MHD Couette flow Through Porous Medium with transpiration cooling

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Abstract:

In this paper we consist the problem of Couette flow between two horizontal parallel porous flat plates of an electrically conducting viscous incompressible fluid. The stationary plate is subjected to a transverse sinusoidal injection of the fluid and its corresponding removal by the constant suction through the other plate, in uniform motion and because of injection velocity the flow becomes three-dimensional. A magnetic field of uniform strength is applied normal to the planes of the plates. The effect of injection/suction velocity and the magnetic field on the flow field, skin friction and heat transfer are reported and discussed in detail.

Introduction

The problem of MHD flow through porous medium has very important in recent year particularly in the field of agricultural engineering to study the underground water resources, seepage of water river beds, in chemical engineering for filtration purification process; in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs.

The purpose of the present paper is to study the hydromantic effects of electrically conducting three-dimensional flow of viscous incompressible fluid through a porous medium, which is bounded by an infinite vertical porous plate with constant temperature.

The purpose of the present paper is to study the hydro magnetic effects of electrically conducting fluid through a porous medium, which is bounded by an infinite porous plate. Ahamadi et all (1971) discussed the Study of unsteady MHD flow of conducting fluid through porous medium. Raptis (1983) discussed about the unsteady flow through a porous medium bounded by an infinite porous plate subjected to a constant suction variable temperature. Dutta (1985) discussed the Temperature field in the flow over stretching surface with uniform heat flux. Massoudi (1992) used a perturbation technique to solve the stagnation point flow and heat transfer of a non-Newtonian fluid of second grade. Tokis (1986) discussed Un study MHD free convective flows in a rotating fluid. Yong (2000) study the Unstudy MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Yang et all (2001) discussed Numerical solution of thermal fluid instability between two horizontal parallel plates. Ahamad et all (2003) discussed the MHD effects on free convection and mass transfer flow through porous media between vertical wavy wall and a parallel flat wall. Attia (2003) study the Hydro magnetic stagnation point flow with heat transfer over a permeable surface. Muhammad (2005) discussed the Effects of Hall current and heat transfer on flow due to a pull of eccentric rotating.

Basic equation

In this modal we consider 3-D flow viscous incompressible fluid through a porous medium, which is bounded by a vertical infinite porous plates. A coordinates system with plate lying vertically on x-z plane such that x-axis is taken along the plate in the direction of flow and y-axis perpendicular to the plane of the plate and direction in to the fluid which is flowing with free stream velocity U and lower plate is to have a transverse sinusoidal injection velocity of the form

$$V'(Z') = V\left(1 + \varepsilon \cos\frac{\pi Z'}{a}\right)$$

… (1)

Where ε is a positive constant quantity (<<1). The distance *a* between the plates is taken equal to the wavelength of the injection velocity. The lower and upper plates are assumed to at constant temperatures T_0 and T_1, respectively, with $T_1 > T_0$.

The problem is governed by the following non-dimensional equations:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

(2)

The momentum equations are

$$\mathrm{v}\frac{\partial u}{\partial y}+\mathrm{w}\frac{\partial u}{\partial z}=\frac{1}{\lambda}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{M^2}{\lambda}u$$

(3)

$$\mathrm{v}\frac{\partial v}{\partial y}+\mathrm{w}\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{\lambda}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{1}{k'}v$$

(4)

$$\mathrm{v}\frac{\partial w}{\partial y}+\mathrm{w}\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{\lambda}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{M^2}{\lambda}w$$

(5)

And the energy equation is

$$\mathrm{v}\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{1}{\lambda p_r}\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)$$

(6)

Where equation (2) is because of conservation of mass, equation (3), (4) and (5) is because of conservation of momentum (i.e. Navier-stokes equation of motion) while equation (6) is because of energy conservation. Here the non-dimensional variables are:

$$\left.y=\frac{y'}{a},\quad z=\frac{z'}{a},\quad u=\frac{u'}{U},\quad v=\frac{v'}{v},\ w=\frac{w'}{v},\quad p=\frac{P'}{\rho v^{2}},\quad \theta=\frac{T'-T_0}{T_1-T_0},\ k'=\frac{a v}{v k}\right\} \qquad (7)$$

