MHD free convection couette flow of a viscous incompressible fluid through a porous medium

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Abstract:

This paper Analyzed the effect of MHD free convection couette flow through of a viscous incompressible fluid past an infinite vertical plate through porous with account viscous dissipative heat, of an uniform transverse magnetic field. The velocity distribution, the temperature distribution, coefficient of skin friction and rate of heat transfer has been investigated. The problem is governed by a coupled non-linear system of partial differential equation. In this case exact solution are not possible, by the explicit finite difference method.

Key Words: Magnetic parameter (M), porosity parameter (k) and Time (t) velocity and skin friction are studied with the help of graph.

Introduction:

The problems of free convective and heat transfer flows through a porous medium under the influence of a magnetic field have been attracted the attention of a number researchers because of their possible applications in many branches of science and technology, such as applications in transportation cooling of reentry vehicles and rocket boosters, film vaporization in combustion chambers. Flow through porous medium have numerous and geophysical applications. In view of these applications, many researchers have studies,

B.K. Dutta et al. [1985] discussed Temperature field in flow over a stretching surface with uniform heat flux. Gupta et al. [1977] studied the Heat and mass transfer on stretching sheet with suction or blowing. Raptis et al. [1983] studied the flow of a viscous fluid through a porous medium by a vertical surface. Perdikis [1983] studied the flow of a viscous fluid through a porous medium by a vertical surface. Singh et al. [1985] study the Unsteady free convection flow through a porous medium. Raptis [1983] study the Unsteady flow through porous medium bounded by an infinite porous plate subject to a constant suction and variable temperature. Yong [2000] study the Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Ahamad [2003] discussed the MHD effects on free convection and mass transfer flow through porous media between vertical wavy wall and a parallel flat wall. Kumar [2004] discussed the Hall current effect on MHD free convection flow through porous media past a semi-infinite vertical plate with mass transfer. Muhammad [2005] discussed the Effects of Hall current and heat transfer on flow due to a pull of eccentric rotating.

Mathematical analysis:

The effect of MHD free convection couette flow through of a viscous incompressible fluid past an infinite vertical plate through porous with account viscous dissipative heat, of an uniform transverse magnetic field. The problem is governed by a coupled non-linear system of partial differential equation. So we employ explicit finite difference method for its solution.

$$\frac{\partial u^*}{\partial t^*} = \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{1}{\rho}\frac{\partial p}{\partial y^*} + g\beta(T^* - T_\infty^*) - \frac{\sigma \mu_e^* H_0^*}{\rho} u^* - \frac{\upsilon}{k} u^*$$

…(I)

And

$$\rho C_P \frac{\partial T^*}{\partial t^*} = k \frac{\partial^2 T^*}{\partial y^{*2}} + \mu \left(\frac{\partial u^*}{\partial t^*} \right)^2 \qquad \text{…(II)}$$

Where u^* is the velocity of the fluid, k is the thermal conductivity of fluid, ρ is the density of fluid, υ is the kinematics viscosity, β is the coefficient of volume of expansion, T_w^* is the temperature of lower plate, T_∞^* is the temperature of fluid far away from the plate. μ Is the viscosity of fluid, σ is the electrical conductivity of the fluid, μ_e is the magnetic permeability, C_P is the specific heat of constant pressure, k^* is the permeability of porous medium.

The initial boundary conditions are

$$\left.\begin{aligned} t^* \le 0,\ u^* = 0,\ \ T^* = T_\infty^* \quad for\,all\, y^* \\ t^* > 0,\ u^* = 0,\ \ T^* = T_w^* \quad at\, y^* = 0, \\ u^* \to 0,\ \ T^* = T_\infty^* \quad at\, y^* \to 0, \end{aligned}\right\} \qquad \text{…(III)}$$

The non-dimensional quantities are

$$t = \frac{t^*}{T_R}, \quad y = \frac{y^*}{L}, \quad u = \frac{u^*}{U_0} \; and \quad \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*},$$

$$\Delta T = T_w^* - T_\infty^*,$$

$$U_0 = (\upsilon g \beta \Delta T)^{\frac{1}{3}} \quad is\, the\, \mathrm{Re}\; ference velocity,$$

$$L = \left(\frac{g\beta\Delta T}{\upsilon^2}\right)^{-1/3} \quad is\, the\, \mathrm{Re}\; ference length,$$

$$T_R = \frac{(g\beta\Delta T)^{-2/3}}{\upsilon^{-1/3}} \quad is\, the\, \mathrm{Re}\; ference time, \qquad \ldots(IV)$$

$$P_r = \frac{\mu C_P}{K} \quad is\, the\; prandtl\; numbers,$$

$$E = \frac{U_0^2}{C_P \Delta T} \quad is\, the\; ec\,\mathrm{ker}\,t\; number,$$

$$M = \frac{\sigma \mu_e^2 H_0^2 T_R}{\rho} \quad is\, the\, magnetic\; parameter,$$

$$K = \frac{K^*}{\upsilon T_R} \quad is\, the\; porosity\; parameter,$$

Using the equation (IV), the equations (I) and (II) reduced in the following form

