Modified Dugdale Approach to Cohesive Quadratic Load Distribution Arresting Crack Opening

Ravinder Kumar[1]* Dr. Pardeep Goel[2]

[1] Research Scholar, Department of Mathematics, CMJ University, Meghalaya

[2] Research Supervisor, Department of Mathematics, CMJ University, Meghalaya

Abstract – The Dugdale model for a single straight slit in an infinite plane was applied to the case of two straight cracks suggested by Theocaris in an infinite plane (55). It is further adjusted by him using a stepwise approximation to obtain the solution of two straight collinear cracks where the formed plastic zones are closed by variable load spread across their rims.

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INTRODUCTION

An continuous, homogeneous, isotropic, elastic-perfectly continuous plastic disk, bounded by the xoy axis, is split between two straight hairline cracks L1 and L2. Such fair and collinear cracks occur on the ox-axis, and are positioned symmetrically along the oy-axis. The crack L1 lies from (— b, 0) to (— a, 0) and L2 lies from (a, 0) to (b, 0) unidirectional stress, is added perpendicularly to the rims of the cracks L1 and L2 at infinite boundary. While the faces of the cracks expand creating narrow patches of plastic instead of the tips of the cracks. In the four tips – b, -a, a and b, the plastic zones formed are denoted by and, respectively. The plastic zone occupies the [b, d] region; the plastic zone occupies the [c, a] interval; the plastic zone occupies [-a, -c] and the [-d, -b] interval.

-- rim of plastic zones (I = 1, 2, 3, 4) is subject to the distribution of compressive tension, and Pxy = 0. Every position on the surface is denoted by t, and the plate's yield point tension. This prevents the cracks from further widening.

The complete configuration as shown in Figure 4.1

Conclusion

At the tips of the cracks obtained for two part problems, the result of the above mentioned problem is obtained using the theory of super positions of stress strength variables. These problems are extracted correctly from the issue mentioned in section 4.1. Both problems are called Problems I and II. This are listed below and clarified below.

MATERIAL AND METHOD:

Problem I

An infinite, homogeneous, isotropic, elastic-perfectly plastic plate occupies xoy plane. The plate is cut along two straight cracks R_1 : $\Gamma_4 UL_1 U\Gamma_3$ and R_2 : $\Gamma_2 UL_2 U\Gamma_1$. The cuts R_1 and R_2 occupy the interval [-d, -c] and [c, d] respectively.

The boundary conditions of problem are

(i) No stresses are acting on the rims of the cracks R_1 and R_2.

(ii) The stress prescribed at infinite boundary is

$$P_{yy} = \sigma_\infty,\ P_{xy} = 0,\ P_{xx} = 0 .$$

(iii) The displacements are single valued on and around the cracks.

This problem is the same as stated in section 3.2.1 and depicted in figure 3.2 of chapter 3. We recapitulate the solution from 3.2.1 make this chapter self-sufficient.

The complex potential $\phi^I(z)$, of interest may directly be written as

$$\phi^I(z) = \frac{\sigma_\infty}{2}\left[\frac{1}{iX(z)}\left\{z^2 - \frac{d^2 E(k')}{K(k')}\right\} - \frac{1}{2}\right], \qquad (4.2.1\text{-}1)$$

Where

$$X(z) = \{(z^2 - c^2)(d^2 - z^2)\}^{1/2} \qquad (4.2.1\text{-}2)$$

And complementary modulus

$$k' = (1 — c^2 / d^2)^{1/2}$$

The opening mode stress intensity factor at interior tip z = c is

$$(K_I^I)_c = \frac{\sigma_\infty d^2}{i}\sqrt{\frac{\pi}{c(d^2 — c^2)}}[\frac{c^2}{d^2} — \frac{E(k')}{K(k')}] \qquad (4.2.1\text{-}3)$$

And at the exterior tip z = d as

$$(K_I^{II})_d = \sigma_\infty\, d\sqrt{\frac{\pi d}{(d^2 — c^2)}}[1 - \frac{E(k')}{K(k')}] \qquad (4.2.1\text{-}4)$$

Problem II

xoy plane is surrounded by a homogeneous, isotropic, and elastic-perfectly fluid limitless line. On the limitless plate ox-axis sit two hairline cracks L1 and L2. The cracks L1 and L2 hold the [-b, -a] and [a, b] positions, respectively. Uniform friction Dynamically applied unidirectional (parallel to oy axis), which allows the cracks to expand rims. These are denoted by and and lie ahead of the corresponding tips b, a, -a and -b. The time in which the plastic region occupies the actual axis is [b, d]; by is [c, a]; by is [-a, -c] and by is [-d, -b].

Every rim of plastic zones I = 1, 2, 3, 4) is subjected to quadratic ally varying stress distribution and denotes the yield point tension, and t is a point on the rim of any of the plastic zones.

Issue Configuration II is seen in Figure 4.2.

The mathematical model of the above problem is obtained assuming that UL1U (= R1), UL2U (= R2) and lying on the ox axis of the infinite plate essentially form two cracks R1 and R2. The limit conditions of the issue can be restated as

(a) The cracks R_1 and R_2 are loaded along the rims of r_i (i = 1, 2, 3, 4)k by stress distribution

$$P_{yy} = t^2\sigma_{ye,}\ P_{xy} = 0,\ P_{xx} = 0$$

(b) The rims of L_1 and L_2 are stress free.

(c) No stresses are acting at infinite of the plate.

