity and temperature distribution along with the effects of Hartmann number (M) and suction parameter (R)

2. Mathematical Analysis:

We are considering a 3-D flow of a viscous incompressible fluid through a porous medium, which is bounded by a vertical infinite porous plate. The coordinate system with plate lying vertically on X^*-Z^* plane, where upper plate is at distant *a* apart such that X^*-axis is taken along the plate in the direction of the plane and Y^*- axis is perpendicular to the plane. The lower and upper plate's temperature considered to be T_0^* and T_1^* respectively with ($T_1^* > T_0^*$) and the upper plate is subjected to a constant velocity U along X^*- axis, while the lower plate to a transverse sinusoidal injection velocity of the following forms

$$V^*(z^*) = V^*\left(1+\varepsilon\cos\frac{\pi Z^*}{a}\right) \qquad \text{(I)}$$

Denoting the velocity component u^*, v^*, w^* in the X^*, Y^* and Z^* direction respectively and temperature by T^*, then problem is governed by the following equations.

Continuity equation:

$$\frac{\partial v^*}{\partial y^*}+\frac{\partial w^*}{\partial z^*}=0 \qquad \text{(Ii)}$$

The momentum equations are

$$v^*\frac{\partial u^*}{\partial y^*}+w^*\frac{\partial u^*}{\partial z^*}=\nu\left(\frac{\partial^2 u^*}{\partial y^{*2}}+\frac{\partial^2 u^*}{\partial z^{*2}}\right)+g\beta(T_1^*-T_1^*)-\sigma\frac{\beta_0^2 u^*}{\rho} \qquad \text{(Iii)}$$

$$v^*\frac{\partial v^*}{\partial y^*}+w^*\frac{\partial v^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\nu\left(\frac{\partial^2 v^*}{\partial y^{*2}}+\frac{\partial^2 v^*}{\partial z^{*2}}\right)-\frac{\nu}{k}v^* \qquad \text{(Iv)}$$

$$v^*\frac{\partial w^*}{\partial y^*}+w^*\frac{\partial w^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}+\nu\left(\frac{\partial^2 w^*}{\partial y^{*2}}+\frac{\partial^2 w^*}{\partial z^{*2}}\right)-\frac{\sigma\beta_0^2 w^*}{\rho} \qquad \text{(V)}$$

And the energy equation is

$$v^{*}\frac{\partial T^{*}}{\partial y^{*}}+w^{*}\frac{\partial T^{*}}{\partial z^{*}}=\alpha\left(\frac{\partial^{2}T^{*}}{\partial y^{*2}}+\frac{\partial^{2}T^{*}}{\partial z^{*2}}\right) \qquad \text{(Vi)}$$

While the boundary conditions of this modal are

$$\left.\begin{array}{l} \text{at } y^{*}=0,\ u^{*}=0,\quad v^{*}(z^{*})=v\left(1+\varepsilon\cos\frac{\pi z^{*}}{a}\right),\ w^{*}=0,\ T^{*}=T_{0} \\ \text{at } y^{*}=a,\ u^{*}=U,\ v^{*}=v,\qquad w^{*}=0,\ T^{*}=T_{1}^{*} \end{array}\right\} \qquad \text{(Vii)}$$

Now we introduce following non-dimensional variables, which change the governing equations in the dimensionless form

$$\left. y=\frac{y^{*}}{a},\quad z=\frac{z^{*}}{a},\quad u=\frac{u^{*}}{U},\quad v=\frac{v^{*}}{v},\ w=\frac{w^{*}}{v},\quad p=\frac{P}{\rho v^{2,}},\quad \theta=\frac{T^{*}-T_{0}^{*}}{T^{*}_{1}-T_{0}^{*}}\right\} \qquad \text{(Viii)}$$

In this model we use some special numbers/ parameters, which are along with their notations as follows:

Suction parameter $(R)=\frac{av}{\nu}$, Hartmann number $(M)=\frac{\beta_0}{V}\sqrt{\frac{\sigma}{\mu}}$,

Prandtl number $(P_r)=\frac{\nu}{\alpha}$, Grashof number $(G_r)=\frac{g\beta\rho^{2}(T_0^{*}-T_1^{*})a^{3}}{\mu^{2}}$,

Where β_0 -magnetic field component along Y^{*}- axis, k- permeability of the porous medium, U- velocity along X^{*}-axis, α- thermal diffusivity, β- volumetric coefficient of the thermal expansion, ρ - density of fluid, ν - kinematics viscosity, μ -viscosity and θ -temperature.