The boundary conditions to this problem in dimensionless form are as:

$$\left.\begin{array}{ll}\mathrm{u}=0,\quad v(z)=1+\varepsilon\cos\pi z,\ \mathrm{w}=0,\quad \mathrm{T}=0,\quad for\ y=0. \\ \mathrm{u}=1,\quad \mathrm{v}=1,\quad \mathrm{w}=0,\quad T=1,\quad for\ y=1.\end{array}\right\}$$

(8)

Mathematical Analysis:

As we know that the amplitude of injection velocity ε is very smalls therefore we can assume the following form the solutions

$$f(y,z) = f_0(y) + \varepsilon f_1(y,z) + \varepsilon^2 f_2(y,z) + ... \qquad (9)$$

Where *f* stands for any of *u, v, w, p,* and *T* function. When ε=0, the problem reduced to two-dimensional with constant injection and suction at both the plates in presence of transverse uniform magnetic field. The solution of two-dimensional problem is

$$u_0(y) = \frac{e^{s_1 y} - e^{s_2 y}}{e^{s_1} - e^{s_2}}, \qquad w = 0,$$

(10)

$$v_0(y) = \frac{1}{e^{t_1} - e^{t_2}} \left[(e^{t_1 y} - e^{t_2 y}) + e^{t_1 + t_2 y} - e^{t_2 + t_1 y} \right] \qquad (11)$$

$$T_0(y) = \frac{e^{\lambda P_r y} - 1}{e^{\lambda P_r} - 1} \qquad (12)$$

Where

$$s = \left[\lambda \pm \sqrt{\lambda^2 + 4M^2}, \right] \quad t = \frac{\lambda k' \pm \sqrt{\lambda^2 k'^2 + 4k'\lambda}}{2k'}$$

When $\varepsilon \neq 0$, then equation (9) becomes in forms,

$$\left.\begin{aligned} u(y,z) &= u_0(y) + \varepsilon u_1(y,z) \\ v(y,z) &= v_0(y) + \varepsilon v_1(y,z) \\ w(y,z) &= w_0(y) + \varepsilon w_1(y,z) \\ p(y,z) &= p_0(y) + \varepsilon p_1(y,z) \\ T(y,z) &= T_0(y) + \varepsilon T_1(y,z) \end{aligned}\right\} \qquad (13)$$

Substituting the equation (13) in to the equation (2) to (6) and comparing the coefficient of identical power of ε, and neglecting the coefficient $\varepsilon^2,\varepsilon^3$ etc. the following first order equations obtained:

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0$$

(14)

The momentum equations are

$$v_1 \frac{\partial u_0}{\partial y} + w \frac{\partial u_1}{\partial z} = \frac{1}{\lambda}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - \frac{M^2}{\lambda} u_1$$

(15)

$$\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{\lambda}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{1}{k'} v_1$$

(16)

$$\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{\lambda}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - \frac{M^2}{\lambda} w_1$$

(17)

And the energy equation is

$$v_1 \frac{\partial T_0}{\partial y} + \frac{\partial T_1}{\partial y} = \frac{1}{\lambda p_r}\left(\frac{\partial^2 T_1}{\partial y^2} + \frac{\partial^2 T_1}{\partial z^2}\right)$$

(18)

The corresponding boundary conditions are:

$$\left.\begin{array}{l} u_1 = 0, \quad v_1 = \cos \pi z, \; w_1 = 0, \quad T_1 = 0, \quad for\ y = 0. \\ u_1 = 1, \quad v_1 = 1, \quad w_1 = 0, \quad T_1 = 1, \quad for\ y = 1. \end{array}\right\} \qquad (19)$$

Cross Flow solution:

For the cross flow solution we assume the following form of v_1, w_1 and p_1:

$$\left.\begin{aligned} v_1(y,z) &= v_*(y)\cos\pi z \\ w_1(y,z) &= -\frac{1}{\pi}v'_*(y)\sin\pi z \\ p_1(y,z) &= p_*(y)\cos\pi z \end{aligned}\right\} \tag{20}$$