Using boundary conditions (a), (b) and equation (2.5-5) of chapter 2 following two Hilbert problems are obtained.

$$[\phi^{II}(t) + \Omega^{II}(t)]^+ + [\phi^{II}(t) + \Omega^{II}(t)]^- = t^2\sigma_{ye}, \qquad (4.2.2\text{-}1)$$

$$[\phi^{II}(t) — \Omega^{II}(t)]^+ + [\phi^{II}(t) — \Omega^{II}(t)]^- = 0, \qquad (4.2.2\text{-}2)$$

Where

$$\Gamma = \overset{4}{\underset{i=1}{U}}\Gamma_1$$

Superscript II denotes that the potentials refer to problem II.

The solution of equations (4.2.2-1) and (4.2.2.-2) may be written using equation (2.5-16) and (2.5-17) as

$$\phi^{II}(z) = \phi_0(z) + \frac{1}{X(z)}[C_0 z^2 + C_1 z + C_2], = \Omega^{II}(z), \qquad (4.2.2\text{-}3)$$

Where

$$\phi_0(z) = \frac{\sigma_{ye}}{2\pi i X(z)}\int_r \frac{t^2 X(t)}{(t — z)}dt. \qquad (4.2.2\text{-}4)$$

And X(z) is same as defined by (4.2.1-2). The constants C_i(I = 0, 1,2) are determined using condition (c) stated above and the condition of single valuendness of displacement around cracks. This gives

$$C_0 = 0 \qquad (4.2.2\text{-}5)$$

$$C_1 = \frac{\sigma_{ye}}{\pi i}[—\frac{a^2 + 2c^2 + 2d^2}{3a}X(a) + \frac{b}{3}X(b) + \frac{c^2 + d^2}{a}X(a) + \frac{c^2 + d^2}{2a}X(a)] \qquad (4.2.2\text{-}6)$$

And

$$C_2 = 0 \qquad (4.2.2\text{-}7)$$

Evaluating integral of the equation (4.2.2-4) complex potential $\phi_0(z)$ may be written as

$$\phi_0(z) = —\frac{z\sigma_{ye}}{\pi i X(z)}[\frac{d}{3}\{2(c^2 + d^2)H_o — c^2 G_0\} — \frac{a^2 + 2c^2 + 2d^2}{3a}X(a) + \frac{b}{3}X(b)$$
$$— (c^2 + d^2 — z^2)\{dH_0 — \frac{X(a)}{a}\} — \frac{X^2(z)}{d}G_1 + \frac{z^2 X^2(z)}{d}P_0], \qquad (4.2.2\text{-}8)$$

Where

$$H_0 = E(u_1) + E(u_2), \qquad (4.2.2\text{-}9)$$

$$G_0 = F(u_1) + F(u_2), \qquad (4.2.2\text{-}10)$$

$$G_1 = u_1 + u_2 \qquad (4.2.2\text{-}11)$$

$$P_0 = [\frac{1}{z^2}\{u_1 + \frac{c^2}{(z^2 — c^2)}II(u_1, \alpha_1^2)\} + \frac{1}{(d^2 — z^2)}II(u_2, \alpha_2^2)], \qquad (4.2.2\text{-}12)$$

And $F(u_1)$, $F(u_2)$, $E(u_1)$, $E(u_2)$, II (u_1, α_1^2)] and II (u_2, α_2^2) are normal elliptic integrals of the first, second and kind respectively. Also

$$u_1 = \sin^{-1}\frac{d}{a}\sqrt{\frac{(a^2 - c^2)}{(d^2 - c^2)}},\ u_2 = \sin^{-1}\sqrt{\frac{(d^2 - b^2)}{(d^2 - c^2)}},$$

X(z) is same as defined by equation (4.2.1-2). These yield

The opening mode stress intensity factor (SIF), K_I^{II}, at the interior tip z = c is obtained substituting value of $\phi^{II}(z)$, from equation (4.2.2-3 to 12), for $\phi(z)$ in equation (2.6-1) one obtains.

$$(K_I^{II})_c = -\frac{2\sigma_{ye}}{i}\sqrt{\frac{c}{\pi(d^2 - c^2)}}[\frac{d}{3}\{2(c^2+d^2)H_0 - c^2G_0\} - d^3H_0 - \frac{(3c^2+d^2)}{2a}X(a)], \quad (4.2.2\text{-}13)$$

And SIF exterior tip z = d may be written as

$$(K_I^{II})_d = -2\sigma_{ye}\sqrt{\frac{d}{\pi(d^2 - c^2)}}[\frac{d}{3}\{2(c^2+d^2)H_0 - c^2G_0\} - c^2dH_0 - \frac{(3d^2+c^2)}{2a}X(a)], \quad (4.2.2\text{-}14)$$

CONCLUSION

xoy plane is surrounded by a homogeneous, isotropic, and elastic-perfectly fluid limitless line. On the limitless plate ox-axis sit two hairline cracks L1 and L2. The cracks L1 and L2 hold the [-b, -a] and [a, b] positions, respectively. Uniform friction Dynamically applied unidirectional (parallel to oy axis), which allows the cracks to expand rims. These are denoted by and and lie ahead of the corresponding tips b, a, -a and -b. The time in which the plastic region occupies the actual axis is [b, d]; by is [c, a]; by is [-a, -c] and by is [-d, -b].

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$$P_{yy} = t^2\sigma_{ye},\ P_{xy} = 0,\ P_{xx} = 0$$

The rims of L_1 and L_2 are stress free.

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Corresponding Author

Ravinder Kumar*

Research Scholar, Department of Mathematics, CMJ University, Meghalaya