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \frac{T_R}{\mu}\frac{\partial p}{\partial y} + \theta - \left(M + \frac{1}{K}\right)u \qquad \ldots(V)$$

And

$$P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + EP_r \left(\frac{\partial u}{\partial y}\right)^2 \qquad \ldots(VI)$$

The non-dimensional initial boundary conditions is

$$\left.\begin{array}{l} t \le 0, \quad u = 0, \quad \theta = 0, \quad for\, all\; y \\ t < 0, \quad u = 0, \quad \theta = 1, \quad at\; y = 0 \end{array}\right\} \qquad \ldots(VII)$$

Numerical solution:

Now define a new dimensional variable

$$\eta = \frac{y}{1+y} \quad \Rightarrow y = \frac{\eta}{1-\eta} \qquad \text{...(VIII)}$$

Using the equation (VIII), the equation (V) and (VI) reduced in the form

$$\frac{\partial u}{\partial t} = (1-\eta)^4 \frac{\partial^2 u}{\partial \eta^2} - 4(1-\eta)^3 \frac{\partial u}{\partial \eta} - (1-\eta)^2 \frac{T_R}{\mu} \frac{\partial p}{\partial \eta} + \theta - Mu - \frac{1}{k} u \qquad \text{...(IX)}$$

And

$$P_r \frac{\partial \theta}{\partial t} = (1-\eta)^4 \frac{\partial^2 \theta}{\partial \eta^2} - 4(1-\eta)^3 \frac{\partial u}{\partial \eta} + P_r E (1-\eta)^4 \left(\frac{\partial u}{\partial \eta}\right)^2$$

...(X)

And the initial boundary conditions are

$$\left.\begin{array}{l} t \le 0, \quad u = 0, \quad \theta = 0, \quad for\,all\,\eta \\ t > 0, \quad u = 0, \quad \theta = 1, \quad at\,\eta = 0 \\ and \quad u = 0, \quad \theta = 0, \quad at\,\eta = 1 \end{array}\right\} \qquad \text{...(XI)}$$

Equations (IX) and (X) are coupled non-linear partial differential equations and are to be solved by using the initial boundary condition (XI). However exact or approximate solution is not possible for the set of equations and solves these equations by finite difference method.

Using the finite difference method the equations (IX) and (X) becomes in the form

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t} = \theta_{i,j} + (1-\eta_{i,j})^4 \left\{ \frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{(\Delta\eta)^2} \right\} - 4(1-\eta_{i,j})^3 \left(\frac{u_{i+1,j}-u_{i,j}}{(\Delta\eta)} \right) - \left(M + \frac{1}{k} \right) u_{i,j} - (1-\eta_{i,j})^2 \frac{T_R}{\mu} \left(\frac{p_{i+1,j}-p_{i,j}}{(\Delta\eta)} \right)$$

…(XII)

And

$$p_r \left(\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t} \right) = (1-\eta_{i,j})^4 \left\{ \frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}}{(\Delta\eta)^2} \right\} - 4(1-\eta_{i,j})^3 \left(\frac{\theta_{i+1,j}-\theta_{i,j}}{(\Delta\eta)} \right) + (1-\eta_{i,j})^4 p_r E \left(\frac{u_{i+1,j}-u_{i,j}}{(\Delta\eta)} \right)^2$$

…(XIII)

Here the index *i* represent to the moment of η. *j* represent to the moment of time *t* and the square mesh $\Delta\eta = .1$ and $\Delta t = .00125$

The initial condition (XI) we have the following form

$$\left. \begin{array}{ll} u(0,0) = 0, & \theta(0,0) = 1, \\ u(i,0) = 0, & \theta(i,0) = 0,\ for\,all\,i\,except \end{array} \right\}$$

…(XIV)

And the boundary condition (XI) are expressed in the finite difference form as follows

$$\left. \begin{array}{ll} \theta(0,j) = 0, & \theta(0,j) = 1,\ forall\ j \\ u(1,j) = 0, & \theta(1,j) = 0,\ forall\ j \end{array} \right\}$$

…(XV)