Where $*$ denote the differentiation with respect to y. substituting equation (20) in equations (16) and (17). We get the following ordinary differential equations:

$$v''_* - \lambda v'_* - \alpha^2 v_* = \lambda p'_* \tag{21}$$

$$v'''_* - \lambda v''_* - (\pi^2 + M^2)v'_* = \lambda\pi^2 p_* \tag{22}$$

Now using the transformed boundary conditions the equations (21) and (22) obtained in the following form

$$v_1(y,z) = \frac{1}{D}\left(\sum_{i=1}^{4} D_i e^{r_i y}\right)\cos\alpha z \tag{23}$$

$$w_1(y,z) = -\frac{1}{\pi D}\left(\sum_{i=1}^{4} D_i r_i e^{r_i y}\right)\sin\pi z \tag{24}$$

$$p_1(y,z) = \frac{1}{\lambda\pi^2 D}\left[\sum_{i=1}^{4} D_i \left\{r_i^3 - \lambda r_i^2 - (\pi^2 + M^2)r_i\right\} e^{r_i y}\right]\cos\pi z \tag{25}$$

Where

$$r_1 = \frac{1}{2}\left[p_1 + \sqrt{p_1^2 + 4\pi^2}\right],\quad r_2 = \frac{1}{2}\left[p_1 - \sqrt{p_1^2 + 4\pi^2}\right],$$

$$r_3 = \frac{1}{2}\left[p_2 + \sqrt{p_2^2 + 4\pi^2}\right],\quad r_4 = \frac{1}{2}\left[p_2 - \sqrt{p_2^2 + 4\pi^2}\right],$$

$$D = \left\{(r_2 - r_1)(r_1 - r_3)(e^{r_1+r_2} + e^{r_1+r_3})\right\}$$
$$+ \left\{(r_3 - r_2)(r_1 - r_4)(e^{r_2+r_3} + e^{r_1+r_4})\right\} + \left\{(r_2 - r_4)(r_3 - r_1)(e^{r_3+r_1} + e^{r_2+r_4})\right\}$$

$$D_1 = r_2(r_1 - r_3)e^{r_1+r_3} + r_3(r_2 - r_4)e^{r_2+r_4} + r_4(r_3 - r_1)e^{r_3+r_1}$$
$$D_2 = r_2(r_3 - r_4)e^{r_3+r_4} + r_3(r_4 - r_1)e^{r_1+r_4} + r_4(r_1 - r_2)e^{r_1+r_2}$$
$$D_3 = r_1(r_4 - r_2)e^{r_2+r_4} + r_2(r_1 - r_4)e^{r_1+r_4} + r_4(r_2 - r_1)e^{r_2+r_1}$$
$$D_3 = r_1(r_2 - r_3)e^{r_2+r_3} + r_2(r_3 - r_4)e^{r_3+r_4} + r_3(r_4 - r_2)e^{r_2+r_4}$$
$$\alpha^2 = \left(\pi^2 + \frac{\lambda}{k'}\right)$$

Main flow solution:

We consider the equations of the main flow component $u_1(y,z)$ and temperature field $T_1(y,z)$, in the following form:

$$u_1(y,z) = u_*(y)\cos\pi z \tag{26}$$

$$T_1(y,z) = T_*(y)\cos\pi z \tag{27}$$

Substituting these equations in equations (15) and (18) respectively. We obtain the ordinary differential equations in the following form:

$$u_*'' - \lambda u_*' - (\pi^2 + M^2)u_* = \lambda v_* u_0' \tag{28}$$

$$T_*'' - \lambda p_r T_*' - \pi^2 T_* = \lambda p_r v_* T_0' \tag{29}$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} u_* = 0,\ T_* = 0,\ \ for \ \ y = 0. \\ u_* = 0,\ T_* = 0,\ \ for \ \ y = 1. \end{array}\right\} \tag{30}$$

Using the boundary condition (30) and equation (26) and (27) in the equations (28) and (29), we get

$$u_1(y,z) = \left[\sum_{i=1}^{2} K_i e^{n_i y} + \frac{\lambda}{D(e^{m_1} - e^{m_2})}\left\{\sum_{i=1}^{2} \frac{m_1 D_i e^{(m_1+r_i)y}}{r_i(3m_1 - \lambda)} + \sum_{i=3}^{4} \frac{D_i e^{(m_1+r_i)y}}{r_i} - \sum_{i=1}^{2} \frac{D_i e^{(m_2+r_i)y}}{r_i} - \sum_{i=3}^{4} \frac{m_2 D_i e^{(m_2+r_i)y}}{r_i(3m_2 - \lambda)}\right\}\right]\cos\pi z \tag{31}$$