Here in the light of (viii) the boundary conditions (vii) and solving the equations (ii) to (vi), becomes in the following form:

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad \text{(Ix)}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{1}{R}\left(\frac{\partial^{2}u}{\partial y^{2}}+\frac{\partial^{2}u}{\partial z^{2}}\right)+\frac{G_{r}\nu}{aRU}-\frac{M^{2}u}{R} \qquad \text{(X)}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial P}{\partial y}+\frac{1}{R}\left(\frac{\partial^{2}v}{\partial y^{2}}+\frac{\partial^{2}v}{\partial z^{2}}\right)-\frac{a^{2}v}{Rk} \qquad \text{(Xi)}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial P}{\partial z}+\frac{1}{R}\left(\frac{\partial^{2}w}{\partial y^{2}}+\frac{\partial^{2}w}{\partial z^{2}}\right)-\frac{M^{2}w}{R} \qquad \text{(xii)}$$

And temperature equation become:

$$v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}=\frac{1}{RP_{r}}\left(\frac{\partial^{2}\theta}{\partial y^{2}}+\frac{\partial^{2}\theta}{\partial z^{2}}\right) \qquad \text{(Xiii)}$$

Then the boundary condition (7) reduce in the dimensionless form:

3. Numerical Solution:

In this model we apply the finite difference method to solve the governing non-linear equations. First we transformed the first and second order derivatives by using the forward and central differences formulas. Here i, j represent the moment along Y and Z direction respectively, and let, there is a square mesh ($\Delta y=\Delta z=h$), then the equations (ix) to (xiii) along with the boundary conditions are as:

$$u_{i,j+1} = \frac{1}{(Rhw_{i,j} - 1)}\left[\begin{array}{r} (-Rhv_{i,j})u_{i+1,j} + \{Rh(v_{i,j} + w_{i,j}) - (4 + M^2h^2)\}u_{i,j} \\ + (u_{i-1,j} + u_{i,j-1}) + \frac{\nu hG_r}{aRU} \end{array} \right] \quad \text{(Xiv)}$$

$$v_{i,j+1} = \frac{1}{(Rhw_{i,j} - 1)}\left[\begin{array}{r} (-Rhv_{i,j})v_{i+1,j} + +\left\{Rh(v_{i,j} + w_{i,j}) - \left(4 + \frac{a^2h^2}{K}\right)\right\}v_{i,j} \\ + (v_{i-1,j} + v_{i,j-1}) + +Rh\Delta P \end{array} \right] \quad \text{(Xv)}$$

$$w_{i,j+1} = \frac{1}{(Rhw_{i,j} - 1)}\left[\begin{array}{r} (-Rhv_{i,j})w_{i+1,j} + \{Rh(v_{i,j} + w_{i,j}) - (4 + M^2h^2)\}w_{i,j} \\ + Rh\Delta P \end{array} \right] \quad \text{(Xvi)}$$

And temperature equation

$$\theta_{i,j+1} = \frac{1}{(RhP_r w_{i,j} - 1)}\left[\begin{array}{r} (-RhP_r v_{i,j})\theta_{i+1,j} + \{RhP_r(v_{i,j} + w_{i,j}) - 4\}\theta_{i,j} \\ + (\theta_{i-1,j} + \theta_{i,j-1}) \end{array} \right] \quad \text{(Xvii)}$$

1. Results and Discussion:

The main flow velocity component u, applied through the porous plate at rest, due to the transverse sinusoidal injection velocity is obtained from equation (xiv). We described the fig- (a) from the equation (xiv), which shows the flow component u decreases with the increases of Hartmann number M, injection parameter R and permeability k of the porous medium.