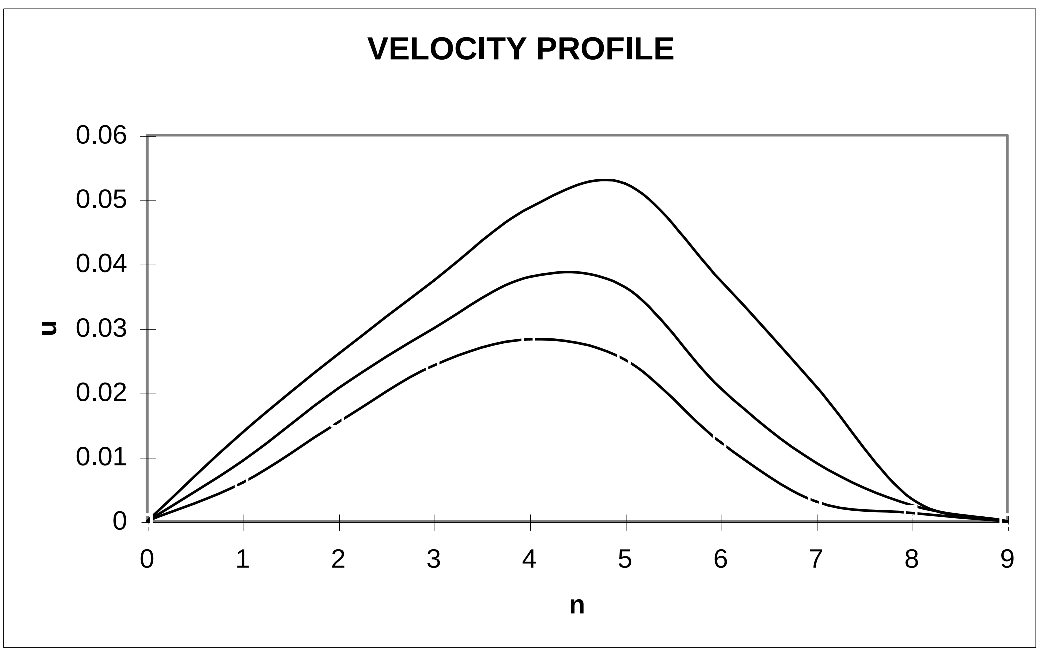
We now calculate the skin friction from the velocity field. It is given in the non-dimensional form as

$$\tau = -\left(\frac{du}{d\eta}\right), \quad where \quad \tau = \frac{\tau^1}{\rho U_0^2} \qquad \ldots(XVI)$$

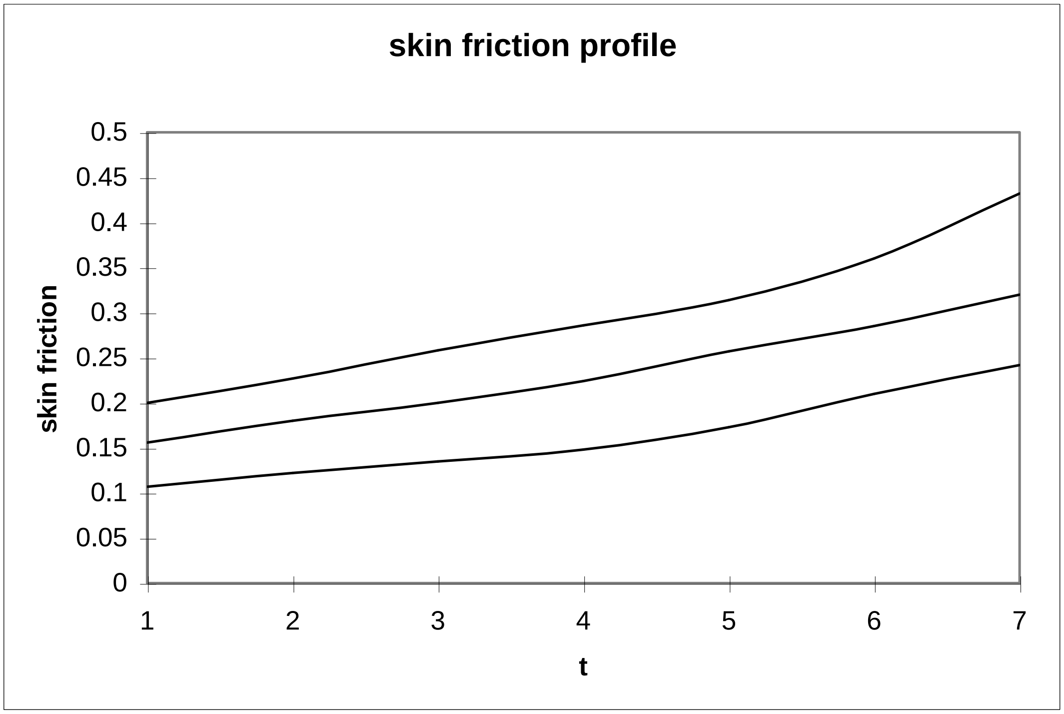
Calculate the skin friction by Newton's interpolation formula for four points.

Result and Discussion:

The velocity profile represent in fig-1 having at the value at p_r =. 7, E = .25 and different value of Magnetic parameter (M), porosity parameter (K) and time (t). From fig-1, the velocity of fluid increases first near the plate and then the trend gets reversed as η increases. It is also noticed that velocity increases with the increase in K and t, but it decreases with increase in M.



(Fig.- 1)



(Fig.- 2)

The skin friction profile represent in fig-2 for the same values as taken in case of velocity. It is noticed that skin friction increases with the increases the time. It is also observed that skin friction increases with the increase in K, but it decreases with the increase in M.

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