$$T_1(y,z)=\left[\sum_{i=1}^{2}N_i e^{s_i y}+\frac{\lambda^2 p_r^2}{D(e^{\lambda p_r}-1)}\left\{\sum_{i=1}^{2}\frac{D_i e^{(\lambda p_r+r_i)y}}{r_i(m_1+\lambda p_r)}+\sum_{i=3}^{4}\frac{D_i e^{(\lambda p_r+r_i)y}}{r_i(m_2+\lambda p_r)}\right\}\right]\cos\pi z \tag{32}$$

$$K_1=A\left[\sum_{i=1}^{2}\frac{D_i m_1\left(e^{n_2}-e^{(m_1+r_i)}\right)}{r_i(3m_1-\lambda)}+\sum_{i=3}^{4}\frac{D_i\left(e^{n_2}-e^{(m_1+r_i)}\right)}{r_i}-\sum_{i=1}^{2}\frac{D_i\left(e^{n_2}-e^{(m_2+r_i)}\right)}{r_i}-\sum_{i=3}^{4}\frac{D_i m_2\left(e^{n_2}-e^{(m_2+r_i)}\right)}{r_i(3m_2-\lambda)}\right]$$

$$K_2=-A\left[\sum_{i=1}^{2}\frac{D_i m_1\left(e^{n_1}-e^{(m_1+r_i)}\right)}{r_i(3m_1-\lambda)}+\sum_{i=3}^{4}\frac{D_i\left(e^{n_1}-e^{(m_1+r_i)}\right)}{r_i}-\sum_{i=1}^{2}\frac{D_i\left(e^{n_1}-e^{(m_2+r_i)}\right)}{r_i}-\sum_{i=3}^{4}\frac{D_i m_2\left(e^{n_1}-e^{(m_2+r_i)}\right)}{r_i(3m_2-\lambda)}\right]$$

$$N_1=B\left[\sum_{i=1}^{2}\frac{D_i m_1\left(e^{s_2}-e^{(\lambda p_r-r_i)}\right)}{r_i(m_1+\lambda p_r)}+\sum_{i=3}^{4}\frac{D_i\left(e^{s_2}-e^{(\lambda p_r-r_i)}\right)}{r_i(m_2+\lambda p_r)}\right]$$

$$N_1=-B\left[\sum_{i=1}^{2}\frac{D_i m_1\left(e^{s_1}-e^{(\lambda p_r-r_i)}\right)}{r_i(m_1+\lambda p_r)}+\sum_{i=3}^{4}\frac{D_i\left(e^{s_1}-e^{(\lambda p_r-r_i)}\right)}{r_i(m_2+\lambda p_r)}\right]$$

Where

$$A=\frac{\lambda}{D(e^{m_1}-e^{m_2})(e^{n_1}-e^{n_2})},\qquad B=\frac{\lambda^2 p_r^2}{D(e^{\lambda p_r^2}-1)(e^{s_1}-e^{s_2})},$$

$$n_1=\frac{1}{2}\left[\lambda+\sqrt{\lambda^2+4(\pi^2+M^2)}\right],\qquad n_1=\frac{1}{2}\left[\lambda-\sqrt{\lambda^2+4(\pi^2+M^2)}\right],$$

$$s_1=\frac{1}{2}\left[\lambda p_r+\sqrt{\lambda^2 p_r^2+4\pi^2}\right],\qquad s_2=\frac{1}{2}\left[\lambda p_r-\sqrt{\lambda^2 p_r^2+4\pi^2}\right]$$

Results and discussion:

We may now obtain the expression for the skin-friction components τ_x *and* τ_z is the main flow and transverse direction respectively, as

$$\tau_x = \frac{\tau_x^{/} a}{\mu U} = \left(\frac{du_0}{dy}\right)_{y=0} + \varepsilon\left(\frac{du_*}{dy}\right)_{y=0} \cos\pi z$$

$$= \frac{m_1 - m_2}{e^{m_1} - e^{m_2}} + \varepsilon\left[\sum_{i=1}^{2} K_i n_i + \frac{\lambda}{D(e^{m_1} - e^{m_2})}\left\{\sum_{i=1}^{2} \frac{m_1 D_i e^{(m_1+r_i)y}}{r_i(3m_1 - \lambda)} + \sum_{i=3}^{4} \frac{D_i e^{(m_1+r_i)y}}{r_i} - \sum_{i=1}^{2} \frac{D_i e^{(m_2+r_i)y}}{r_i} - \sum_{i=3}^{4} \frac{m_2 D_i e^{(m_2+r_i)y}}{r_i(3m_2 - \lambda)}\right\}\right] \quad (33)$$

$$\tau_z = \frac{\tau_z^{/} a}{\mu V} = \varepsilon\left(\frac{dw_1}{dy}\right)_{y=0} = -\frac{\varepsilon}{\pi D}\left(\sum_{i=1}^{4} D_i r_i^2\right)\sin \pi z \quad (34)$$