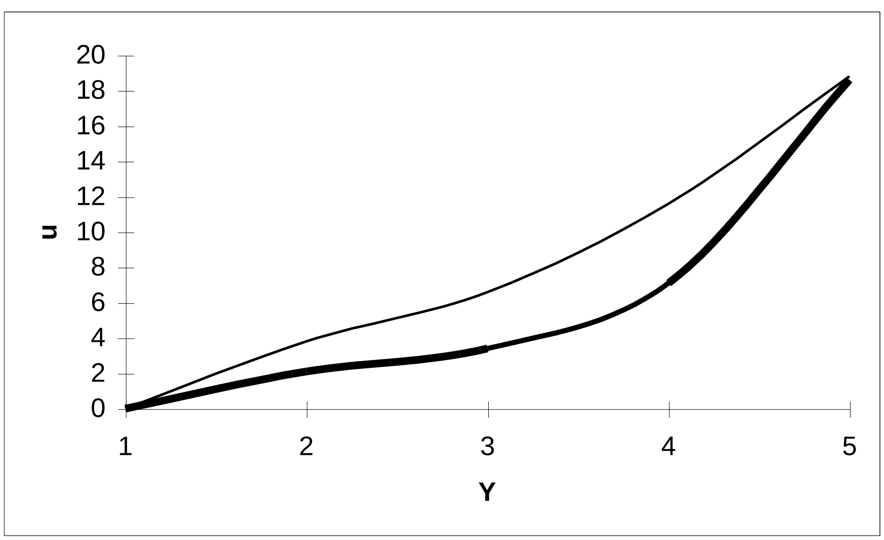
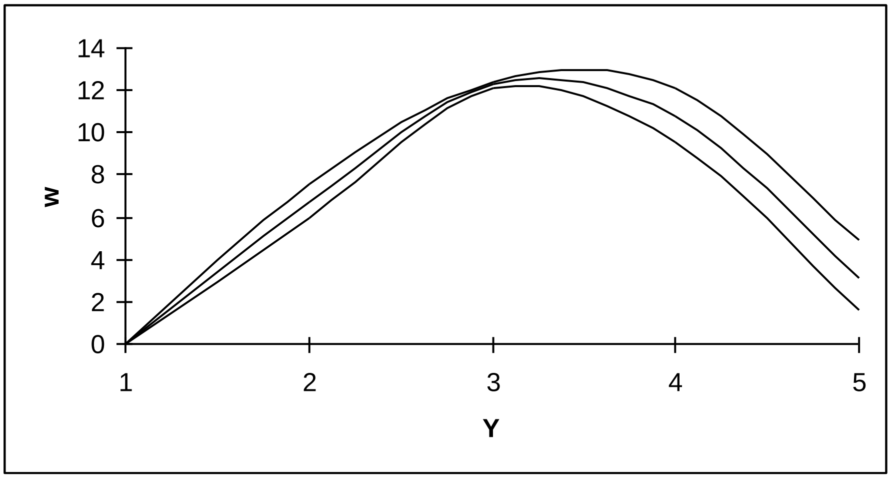


Fig. (a) Main flow velocity profile for z=0



. Fig. (a) Main flow velocity profile for Z=0.5

We described velocity profile for the small and large value of permeability of the porous medium through the fig.-(b), which shows the velocity decrease with the increase of the injection suction parameter R, it is also observed that increase in the permeability of the porous lead to an increase in the flow velocities.

The fig(c) shows that the rate of heat transfer coefficient at the stationary porous plate. In this case the values of Prandtl number Pr are chosen as 0.5 approximately, which represents air and water at $15^0 C$. Here it is found that the rate of heat transfer is much lower in the case of water than in air.

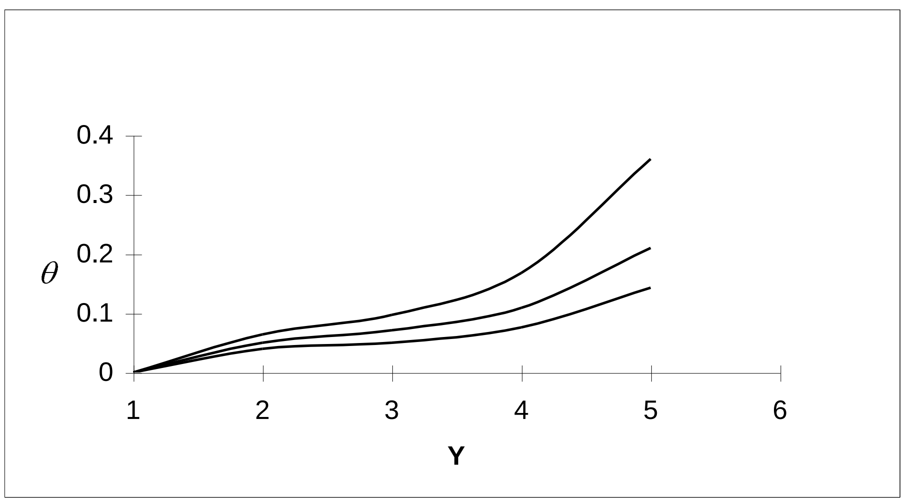


Fig. (c) Temperature distribution

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