We may calculate the heat transfer coefficient in terms of the Nusselt number

$$Nu = \frac{-q' a}{k(T_1 - T_0)} = \left(\frac{dT_0}{dy}\right)_{y=0} + \varepsilon\left(\frac{dT_*}{dy}\right)_{y=0} \cos\pi z$$

$$= \frac{\lambda p_r}{e^{p_r} - 1} + \varepsilon\left[\sum_{i=1}^{2} N_i s_i + \frac{\lambda^2 p_r^2}{D(e^{\lambda p_r} - 1)}\left\{\sum_{i=1}^{2} \frac{D_i(\lambda p_r + r_i)}{r_i(m_1 + \lambda p_r)} + \sum_{i=3}^{4} \frac{D_i(\lambda p_r + r_i)}{r_i(m_2 + \lambda p_r)}\right\}\right]\cos\pi z$$

(35)

The From figure-1 it is clear that the main flow velocity decreases with increases Hartmann number M, and injection parameter λ. Cross flow velocity component *w* due to the transfer sinusoidal injection velocity distribution applied through out the porous plate at rest. The cross flow velocity profile is shown through the figure-2. Here it is observed from this figure that while increasing the Hartmann number (M) or the injection parameter (λ), the velocity component *w* first decreasing up to the middle of channel and increasing thereafter.

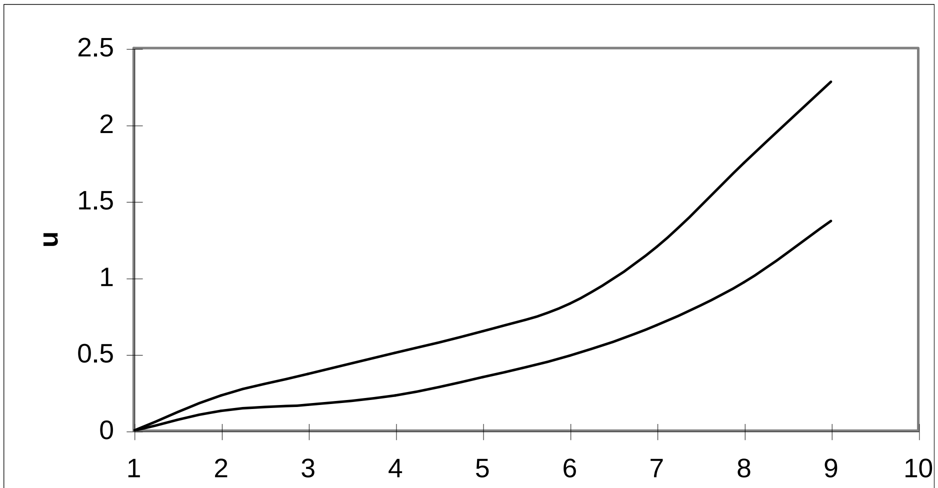

fig-1. Main flow velocity profile for z=0

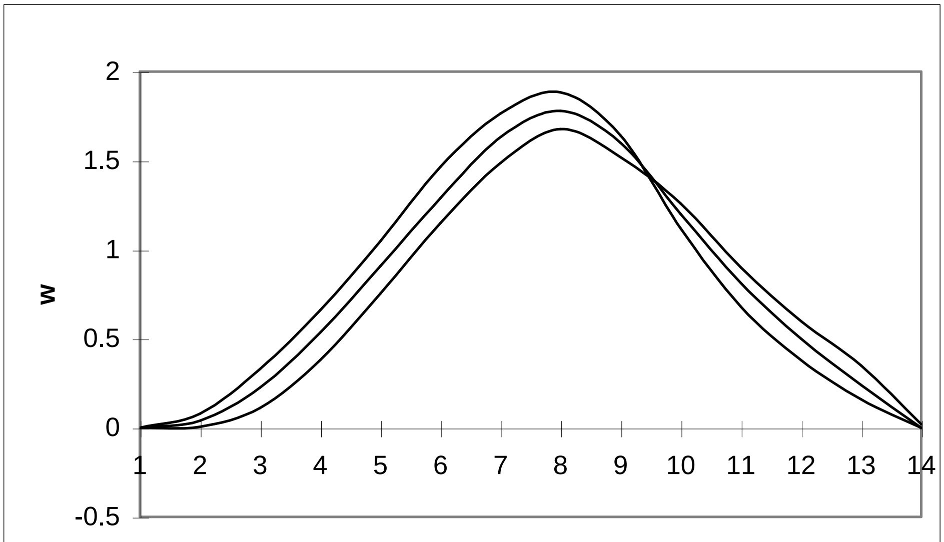

fig-2. cross flow velocity profile for z=0

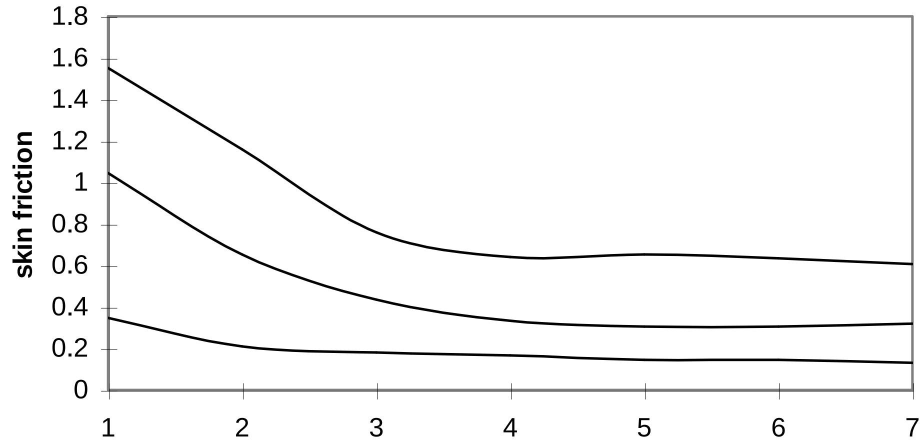

fig-3 main flow and trasvers components of skin friction for z=0 and z=.5

The skin-friction components τ_x *and* τ_z in the main flow and transverse direction, respectively, are presented through the figure-3. This figure shows that τ_x *and* τ_z decrease with increasing λ. It is also noticed that with increasing Hartman number (M), the skin-friction component τ_x decrease, however, τ_z increasing.

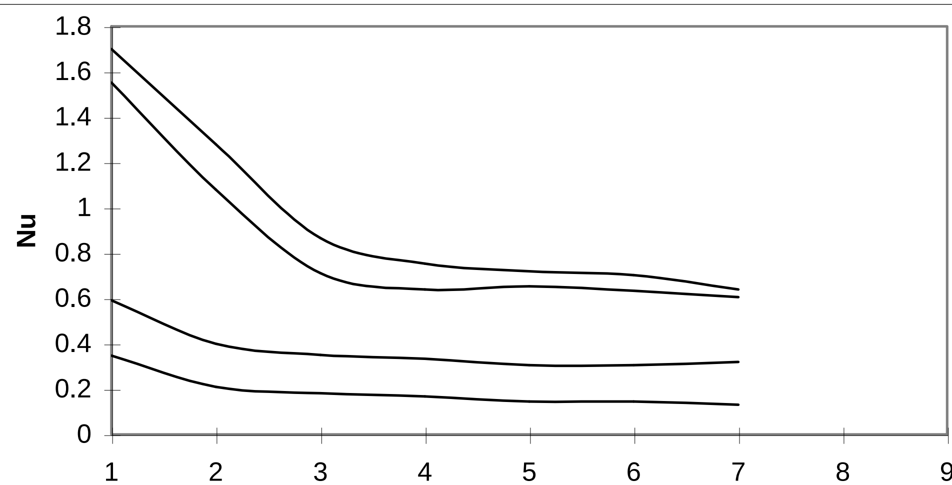

fig-4 Nusselt number Nu for z=0 and z=.5

The rate of heat transfer coefficient at stationary porous plate in term of the Nusselt Number is shown through the figure-4. The value the prandtl number P_r is chosen as 0.7 and 7 approximately which represent air and water respectively at 20^0C. The Nusselt number is also observed to be decreasing with the injection/section parameter λ.

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